A Theoretical Model for Predicting Axial Compressive Strain of FRP-Confined Concrete

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Abstract: The axial compressive strength and strain of structural elements made of reinforced concrete are enhanced by the external confinement provided by fiber-reinforced polymer (FRP) sheets. There is still a need for more research into estimating axial compressive strain even though numerous studies have suggested analytical approaches to predict the axial compressive strength of concrete structural elements. This is a result of earlier strain models' inadequate accuracy. Furthermore, rudimentary modelling techniques and small, noisy databases were used in the development of these models. To suggest a more realistic strain model and compare it with earlier models, a more rigorous methodology is therefore required. The goal of this study is to present a strain model for FRP-confined concrete members by analytical modeling based on a large database containing 570 sample points. When the models were assessed using statistical parameters, it was discovered that the estimations of the freshly proposed models were more accurate than those of the previous models. The estimations' relative study provides significant support for the recommended analytical model's applicability and accuracy in forecasting the axial-strain of CFRP-confined concrete compression members.
Keywords: confined concrete; CFRP; axial strain; analytical model; strain model; database

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1.INTRODUCTION

The primary reason fiber reinforced polymers (FRPs) are being used more often is to improve concrete's axial strength and axial strain. In addition, FRP is frequently used to improve the flexural strength of structural components [1]. Other usages of FRPs is the strengthening, repairing and enhancement of the ductility of reinforced concrete columns in the corrosive and aggressive environments [2]. The FRP composites are durable, lightweight, corrosion-resistant, electromagnetic resistant, chemical resistant, and have high tensile strengths [3-5]. The FRPs are the most suitable materials to be used in the marine structures and the coastal environments due to their high corrosion resistance property [6]. These composites are very efficient in providing the confinement mechanism. The confinement mechanism depends upon their thickness, elastic modulus, the number of wraps, strength of confined material and the angle of orientation wrap to the structural element [7]. The lateral confining impact of structural concrete with the FRPs has become an advanced and the most popular technology to enhance the efficiency and strength [8]. The FRPs are being favored over steel jackets due to the advantages of easy installation, easy handling, slight disturbance of the structural elements, and reduced construction time [9].

The strengthening approach for the FRP-confined concrete structural elements plays a vital role to improve both ductility as well as strength of the concrete elements [10, 11]. The serviceability and the strength of concrete structures are significantly reduced due to the earthquake that needs rehabilitation and retrofitting to improve their strength and ductility [12]. The axial strength of bridge piers is considerably enhanced by the lateral confinement due to FRP wraps [13-15]. The FRP confinement prevents the lateral expansion of core material, which is the main reason for the enhancement in the ductility, as well as the strength of concrete elements. This lateral confinement mechanism is more efficient when the concrete core material is subjected to triaxial stresses [16]. As compared with the confinement of steel ties or spirals, FRPs provide continuous confinement action for the whole cross-section of the members along with the easiness of installation [17]. The confining impact due to FRPs depends on the shape of the confined concrete structural member. This effect is more efficient for the structural members with a minimum radius of corners [18].

Using a vast database of 570 experimental sample points, the current study seeks to analyze the FRP confinement mechanism in concrete and introduce novel models to estimate the axial compressive strains of such members. These strain models were created using regression analysis, **a novel technique**. Using the created database, the models that are currently in use for the axial strains of FRP-confined concrete were assessed. It is anticipated that the recently created model, which is based on this extensive database, will precisely forecast the axial compression strains of concrete compression elements that are FRP-confined. This development will help with comparable element analysis and design in the construction industry.

2. ANALYTICAL MODELING

2.1. Development of Database

570 sample points were compiled into a sizable database from the earlier research that were released. The database was expanded to include all the variables that could impact the axial compressive strain of FRP confined concrete compression elements. A few sample points presented poor predictions for the test results using the previous strain models. Thus, those sample points were removed from the database so that the saturation of root mean square error (RMSE) could be avoided. The statistical information of all the parameters (height H, and diameter D) included in the database was given in Table 1.

