An overview of various control strategies for robotic manipulators

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Abstract. This paper outlines linear and nonlinear systems, presenting some key features of each while pointing out the major problems brought by the nonlinearities into control applications. Linear systems exhibit proportional and additive relationships and are comparatively easier to model and control. On the contrary, the nonlinear systems possess complex and unpredictable patterns, thus often forcing their simplification. Linearization, performed via Jacobian-based approximations and feedback linearization, is very important in allowing the application of linear control strategies to be employed for nonlinear systems. The paper presents proportional-integral-derivative (PID) control and Computed torque control (CTC) as the major control methods for manipulation, showing the corresponding design principles and also discusses other control techniques in brief.

Keywords: Linear Systems, Non-linear systems, Control techniques

1 Introduction

Control systems can be understood with a simple analogy. Imagine a person trying to catch a ball; in this scenario the person's eyes are sensors which sense crucial information in this case the position and trajectory of the ball; the brain is the controller which receives the signals from the eyes and uses this information to command the actuator, which in this case is the hands of the person to catch the ball. So, control systems can be defined as a device which commands some plant to act in a way so as to get the desired result.

When designing a control system, it has been observed that design of control systems for linear systems is relatively easy as compared to for non-linear systems. This is because non-linear systems are mathematically complex and computationally expensive to analyse [1]. But a major concern is that majority of the systems needed to be controlled are in fact non-linear systems and thus, they are first linearised before any control strategy is applied on them.

A comprehensive study of the major differences between a linear and nonlinear control system, covering the major features that distinguish the two systems from each other has been provided here.

The reasons for the prevalence of nonlinear behavior in real-world systems are further investigated, namely physical constraints, dynamic interactions, and environmental factors that account for the non-linearity in engineering systems [2]. Finally, problems posed by nonlinear dynamics in the way of control design, pointing out that a simplification of such systems would be a necessity for successful implementation of control methods has been discussed [3].

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2 Challenges in control systems

2.1 Occurrence of non-linear systems

Before understanding different ways to linearize non-linear systems it is important to understand why there are non-linear systems. As per Yu et. al. [4], non-linear systems occur due to the presence of non-linear elements within a system or network, where the input and output are not superimposed and uniform. This non-linearity leads to behaviours that are fundamentally different from external excitations, causing small changes in parameters to result in qualitative changes in motion. The failure of the superposition principle in such systems contributes to their complexity, variability, and the rich diversity observed in the material world. Also, according to Stewart et. al. [5] non-linear systems occur because complex behaviour can arise from simple mathematical models, particularly when the dynamics involve interactions among many elements. In non-linear dynamics, the interplay between linear and non-linear components can lead to phenomena such as bifurcations and chaos, where small changes in parameters can result in significant changes in behaviour. The emergence of low-degree-of-freedom systems on a macroscopic scale from large numbers of microscopic elements also contributes to the occurrence of non-linear systems.

2.2 Linearisation of non-linear systems

Linear systems are characterized by mainly 2 properties [6]. Namely, superposition and homogeneity. Superposition is defined as in (1), and homogeneity is defined as in (2) where f(x) is defined as a function of x.

$$\boldsymbol{f}(\boldsymbol{x_1}+\boldsymbol{x_2})=\boldsymbol{f}(\boldsymbol{x_1})+\boldsymbol{f}(\boldsymbol{x_2}) \tag{1}$$

$$\boldsymbol{f}(\boldsymbol{a}\times\boldsymbol{x_1})=\boldsymbol{a}\times\boldsymbol{f}(\boldsymbol{x_1}) \tag{2}$$

Now that we have a basic understanding of linear systems it is important to understand why is linearisation needed for non-linear systems. According to Perduková et. al. [7] linearization is essential for control systems as it simplifies the analysis and design process of nonlinear systems, enabling the application of established linear control techniques. To understand it in a basic manner it is to be understood that for applying majority of the control techniques to a system it is necessary that the system’s equation can be written in a state-space form which is as shown in (3).

$$\begin{aligned}\dot{\mathbf{x}}&=\mathbf{A}\times\mathbf{x}+\mathbf{B}\times\mathbf{u}\\ \mathbf{y}&=\mathbf{C}\times\mathbf{x}+\mathbf{D}\times\mathbf{u}\end{aligned} \tag{3}$$

In the state-space representation, A represents the system matrix, B signifies the control input matrix, C indicates the output matrix, and D stands for the feedforward matrix. It can be inferred that only linear systems can be represented in state space form. State-space representation is discussed in detail in this paper because it allows the user to handle both Single Input Single Output (SISO) as well as Multi Input Multi Output (MIMO) cases in most of the scenarios.

