Optimization of Impact Resistant Throwable Unmanned Ground Vehicle Using Mathematical Modeling Techniques

Hamza Sohail[1*] , *Amir Hamza*[1], *Nasir Rashid*[1], *Muhammad Saad Ali*[2], *Taha Ghani*[2]

[1, 2] Department of Mechatronics Engineering, National University of Sciences and Technology (NUST), CEME, 43701 Rawalpindi, Pakistan

Abstract. Unmanned Ground Vehicles are used for various reasons. One of them is used in search and rescue operations which are known as Throwable Unmanned Ground Vehicles. It is a type of Unmanned Ground Vehicle which can sustain falls from several meters. Such vehicles are easily deployable, lightweight and impact resistant. Wheels play an important role since they absorb most of the impact. To optimize the performance of such vehicle, mathematical modeling is necessary. In this research paper, the main goal is to model the system mathematically. Parameters like impact forces, elasticity, Young's modulus and duration of impact depend highly on material properties and structure of wheels. Results show the effect of certain parameters on wheel's design and materials. For that purpose three materials are chosen PCTPE, carbon fiber, and vulcanized rubber for wheels. Impact forces, and duration of time are plotted against height of fall and diameter of the wheel for these materials.

Keywords. *Throwable Unmanned Ground Vehicles; Optimization; Composite Materials; Impact Force; Mathematical Modeling; Land Vehicle*

1 Introduction

Compact Unmanned Ground Vehicles are designed to provide us with specific tasks, including search and rescue operations. It can work in environments where human life is compromised. It also has other purposes in domestic areas, such as inspection of sewage pipes, etc. To make it easily deployable, size and weight should be reduced. Such vehicles are impact-resistant to avoid any harm from harsh environments. For that purpose, wheels play an important role. Wheels play an important role in absorbing impact. For that purpose, different materials have to be studied to achieve the desired goal.

PCTPE stands for Plasticized Copolyamide TPE. It can resist impact and provide durability [1]. Also, its elongation at break is 497%. Carbon fiber provides toughness due to its high ultimate tensile strength which is 3.5 GPa. Vulcanized rubber is processed by the molding process. Due to its high elasticity, it is considered a potential material for the manufacturing of wheels [2].

To model the system mathematically, first of all, a calculation of impact velocity is a must. Impact velocity is calculated by using Newton's third equation of motion [3]. Using the concept of simple physics law of conservation of momentum is applied on free falling bodies. For an ideal body the impact velocity before the impact and after the impact is equal but in opposite in direction [4]. The impulsive force is the rate of change in momentum concerning time [5, 6]. It shows duration of impact affects the impulsive force lesser the duration of impact higher the impulsive force will be and vice versa. Stress Strain curve [7]

* Hamza Sohail: hamza.sohail@ceme.nust.edu.pk

shows how much material is undergone in stress. It is an important feature in impact testing. To optimize the performance of the Throwable Unmanned Ground Vehicle stress strain curve provide a great insight into when a material will break.

This research paper explores the mathematical model and results that show different parameters on which the performance and design of UGV can be modified.

2 Methodology

To optimize the performance of impact-resistant unmanned ground vehicle mathematical modeling is necessary. Certain parameters like impact forces, stresses, strains, elasticity and Young's modulus on a body depends highly on the material and structure we are choosing. Throwable Unmanned Ground Vehicles [1] are made in such a way that maximum impact is absorbed by their wheels without jeopardizing the interior structure [8]. For that purpose, wheels are designed with larger diameter to avoid direct contact of body with ground even if it falls upside down.