Table 1. The statistical details of the database

Parameter	Unit	Min.	Max.	Diff.	Avg.	St. Dev.	COV.
D	(mm)	51.00	406	154.17	41.98	41.98	0.28
H	(mm)	102.0	812	308.0	84.46	84.46	0.28
f'_{co}	MPa	12.41	188.2	45.98	23.26	0.51	12.41
E_f	MPa	10.00	663	163.08	121.33	121.33	0.75
nt	(mm)	0.09	5.90	0.89	1.060	1.06	1.20
ε_{co}	(%)	0.17	1.53	0.27	0.150	23.24	0.51
ε_{cc}	(%)	0.33	4.62	1.40	0.650	34.83	0.47

2.2 Evaluation of Previous Models

When the axial compressive load on the concrete compression members is low, the lateral confinement effect is negligible. However, when the load reaches the peak strength of the members, the lateral confinement effect is activated to develop a lateral pressure that prevents the core material to dilate. The lateral confinement effect develops stress known as confinement stress as shown in Figure 1. The lateral confinement stress (f_l) can be calculated using the following relationship (Eq. 1).

$$f_l = \rho_\varepsilon \rho_k f'_{co} \tag{1}$$

where ρ_k is the stiffness ratio of confinement, ρ_ε is the strain ratio of confinement, and f'_{co} designates the axial strength of unconfined concrete. These confinement ratios can be calculated using the Eq. (2) and Eq. (3) [19].

$$\rho_\varepsilon = \frac{\varepsilon_{h,rup}}{\varepsilon_{co}} \tag{2}$$

$$\rho_k = \frac{2E_f t}{\left(\frac{f'_{co}}{\varepsilon_{co}}\right) D} \tag{3}$$

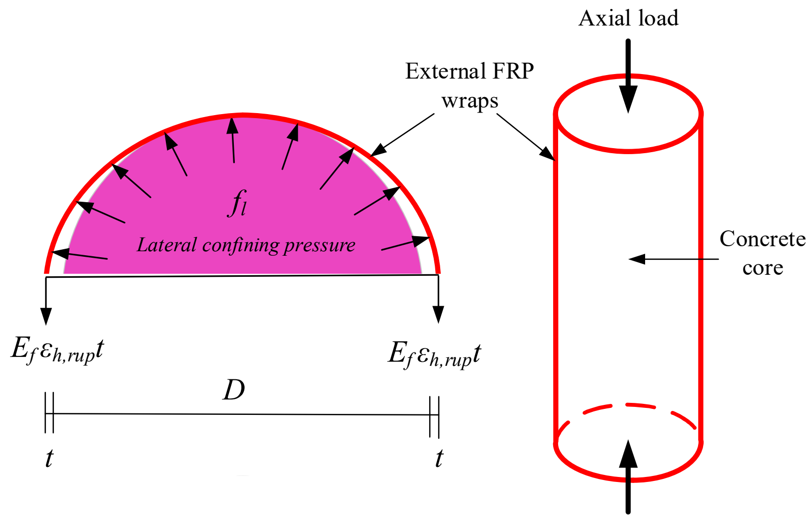


Figure 1. Confinement due to FRP sheets.

In these equations, ε_{co} denotes the compressive strain of unconfined concrete, ‘t’ shows the total thickness of FRPs, E_f depicts Young’s modulus of FRPs, while $\varepsilon_{h,rup}$ depicts the rupture strain of FRPs in the lateral direction. A relationship for this parameter was suggested in the previous work [20].

$$\varepsilon_{h,rup} = \frac{\varepsilon_f}{f'^{0.125}_{co}} \tag{4}$$

here, ε_f is the ultimate tensile strain of FRPs. After incorporating the values of ρ_k and ρ_ε in Eq. (5), the relationship for f_l take the form as given by the following relationship.

$$f_l = \frac{2E_f \varepsilon_{h,rup} t}{D} \tag{5}$$

Fardis and Khalili [17] developed the first ever strain model to analyze the FRP confining impact. The model employed the fact that increase in the circumferential stiffness is associated with the increase in the highest strain of FRP confined concrete. Likewise, Mander et al. [21] established a model to estimate the axial compressive strain of FRP confined concrete elements which were primarily employed by ACI 440-2R-02 recommendations [22]. However, Samaan et al. [23] negated this approach by proposing a strain model by stating that the highest tensile strength of FRPs during the failure phase is greater than the original hoop rupture strain. Similarly, Karbhari and Gao [24] developed a strain model by employing the concept that the FRP wraps attain the corresponding ultimate tensile strength when loaded however, it seems impossible during the actual experimental conditions. By taking into account the original rupture strain of FRPs, Toutanji [25] recommended a novel strain model. In addition, Lam and Teng [26] developed a novel strain model by taking into account the original failure strain of FRP wraps. Subsequently, Teng et al. [27] updated this model by experimenting 18 FRP confined concrete compression elements. All these strain models were evaluated for selection of more generic form of novel model sourced from the established database as shown in Table 2.

