Analysis of the dynamic behaviour of lightweight floor slabs

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Abstract. Floor slabs are one of the integral load-bearing elements in buildings. When it comes to their serviceability limit state (SLS), one of the key aspects is their dynamic behaviour. In case of residential or administrative buildings, the knowledge of dynamic behaviour of slabs is important in terms of hygiene (psyche) of the user. When speaking of buildings such as medical facilities or laboratories that are using delicate machinery, the knowledge of the dynamic behaviour is important in order to prevent vibration induced damage to given machinery. Therefore, the aim of this paper is to provide a brief overview of standards applicable for assessment of dynamic behaviour, present various methods for calculation of first natural frequency, as well as to present outlines for in-situ measurements, all with special regard to voided slabs.

1 Introduction

Construction industry is a major source of air pollution. Studies suggest that construction of buildings and their maintenance is responsible for as much as 30% of global production of greenhouse gasses [1].

Floor slabs are one of the integral load-bearing elements in buildings, as they may represent as much as 90% of total weight of the load bearing structure [2]. Currently, building materials are experiencing a steep increase in their price, as data from the USA show. In average, the price of raw building materials (excluding energy) has risen by 41% between March 2020 and March 2022 [3]. In addition, ecology plays an increasingly important role in every industry, including construction, therefore, it is appropriate to search for more sustainable solutions.

One of such solutions may be the use of lightweight ceiling slabs. In these slabs, dead weight is reduced by either placing of void plastic formers or blocks of low-density material such as polystyrene in the area around the middle of their cross section, where stresses in concrete would be rather minimal. This saves materials in two areas; primary, in the slab itself, which weight may be reduced by as much as 30-35% [4, 5]. Secondary savings occur in downstream load-bearing elements such as walls, columns and foundations, as they need to sustain smaller dead-load. Their weight may decrease by as much as 40% [5]. Other advantages of such slabs are their ability to span larger spans [6], their reduced thickness that

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allows for lower floor-to-floor height. In addition their lower weight is beneficial in seismic design, as seismic loads induced by movement of lighter slabs are smaller [5].

When it comes to design of the slabs for serviceability limit state (SLS), their dynamic behaviour is becoming increasingly problematic, as the spans are larger and structures are subtler, which is even more prevalent in case of voided slabs [6]. Hence, the aim of this paper is to summarise dynamic requirements for floor slabs, stemming from the SLS, as well as to provide a quick overview of various approaches to calculation of dynamic parameters. Last, but not least, it aims to present outlines for in-situ verification of these parameters.

2 Floor vibrations – sources, problems, solutions

Vibrations of floor slabs pose an increasingly more prevalent problem in buildings. In the past, the vibrations were mainly eliminated by robustness of the slabs, stemming from the need to factor in inaccuracies of calculations at given time. This robustness granted greater stiffness therefore higher natural frequencies [6]. Currently, floor slabs are becoming more and more subtle, as all load-bearing elements are generally being designed to be used to their limit thanks to finite element method (FEM), that is allowing for highly precise calculations. In addition, voided slabs are generally used for larger spans, therefore their total stiffness is even lower.

Vibrations in buildings are generally caused by three main factors:
- Human activities – walking, jumping etc.
- Continually working machinery with moving parts – air conditioning, washing machines, presses etc.
- External excitations – wind, traffic, construction sites and other activities in adjacent areas

From the perspective of the usage of the structure, this has various undesirable effects. First, vibrations mainly in up and down direction in an intensity, that can be sensed by humans have bad influence on the psychological well-being of users [6]. Level of annoyance caused by these vibrations perceived by users of the structure is generally dependent on following factors[7, 8]:
- The direction of the vibration (although this paper focuses mainly on vertical vibrations)
- The posture of the affected (lying, sitting, standing)
- Current activity of affected person
- Health and age of the user

In addition to human well-being, vibrations also affect various pieces of machinery. Especially precise machinery is generally sensitive to vibrations, that may cause its inoperability or even damage. Some examples are laboratory equipment such as electron microscopes [9] or hard drives in server storage pools.

3 Criteria for vibrations – overview of standards

There are various relevant standards, either international or European, that can be used to assess the effects of vibrations in buildings on humans. This chapter in its subchapters provides a summary of different applicable standards with brief descriptions of methodology proposed by them. In this chapter, either international (ISO) or standards from EU member states are described.
3.1 ISO 2631-2: 1989

This standard is based on weighting (base) curves, for all three axes of human body. These curves describe human response to vibrations of buildings and take into consideration parameters, that influence the final perception, such as the time of day, direction of vibration and intended use of investigated building. The method to assess the influence of vibrations used in this standard is Root Mean Square (RMS) value of weighted accelerations [7].