Moving on, there are various ways of linearising a non-linear system such as Taylor Series Expansion, Jacobian Matrix and Feedback linearisation respectively. According to Nečasová et. al. [8] Taylor series expansion linearizes nonlinear problems by transforming them into autonomous systems of ordinary differential equations, allowing for effective parallelization and comparison with state-of-the-art solvers. To make use of Taylor series expansion (4) can be used as the primary equation.

$$\mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{x_0}) + \frac{\partial \mathbf{f}}{\partial \mathbf{x}}|_{\mathbf{x}=\mathbf{x_0}} (\mathbf{x} - \mathbf{x_0}) \quad (4)$$

According to Skopec et. al. [9], the Jacobian matrix for linearizing nonlinear equations is derived from the predictive Jacobian matrix, which quantifies how changes in inputs affect state predictions. Equations (5) and (6) can be used to explain how Jacobian matrices work where, A,B,C,D are Jacobian matrices.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \qquad \mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{u}) \quad (5)$$

$$\dot{\Delta \mathbf{x}} = \mathbf{A}\Delta \mathbf{x} + \mathbf{B}\Delta \mathbf{u} \qquad \Delta \mathbf{y} = \mathbf{C}\Delta \mathbf{x} + \mathbf{D}\Delta \mathbf{u} \quad (6)$$

Also, one thing which needs to be understood for Taylor Series Expansion and Jacobian Matrix methods of linearisation is that they can only linearise the non-linear system about an operating point and in case the system leaves the bound of this operating point then the equation of the linear system derived from the above 2 methods will not work in controlling the plant as they will need to be recalculated again. This problem can be tackled by making use of the feedback linearisation technique.

Feedback linearization is defined as a linearization technique in which the non-linear terms of the system are cancelled out by feedback control making the system a linear system effectively [10]. It uses nonlinear state or input transformations to cancel the nonlinearities of the system. Key steps to implement feedback linearisation technique include identifying nonlinear terms in the equations and defining a new input that compensates for these terms.

3 Control Strategies

In practical sense control techniques involve the use of sensors to gather actual real time data of the system, pass it forward to the controller which compares the desired response with actual response and passes a control signal such that the system achieves the desired result. The aim of this paper is to discuss the theoretical aspect of control strategies which involves discussion on what are K_p, K_i, K_d values and what is their significance. To gain a basic knowledge of these terms it is important to understand what PID controller is.

The Proportional-Integral-Derivative (PID) controller is widely recognized as one of the most utilized feedback control systems in both industrial and engineering applications. Its main function is to adjust a system's output so that it aligns with a predetermined setpoint by consistently reducing the error, which represents the difference between the setpoint and the observed process variable. The PID controller accomplishes this by integrating three control components: proportional, integral, and derivative. The proportional component produces an output that is directly related to the current error, offering immediate correction. The integral component aggregates past errors, ensuring that any steady-state inaccuracies are resolved. The derivative component forecasts potential future errors by assessing the error's rate of change, contributing a damping effect that enhances overall stability [11]. The interplay of these three components enables the PID controller to strike a balance between responsiveness, stability, and precision across different systems.

For PID control, the control input u(t) is computed as shown in (7).

$$\boldsymbol{u(t) = K_p e(t) + K_i \int_0^t e(t)\,dt + K_d \frac{de(t)}{dt}} \quad (7)$$

where, K_p = Proportional gain, K_i = Integral gain, K_d = Derivative gain

These gains determine the controller's performance and must be tuned for optimal results. Tuning methods such as the Ziegler-Nichols approach or modern software tools are commonly employed to determine appropriate gain values. The Ziegler-Nichols method, for instance, involves increasing the proportional gain until the system oscillates at a constant amplitude, from which critical parameters are used to compute the gains.