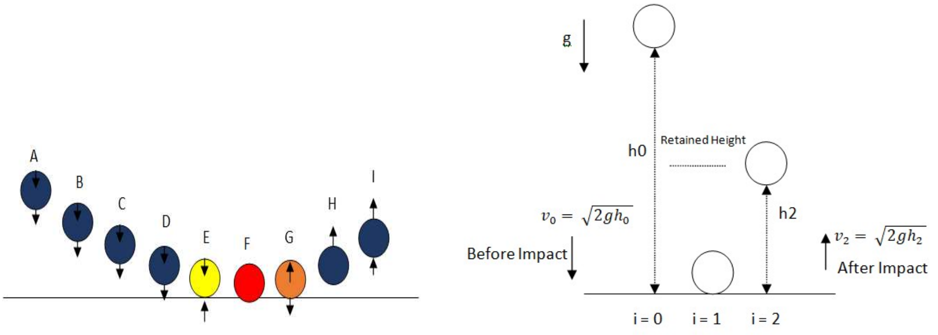

Figure 1(a) Impact force of perfectly elastic wheel, (b) Response in non-ideal scenario

Law of conservation of momentum applies when a body collides. In our case, when impact resistant unmanned ground vehicle falls from a height the momentum is conserved as earth's momentum will be equal but in the opposite direction of a free falling unmanned ground vehicle. The equation $P = mv$ shows the product of mass and velocity which is momentum 'p'.

$$\sum_{i=1}^{2} \mathrm{m_i v_i} = 0 \tag{1}$$

Equation 1 shows law of conservation of momentum. 'm' is the mass of the wheel that we are testing, and 'v' is the impact velocity of the wheel. 'i' is a numerical value. 'i' is 1 for momentum of object and 'i' is 2 for earth's momentum which is equal but in opposite direction to the object as shown in Figure 1 (a). Impact force also known as impulsive force is the rate of change in momentum with respect to time. While optimizing the design mass generally remains constant while changing the velocity to check how much impact force the wheel can absorb and how much stress it can bear while keeping the momentum conserved. Impact force can be calculated by impulse which is P = Ft.

$$F_{impact} = \frac{d(mv)}{dt} \tag{2}$$

Impact force suggests that if we increase the velocity by keeping the mass constant, impact force will also significantly increase and vice versa. If we consider an *ideal case* in which perfectly elastic material for wheel is fallen from a height. To keep the momentum conserved it will bounce back with the same velocity. In this scenario impact velocity before the impact and impact velocity after the impact remains the same.

$$v_{before\ impact} = v_{After\ impact} \quad (3)$$

Figure 1(a) shows no impact force from point A to point D. As soon as the perfectly elastic wheel hits the ground at point E with the velocity of 'v1', it absorbs impact. At point F the body tries to change its shape to absorb the impact such that it does not break. Point F is the point of impact. The velocity is zero at point F. At point G, the velocity is said to be 'v2' which is the velocity after impact. Figure 1(b) shows non-ideal behaviour in which wheel has fallen from a height of h0 with the velocity of v0. As soon as it hits the ground the velocity is zero, at point i = 1. At point i = 2 the velocity is v2, the velocity after impact. At this point the wheel will not retain its height. The retained height is h2.

$$v_i = \sqrt{2gh_i} \quad (4)$$

If we throw the wheel from a greater height the resulting velocity will be greater and vice versa. Impact force during the collision remains constant. During the collision, stress is constant as it is acting on a cross-section area of the body 'A'. But the only effect that happens on a wheel is due to its material properties. When a free falling wheel hits the ground, the duration of impact depends on the density and Young's modulus of the wheel material which is given as follows.

$$\Delta t = 2H\sqrt{\frac{\rho}{E}} \quad (5)$$

ρ is the density of material, E is Young's modulus of the material and H is the height of the wheel that is under impact. Different materials have different impact force due to their different density and Young's modulus.

$$F_{impact} = \frac{mv}{H\sqrt{\frac{\rho}{E}}} \quad (6)$$

Equation 6 shows that an increase in the volume of the wheel decreases the impact force and vice versa. Same case for density, if density is greater less impact force will be exerted on the body upon collision. Now the stress-to-strain ratio is a material property which is Young's modulus of the material which is expressed as follows.

$$E = \frac{Stress}{Strain} \quad (7)$$

Stress-to-strain ratio shows stress is directly proportional to strain. E is a constant and material property of the material. Stress is defined as impact force per unit cross section area of impact. Strain is the ratio of change in length to original length.

$$\Delta L = \frac{LF}{EA} \quad (8)$$

ΔL is the change in length of the wheel. L is the original length. A is the cross-section areas upon impact. Bodies with smaller surface area are more elongated as compared to wheels having larger surface areas. Another material property that defines how much impact a body can bear is its ultimate tensile strength which is the ratio of maximum stress that a wheel can bear to original cross section area of the body.

$$F_{max} = ultimate\ tensile\ strength\ \ X\ A \qquad (9)$$

Material properties of carbon fiber, PCTPE, and Vulcanized rubber are given in table 1.