Table 2. Previously proposed strain models

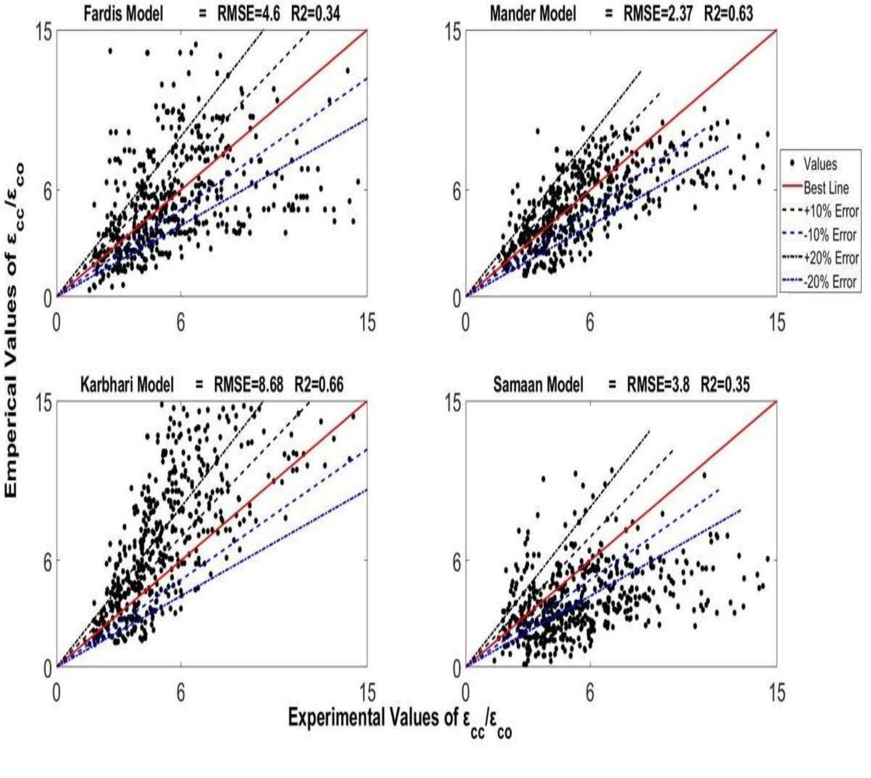
Strain model	Expression
Fardis and Khalili [17] model	$\varepsilon_{cc} = 0.002 + 0.001\dfrac{E_f t}{D f'_{co}}$
Samaan et al. [23] model	$\varepsilon_{cc} = \dfrac{f'_{cc} - f_o}{245.61 {f'_{co}}^{0.2} + 1.3456\dfrac{E_f t}{D}}$ $where\ f_o = 0.872 f'_{co} + 0.371 f_l + 6.258$
Karbhari and Gao [24] model	$\varepsilon_{cc} = \varepsilon_{co} + 0.01\dfrac{f_l}{f'_{co}}$
Mander et al. [21] model	$\dfrac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + 5(\dfrac{f'_{cc}}{f'_{co}} - 1)$
Toutanji [25] model	$\dfrac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + (310.57\varepsilon_{h,rup} + 1.90)(\dfrac{f'_{cc}}{f'_{co}} - 1)$
Lam and Teng [26] model	$\dfrac{\varepsilon_{cc}}{\varepsilon_{co}} = 1.75 + 12\rho_k \rho_\varepsilon^{1.45}$
Teng et al. [27] model	$\dfrac{\varepsilon_{cc}}{\varepsilon_{co}} = 1.75 + 6.5\varepsilon_k^{0.8} \rho_\varepsilon^{1.45}$