Implementing a PID controller involves defining the desired setpoint, measuring the system's output, calculating the error, and applying the PID equation in a real-time loop. The control output is then sent to an actuator to correct the system's behaviour. This simple yet powerful approach makes PID control widely applicable across various domains. For example, in process control, it is used to regulate temperature, pressure, and flow in chemical and manufacturing industries. In motion control, PID controllers manage the speed and position of motors and robotic arms. Other applications include aerospace for flight stabilization, energy systems for load-frequency control, and automation for tasks like conveyor belt regulation [12].

Having understood PID control, it should also be highlighted that it is one of the most basic and easiest control strategies in practice which makes it widely acceptable. However, this does not mean that it is the best approach out there. One possible disadvantage is that it can be only applied to a linear system or a non-linear system only after it has been linearized. This problem can be tackled with Model Based Control.

Model Based Control also known as Computed Torque Control (CTC) is a nonlinear control strategy widely used in robotics and mechanical systems to manage their inherently nonlinear and coupled dynamics. The essence of CTC lies in transforming the nonlinear dynamics into a linear, decoupled system through a carefully designed control law, enabling easier control and analysis of the system behaviour [13]. The governing equation for a robotic manipulator with n-degrees of freedom is expressed as in (8)

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \mathbf{G}(\boldsymbol{\theta}) = \boldsymbol{\tau} \quad (8)$$

Where, M = Inertia matrix, C = Coriolis and Centrifugal forces matrix, G = Gravitational forces matrix, θ = Joint position vector, $\dot{\theta}$ = Joint velocity vector, $\ddot{\theta}$ = Joint acceleration vector, τ = Torque

The control law is defined as in (9).

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}}_{\mathbf{d}} + \mathbf{C}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}}_{\mathbf{d}} + \mathbf{G}(\boldsymbol{\theta}) + \mathbf{u} = \boldsymbol{\tau} \quad (9)$$

Where $\dot{\theta}_d$, $\ddot{\theta}_d$ represent the desired velocity and acceleration respectively u is a proportional-derivative feedback term. Substituting this control law into the dynamics equation of the system which is shown in (9), the CTC becomes (10) which is a decoupled second-order system that can be tuned using the proportional and derivative gain matrices for desired performance [14].

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\mathbf{e}} + \mathbf{K_d}\dot{\mathbf{e}} + \mathbf{K_p}\mathbf{e} = \mathbf{0} \quad (10)$$

CTC is extensively used in robotic manipulators for precise trajectory tracking and control. Other applications include aerospace systems for satellite and drone control, automotive systems for active suspensions, and wearable devices like prosthetics and exoskeletons for motion assistance [15].

In the past decades many other control strategies have also emerged which have found their application in various scenarios. One such example being that of adaptive control which is defined as a control strategy where parameters of the control are changed in response to the changes observed in the environment [16-18]. Majorly three types of adaptive control strategies are applied Full-Order Adaptive Control, Reduced-Order Adaptive Control, and KAMS. [19-20]. Robust control is another field of study which involves accurate and ideal response from system even in the presence of uncertainties and disturbances and find their usage in classical systems all the way to advanced quantum systems [21-22]. However these techniques sometimes tend to give inaccurate responses and are henceforth combined with novel techniques to bridge the gap [23-24].

While robust control techniques maintain system stability in presence of uncertainties and disturbances, Sliding Mode Control (SMC) should also be discussed which involves proper tracking of system along different surfaces in the state space [25-26]. Neural Networks and fuzzy logic control techniques have also gained momentum in the past few years which use artificial intelligence for predicting parameters of control strategies as well as uncertainties and disturbances in the system [27-30]. In the past years, interest has also been observed in ways to find gain values of control strategies through artificial intelligence rather than using other tuning methods [31-33].Also as discussed in Chen et al. [34] Luenberger Observer may also be used for prediction and control of systems.