Table 1 Material Properties

Material	Density Kg/m^3	Young's Modulus MPa	Ultimate Tensile Strength MPa
PCTPE	1113	75.0	34.8
Vulcanized Rubber	950	16.3	28.0
Carbon Fiber	1750	250	3500

3 Results and Discussions

Materials chosen for the desired problem are carbon fiber, PCTPE and vulcanized rubber wheel. Impact force and duration of impact are plotted against height of fall and diameter of wheel. Figure 2(a) shows how impact force is reduced if we increase the height of the wheel in meters while keeping the free-falling velocity, free falling height and mass constant. It is done on three different materials; PCTPE, carbon fiber and vulcanized rubber. All of these have different densities and Young's modulus. Changing the material affects impact force. If we want to reduce the size and weight of our UGV; material and shape should be considered. Figure 2(b) illustrates that vulcanized rubber has greater duration of impact due to its elastic characteristics as compared to PCTPE and carbon fiber. Carbon fiber has the lowest because it is the least elastic of them all. Duration of impact is increased if we increase the diameter of the wheel because larger the wheel larger the energy it will absorb and more time it will take to release that energy.

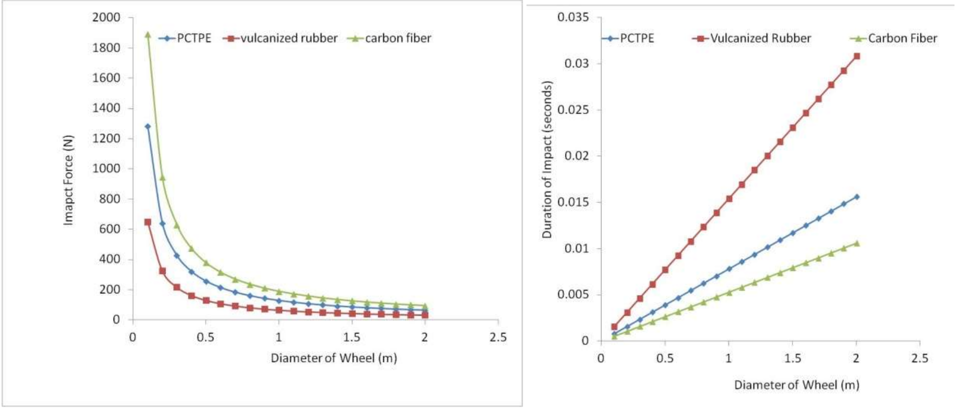

Figure 2 (a) Impact Force VS Diameter of Wheel (b) Duration of Impact VS Diameter of Wheel

Increase in the height of the fall, impact force increases. Greater the height of the fall, the more impact will be on the wheels. Figure 3(b) shows duration of impact is independent of height of fall as it only depends on its material properties like Young's modulus and density as well as height or diameter of the wheel. In the graph shown in figure 3(b) it is clear that material with greater Young's modulus has lower duration of impact as seen in carbon fiber while vulcanized rubber has the highest.