Using the constructed database, the design-oriented strain model was suggested in this work. The statistical functions (sum of square errors (SSE), R^2 and RMSE) were used for the assessment of previous strain models over the database to select a most appropriate form of the newly established model. The assessment of the models recommended by Karbhari and Gao [24], Fardis and Khalili [17], Lam and Teng [26], Mander et al. [21], Toutanji [25], Samaan et al. [23], and Teng et al. [27] were made. The selection of these equations was done because of their wide applications in the literature. The parameters R^2 and RMSE were expressed by Eq. (6) and Eq. (7), respectively.

$$R^2 = (\frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2}})^2 \tag{6}$$

$$RMSE = \sqrt{\frac{\sum(x-y)^2}{n}} \tag{7}$$

In these equations, x is the experimental measurements, n shows the total number of tests in the database, and y are the axial compressive stresses of fiber-reinforced polymer-confined concrete parts predicted theoretically. As RMSE approaches zero, accuracy increases, while R^2 approaching one indicates improved prediction accuracy. The best fit was achieved by minimizing the error between experimental and theoretical results using SSE. As SSE approaches zero, prediction accuracy increases, with the best fit at zero and no relationship at one. The performances of various strain models are depicted in Figure 2 based on R^2 and RMSE.



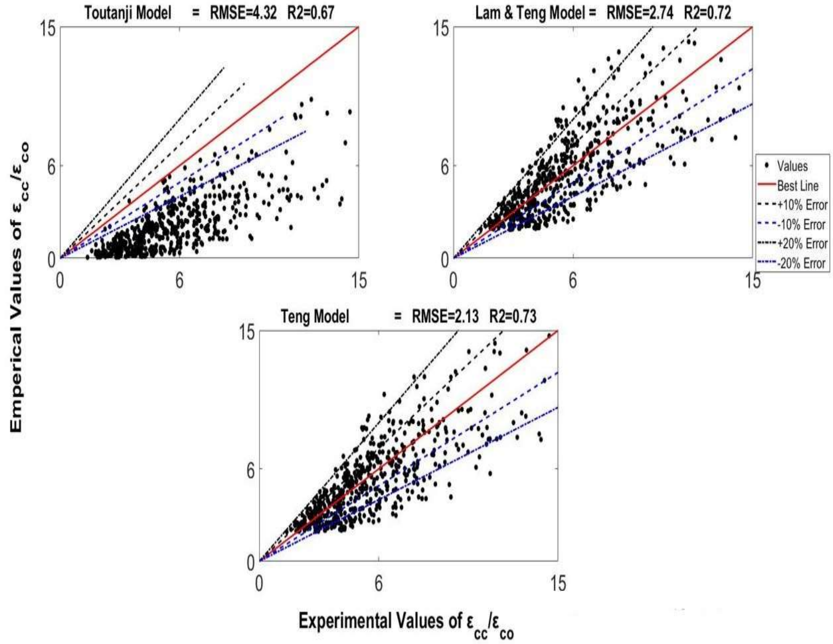


Figure 2. Performance of various strain models over-developed database

By showing a value 0.34 for R^2 and 4.6 for RMSE, the model given by Fardis and Khalili [17] portrayed the good presentation on the developed database. These higher deviations show that the concept of an increase in the maximum strain with circumferential stiffness of FRP confined concrete is not accurate. By showing a value 0.63 for R^2 and 2.37 for RMSE, the model given by Mander et al. [21] exhibited relatively good performance. Therefore, this equation was suggested by ACI 440-2R-02 [22]. The equation given by Karbhari and Gao [24] resulted R^2 = 0.66 whereas RMSE = 8.68, while the Samaan et al. [23] model showed RMSE = 3.80 and R^2 = 0.35. The best performance among all of the studied models was presented by the equation proposed by Teng et al. [27]. This equation gave the values of 0.73 and 2.13 for R^2 and RMSE, respectively.