4 Results and Discussion

In this section the authors have discussed the comparison of PID and CTC on application of two link robotic manipulator whose equations of M,C and G matrices as of (10) have been illustrated in [14]. Figure 1 and Figure 2 shows the response of system when sinusoidal input is given and PID control is applied. Figure 3 shows the response of system under same input conditions and under application of CTC control.

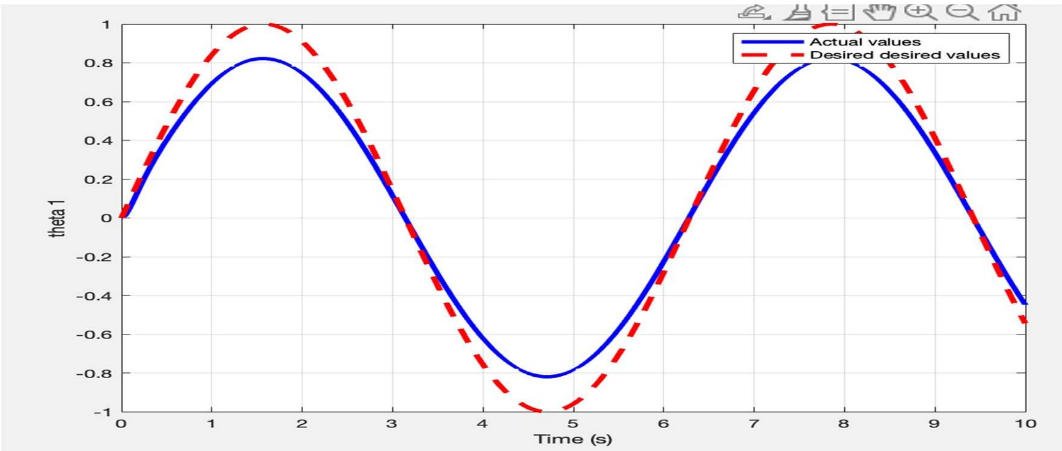

Fig. 1. Link 1 PID response

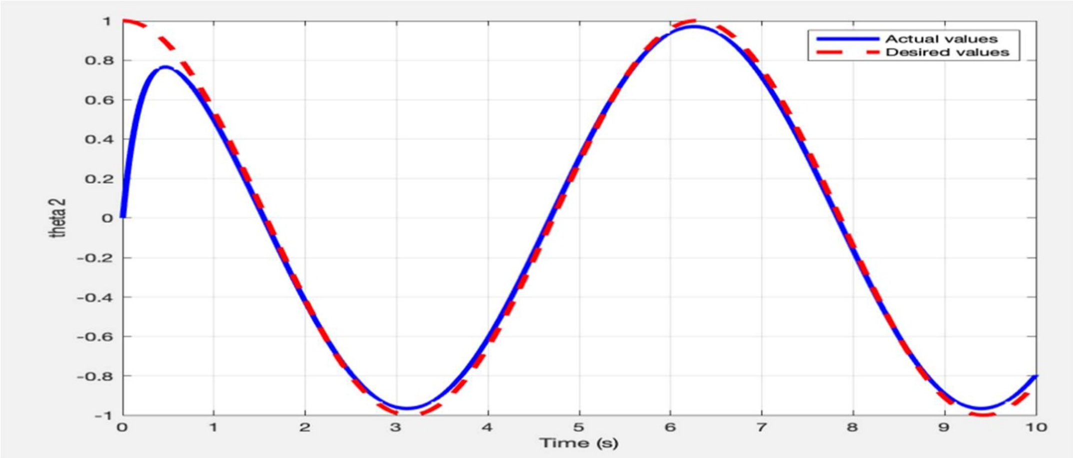

Fig. 2. Link 2 PID response

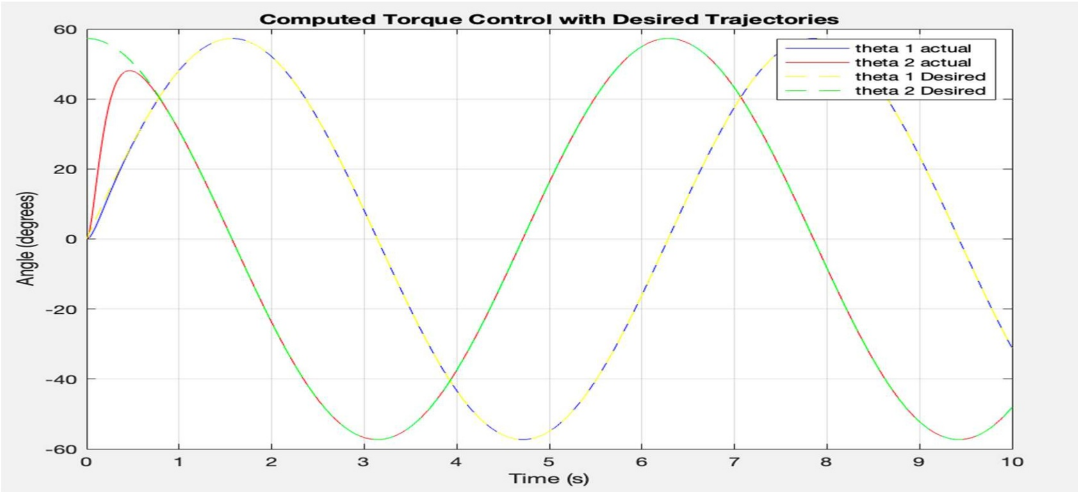

Fig.3. Link 1 and Link 2 CTC response

5 Conclusion

This paper discusses various control techniques and simulation results of PID and CTC techniques have been shown where CTC is observed to outperform PID control. The following 2 control techniques have been illustrated as they are most widely used in the industry.

Conflict of Interest

The authors declare no conflict of interest that could have appeared to influence the work reported in this paper.

References

1. K. Ogata, Modern control engineering (Prentice Hall, India, 2009)
2. A. Isidori, Nonlinear control theory for automation, in Springer Handbook of Automation (Springer, Cham, 2023), pp. 163–187

3. N. S. Nise, Control systems engineering (Wiley, 2020)
4. F. Yu, H. Lin, V.-T. Pham, Editorial: Advances in nonlinear systems and networks. Front. Phys. 11, 1180413 (2023). https://doi.org/10.3389/fphy.2023.1180413
5. T. R. J. Bossomaier, D. G. Green (eds.), Complex systems (Cambridge Univ. Press, 2000). https://doi.org/10.1017/CBO9780511758744.006
6. B. David, Linearization and transformation, pp. 231–274 (2001). https://doi.org/10.1007/978-1-4757-4971-7_6
7. D. Perdukova, P. Fedor, M. Sobek, J. Bačík, A simple linearization method of nonlinear systems based on fuzzy logic. IEEE Access 12, 165441–165457 (2024). https://doi.org/10.1109/ACCESS.2024.3493251
8. G. Nečasová, P. Veigend, V. Šátek, Taylor series method in numerical integration: Linear and nonlinear problems, in IEEE Int. Sci. Conf. Informatics (2022), pp. 239–244. https://doi.org/10.1109/Informatics57926.2022.10083462