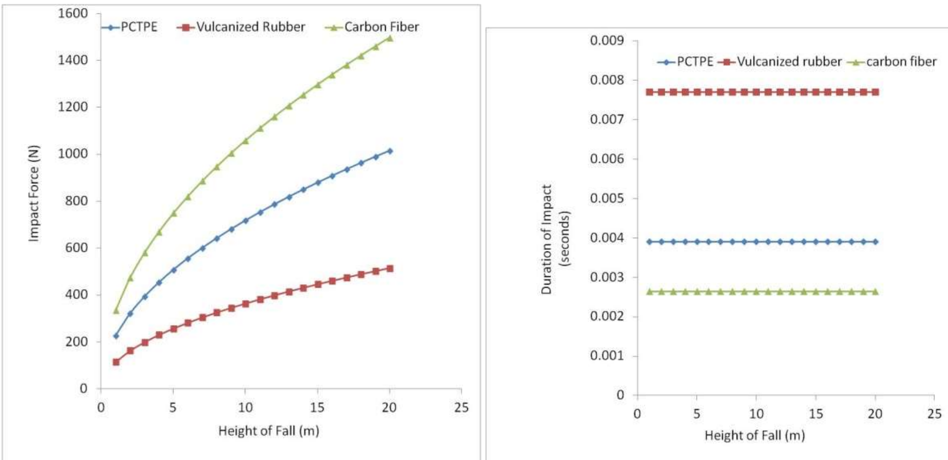

Figure 3 (a) Impact Force VS Height of fall (b) Impact Force VS Height of Fall

Figure 4 shows impact velocity increases as we increases the height of fall in meters. The graph shows a velocity of 19 m/s at 20 meters. Velocity at each point is found by $v_i = \sqrt{2gh_i}$ where i is the height of fall at every point from 1 to 20 meters. Impact velocity is independent of material property. Every material has the same impact velocity with different impact forces and duration of time.

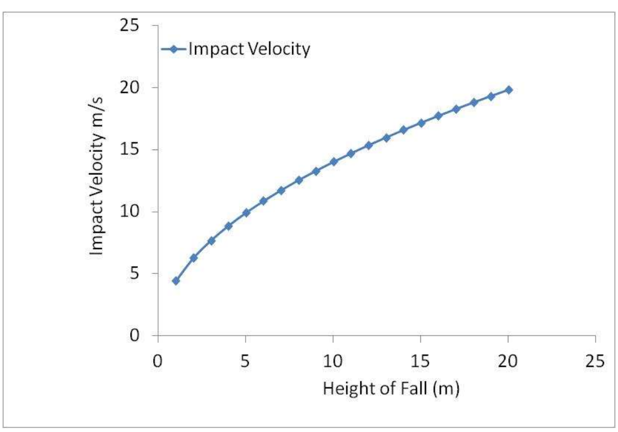

Figure 4 Impact Velocity VS Height of Fall

Figure 5 shows a constant ultimate tensile strength, the graph is plotted against the force of impact and cross section area of impact. It shows how increasing the surface area of the wheel increases the force of impact at which the wheel can sustain itself since the results are at ultimate tensile strength. At cross section area of PCTPE wheel is 0.01 meters and ultimate tensile strength which is a constant parameter of a material, the maximum impact force the wheel can sustain is 340000 Newton of force. If the surface area is increase it can bear greater amount of impact force and vice versa.

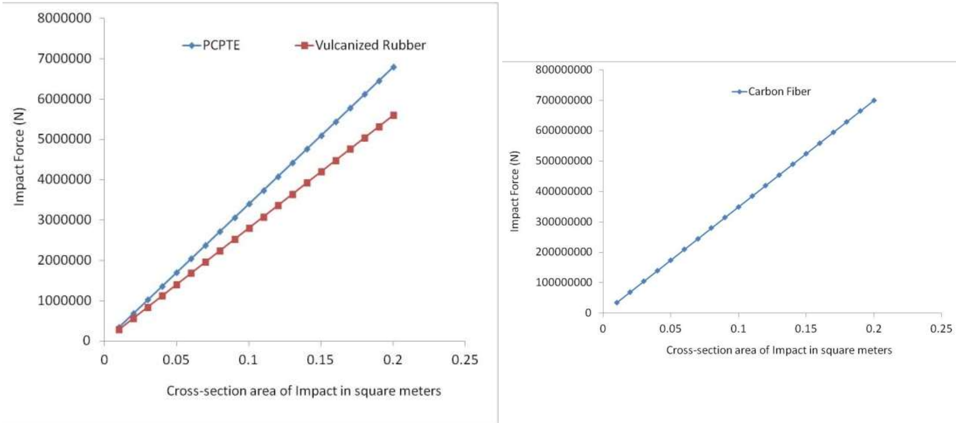

Figure 5 Impact Force VS cross section area of impact (a) PCPTE and Vulcanized Rubber (b) Carbon Fiber

Figure 6 shows while keeping parameters like impact velocity constant we get impact force VS mass of wheel graph that shows if we make a light weight wheel, the impact force exerted on it will be lower and vice versa.