2.3 Proposed Model

Accordingly, the form of the newly developed equation was kept comparable to the form of the model proposed by Teng et al. [27] due to better performance of this model based on the developed experimental database as represented by Eq. (8).

$$\varepsilon_{cc}/\varepsilon_{co} = \alpha + \beta\rho_k^{\lambda_1}\rho_\varepsilon^{\lambda_2} \quad (8)$$

where α, β, $\lambda_1$and λ_2 are the coefficients. After incorporating the values of these constants obtained from the curve fitting using MATLAB with the help of Eq. (2)

and Eq. (3), Eq. (9) is obtained which is the newly proposed nonlinear model in the present work.

$$\varepsilon_{cc}/\varepsilon_{co} = 1.93 + 7.46\left(\frac{2E_f t}{\left(\frac{f'_{co}}{\varepsilon_{co}}\right)D}\right)^{3/4}\left(\frac{\varepsilon_{h,rup}}{\varepsilon_{co}}\right)^{3/4} \quad (9)$$

According to the behavior of the currently available analytical model, the 570 sample points of FRP-confined concrete compression members were anticipated to have RMSE values of 1.42 and 1.93 and R^2 values of 0.85 and 0.74, respectively. These statistical function values provide strong evidence that these models are more dependable than the models that have been previously published. The performance of the recently created analytical model is displayed in Figure

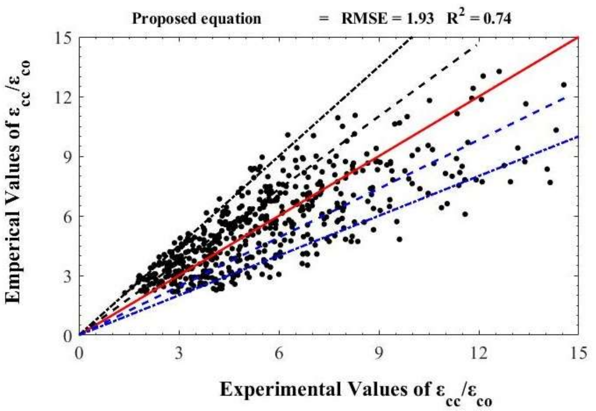


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Figure 3. The suggested analytical strain model's performance

In order to further evaluate the accuracy of the proposed models, the normal distribution/experimental values were plotted and checked against the other models. The Lam and Teng [26] model depict improved performance for the normalizing procedure compared to the earlier models. The proposed predictions of the Lam and Teng [26] model and the recently constructed analytical model were shown to deviate from the unity by 3% and 2.5% respectively. Most other models' normalized projections were conservative. In terms of strain ratio distribution, Toutanji's model [25] showed good performance, with the majority of values falling between 0 and 2. The models that are now being suggested showed similar distributional values with the experimental ratios over various ranges. As a result, the findings unequivocally confirm the newly constructed models' applicability and correctness.

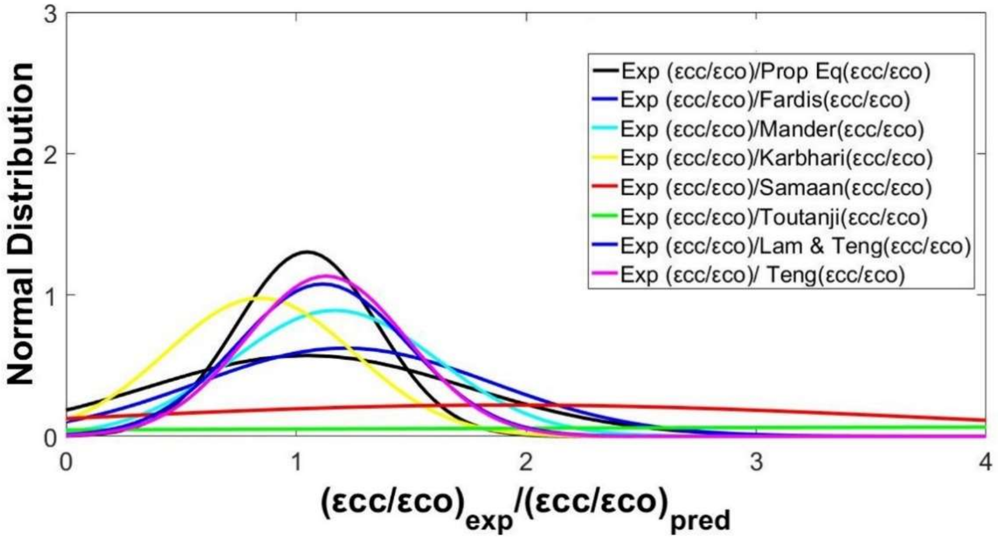

Figure 4. Normal distribution of $(\varepsilon_{cc}/\varepsilon_{co_{exp}})/\ (\varepsilon_{cc}/\varepsilon_{co_{pred}})$ for FRP confined specimens.