9. P. Skopec, T. Vyhlidal, J. Knobloch, Nonlinear model prediction Jacobian approximation using state-independent order reduction, in Int. Conf. Process Control (2023), pp. 42–47. https://doi.org/10.1109/PC58330.2023.10217385
10. T. Phan, H.-N. Duong, V. Y. T. Lu, PID-based control strategies for electro-hydraulic actuation systems. Int. J. Sci. Res. Arch. 13, 3107–3114 (2024). https://doi.org/10.30574/ijsra.2024.13.1.2007
11. K. J. Åström, T. Hägglund, Advanced PID control (ISA, 2006)
12. P. Gupta, S. Kumar, Liquid crystal polymers: Thermo-optical-mechanical coupling and actuations, in 2024 1st Int. Conf. Sustainability and Technological Advancements in Engineering Domain (SUSTAINED) (IEEE, 2024), pp. 831–835
13. M. W. Spong, S. Hutchinson, M. Vidyasagar, Robot modeling and control (Wiley, 2020)
14. J. J. Craig, Introduction to robotics: Mechanics and control, 3rd edn. (Pearson, 2009)
15. J.-J. E. Slotine, W. Li, Applied nonlinear control (Prentice Hall, 1991)
16. D. Landau, R. Lozano, M. M. Saad, A. Karimi, Adaptive control: Algorithms, analysis and applications (2024). https://doi.org/10.48550/arXiv.2406.07073
17. M. Paluszek, S. Thomas, Adaptive control, pp. 127–155 (Apress, 2024). https://doi.org/10.1007/978-1-4842-9846-6_5
18. H. A. Qazi, Adaptive control of a linear first-order discrete-time system. Franklin Open (2023). https://doi.org/10.1016/j.fraope.2023.100044
19. V. Sycheva, A. N. Belikov, Adaptive control system for objects with dynamically varying parameters (2023). https://doi.org/10.1109/PEAMI58441.2023.10299911
20. H. Abubakr, A. Lashab, S. Golestan, A. Abusorrah, M. Rawa, M. Yaqoob, J. C. Vasquez, J. M. Guerrero, Adaptive control as a hierarchical system (2023). https://doi.org/10.23919/EPE23ECCE-Europe58414.2023.10264455
21. P. Rout, A. K. Jha, P. Gupta, B. Singh, S. Choudhury, Failure analysis of composite plate under ballistic impact. Mater. Today Proc. 74, 1008–1011 (2023). https://doi.org/10.1016/j.matpr.2022.11.385
22. H. S. Ruhela, S. Bhardwaj, T. Agrawal, P. Gupta, Explicit dynamics analysis of shin pads using finite element analysis, in Int. Conf. Industrial Problems on Machines and Mechanism (Springer, Singapore,