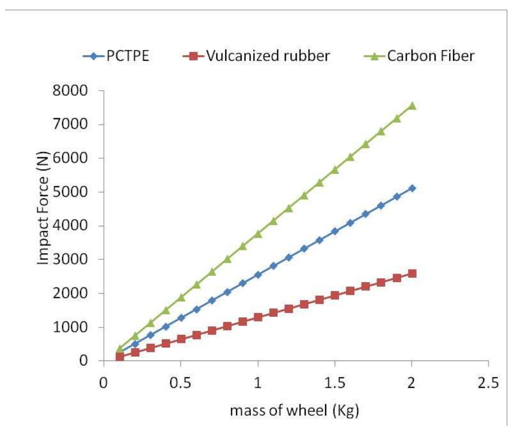

Figure 6 Impact Force VS mass of wheel

4 Conclusions

The results suggest that to optimize the performance of Throwable Unmanned Ground Vehicle height of fall, material density, its Young's modulus, cross section area of impact, and diameter of wheel should be considered. The design of UGV depends on its height of fall as more of it results in greater impact force which results in greater stress acting on it and vice versa. The height of fall is independent of duration of impact. The duration of impact depends on material properties. Material with more Young's modulus has more duration of impact. Greater the size of wheel lower the impact force and chance for it to break. Similarly, if the UGV is falling vertically on wheels then it can bear more impact force due to its larger cross-section area of impact.

References:

1. H. Sohail, A. Hamza, N. Rashid, M. S. Ali and T. Ghani, "Design and Analysis of Throwable Unmanned Ground Vehicle," 2023 International Conference on Robotics and Automation in Industry (ICRAI), Peshawar, Pakistan, 2023, pp. 1-6, doi: 10.1109/ICRAI57502.2023.10089612.

2. Liu, Qinfu, Yude Zhang, and Hongliang Xu. "Properties of vulcanized rubber nanocomposites filled with nanokaolin and precipitated silica." Applied Clay Science 42.1-2 (2008): 232-237.

3. Sivaraj, Priyadharsini. (2018). A Study on Motion of a Free Falling Body in Kinematic Equation. International Journal for Research in Applied Science and Engineering Technology. 6. 3118-3124. 10.22214/ijraset.2018.1431.

4. Kin Son Wong, Hang Wong; Understanding the Law of Conservation of Momentum in One-Dimensional Collisions Between Two Objects Using the Velocity Space Approach. *Phys. Teach.* 1 February 2022; 60 (2): 94–96. https://doi.org/10.1119/5.0024603

5. Website: [online], Available: *http://www.gotstogo.com/misc/engineering_info/bounce*

6. Schilling, Brian & Falvo, Michael & Chiu, Loren. (2008). Force-Velocity, Impulse-Momentum Relationships: Implications for Efficacy of Purposefully Slow Resistance Training. Journal of sports science & medicine. 7. 299-304.

7. Al Baida H, Langlade C, Kermouche G, Ambriz RR. Identifying the stress–strain curve of materials by microimpact testing. Application on pure copper, pure iron, and aluminum alloy 6061-T651. *Journal of Materials Research*. 2015;30(14):2222-2230. doi:10.1557/jmr.2015.186

8. Y. Li, Q. Huang, Y. Huang, L. Zhang, J. Gao and Y. Tian, "A throwable miniature robotic system," *2011 IEEE International Conference on Automation and Logistics (ICAL)*, Chongqing, China, 2011, pp. 114-118, doi: 10.1109/ICAL.2011.6024695.