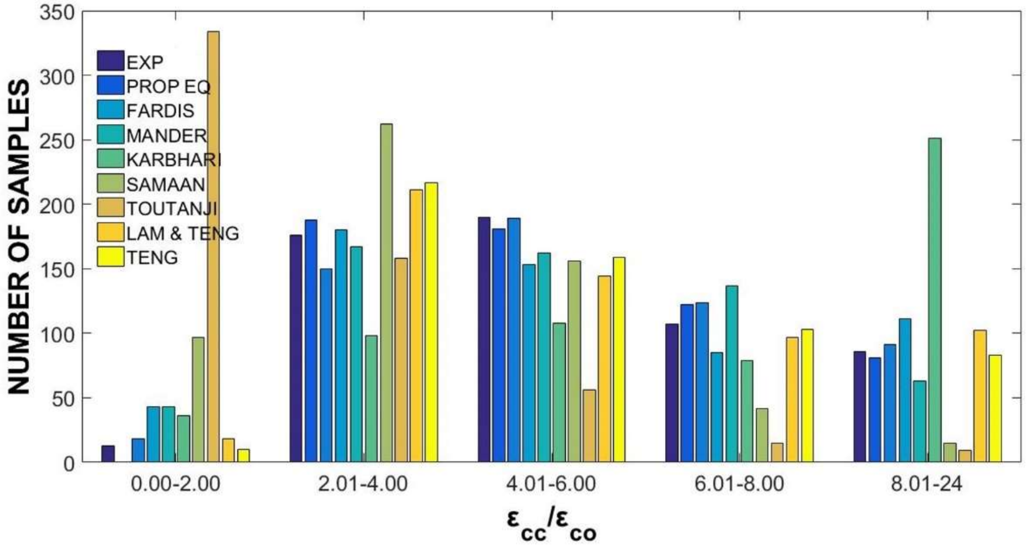

Figure 5. Distribution of $\varepsilon_{cc}/\varepsilon_{co}$ for FRP confined specimens.

3 RESULTS AND DISCUSSION

The FRP wraps are provided to improve the efficiency and strength of concrete experiencing axial compressive strain when the concrete is subjected to axial compressive load. Different models were brought under observation to study this axial compressive strain. The expressions proposed by previous strain models were brought together and compared to analyze the data for 570 testing results. The value of RMSE approaching to zero shows an increase in accuracy while R^2 value approaching to one indicates improved prediction accuracy. The performance of various strain models in Figure 2 provided the validity of the model proposed by Teng et al. [27] to be more accurate with an RMSE value of 2.13 while an R^2 value of 0.73.

So, a newly developed equation represented as Eq. (8) based on this model was further used. After inserting the value of constants using MATLAB with the help of Eq. (2) and Eq. (3), Eq. (9) was obtained which was the newly proposed nonlinear model in the present work. The recently created model displays better results as shown in Figure 3 with an RMSE value of 1.93 and an R^2 value of 0.74. The normal distribution/experimental values were plotted and checked against other model values which showed the correctness of the findings and the reliability that could be placed on these findings as shown in Figure 4. Therefore, the newly developed strain models can be used to predict the axial compressive strain of FRP-confined concrete with improved accuracy and these strain models can help to decide the FRP confinement of concrete in various field-imposed situations to improve the strength and performance of concrete for that particular situation. Thus, FRP wraps for confinement under different field situations can be decided through the proposed model for better precision.

4 CONCLUSION

The current work performs a reliability analysis of the strain models proposed using the general regression analysis with a large database of 570 testing results. The performance of the currently recommended analytical strain model over the large experimental data of 570 tests portrays that its accuracy was higher than the previous strain models for FRP confined concrete with RMSE of 1.93 and R^2 of 0.74. The newly created analytical model's predictions showed a mere 2.5% variation from unity. Thus, these statistical parameters substantiate the superiority of newly developed strain models over the previous models.

5 ACKNOWLEDGEMENTS

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