2022), pp. 683–690E. Lavretsky, K. A. Wise, Robust adaptive control, pp. 469–506 (2024). https://doi.org/10.1007/978-3-031-38314-4_11

23. Z. Ishmatov, Robust control of an electric drive based on inverse dynamics (2023). https://doi.org/10.1109/ACED57798.2023.10143457
24. R. Lán, Sliding mode control and its applications in robot trajectory tracking: Review. Appl. Comput. Eng. 111, 152–158 (2024). https://doi.org/10.54254/2755-2721/111/2024ch0100
25. C. S. T. Dong, Q. T. Nguyen, N. Q. Dũng, H. H. Vo, T. C. Tran, P. Brandštetter, PMSM drive with sliding mode direct torque control, in Springer Proc. (2024), pp. 257–267. https://doi.org/10.1007/978-981-99-8703-0_22
26. F. Sayeh, S. Tamalouzt, D. Ziane, A. Bekhiti, Y. Belkhier, AI-based fuzzy logic control for wind turbine systems. Int. J. Energy Res. (2025). https://doi.org/10.1155/er/5985904
27. P. Picton, Neural and neuro-fuzzy control systems, pp. 185–203 (2024). https://doi.org/10.1201/9781003572886-11
28. F. R. Lima, Advances in fuzzy logic and artificial neural networks. Mathematics 12, 3949 (2024). https://doi.org/10.3390/math12243949
29. S. S. Sefati, A. M. Nor, O. Fratu, S. Halunga, Traffic control in 6G-IoT using RNN and fuzzy logic, in IEEE Conf. (2024), pp. 519–523. https://doi.org/10.1109/ICSCC62041.2024.10690801
30. J. Castro, D. Rosales, C. Calderón, Trajectory control using fuzzy logic and neural networks for an industrial robot arm. IEEE Lat. Am. Trans. 22, 529–538 (2024). https://doi.org/10.1109/TLA.2024.10562261
31. J. V. Devi, R. V. Argiddi, P. Renuka, K. Janagi, B. S. Hari, S. Boopathi, Neural network and fuzzy logic control for autonomous EVs, in IGI Global (2024), pp. 104–127. https://doi.org/10.4018/979-8-3693-1487-6.ch006
32. D. Duc, T. Viet, Adaptive synchronous sliding control for robot manipulator using neural networks. Int. J. Electr. Comput. Eng. (2024). https://doi.org/10.11591/ijece.v14i3.pp2377-2385
33. W. H. Chen, J. Yang, L. Guo, S. Li, Disturbance-observer-based control and related methods—An overview. IEEE Trans. Ind. Electron. 63, 1083–1095 (2015)