3.2 ISO 2631: 2003

Represents an updated version of aforementioned standard. The main difference is present in revised weighting curve, which defines frequency weighting for frequencies between 1 to 80 Hz, where posture of the affected person does not need to be defined [7].

3.3 BS 6472: 1992

This British standard is based on ISO 2631: 2003 and uses the same base curve for frequency weighting of frequency signals. However, the vibrations are evaluated not by using RMS value, but so-called Vibration Dose Value (VDV) [7].

3.4 DIN 4150, Teil 2: 1992

German standard is applicable for cases of either continuous or random vibrations within a frequency range of 1 to 80 Hz, such as in the case of ISO standard. It is based on results of research, that studied the perception of vibrations by humans in both, sitting and standing pose. The standard uses so-called progressive effective value to determine vibration exposure [7].

3.5 VDI 2057, Blatt 1: 2002

Another standard form Germany (created by Association of German Engineers). This standard focuses only on external sources of vibrations and their effects. It uses frequency weighting based on ISO 2631-2:1989 standard and uses RMS values to assess the effect of vibrations on affected individual. In weighting, the standard uses distinguished curves for various postures of human body and scope of effects (well-being, performance, health) [7].

3.6 SBR, 2002

A Dutch standard, that is very similar to aforementioned DIN standard, as it also uses the same method to assess the annoying vibrations. Weighting function for accelerations is the one used in ISO 2631-2 [7].

3.7 Recommendations

Focusing solely on human induced vibrations, it is generally recommended to use One Step RMS (OS-RMS) value as a measure for assessing annoying floor vibrations. The values of OS-RMS are representing harmonic vibrations caused by one relevant step onto the investigated floor slab [7]. These values may be either obtained by measurement, which is viable if an existing structure is investigated, or by calculations and use of design diagrams applicable for parameters of given structure.
4 Dynamic behaviour of slabs

Dynamic parameters of floor slabs in the design stage may be determined by various methods. Some of these methods are simplified, and serve the purpose of approximate, generally preliminary calculations. Such methods are for example Equivalent Plate method, Jänich’s approximation or Bachmann’s method. Description of these individual methods is provided in following sub-chapters. In general, these methods provide a quick and simple mean of calculating first natural frequency of a slab, however they are derived from the behaviour of full cross-section slabs.

4.1 Equivalent Plate Method

This method has been derived by Jeary from Equivalent Beam method by adding the effects of flexural rigidity of a plate and second direction of spanning. Given modification allowed for approximate calculation of first natural frequency of simply supported, two-way slabs.

\[ f_1 = \frac{\pi}{2} \sqrt{\frac{D}{m}} \left( \frac{1}{l_x^2} + \frac{1}{l_y^2} \right), \]  

(1)

Where \( D \) is flexural rigidity of plate, \( m \) is the mass of plate, \( l_x \) and \( l_y \) are spans in the directions of respective axes.

Flexural rigidity is expressed as:

\[ D = \frac{E \cdot h_d^2}{12 \cdot (1 - \nu^2)}, \]

(2)

Where \( h_d \) is the thickness of slab and \( \nu \) is Poisson’s ratio.

4.2 Jänich’s Approximation

An approximation for calculation of first natural frequency presented by Jänich, in comparison with Equivalent Plate Method is a more complex one. In the calculation of the first natural frequency, it considers both the effects of loads imposed on investigated slab, as well as various boundary conditions (free, hinged, or clamped edge). The formula used to calculate first natural frequency is:

\[ f_1 = \frac{\pi}{2} \sqrt{\frac{D \cdot g \cdot K}{h_d \cdot \gamma \cdot N}}, \]

(3)

Where \( D \) is flexural rigidity of slab, \( g \) is gravitational acceleration, \( K \) is a parameter factoring in boundary conditions of slab, \( h_d \) is thickness of slab, \( \gamma \) is volumetric density of slabs material and \( N \) is a parameter factoring in loads imposed on slab. The parameter factoring in boundary conditions of the slab is given in the following formula:

\[ K = \frac{K_1}{l_x^4} + \frac{K_2}{l_x^2 \cdot l_y^2} + \frac{K_3}{l_y^4}, \]

(4)

Where \( l_x \) and \( l_y \) are spans in the directions of respective axes. \( K_1, K_2 \) and \( K_3 \) are parameters dependent on boundary conditions. Their values for chosen boundary conditions are provided in Table 1.
In order to factor the effects of loads imposed on a slab in calculations, following formula is being used:

\[ N = N_0 \left( 1 + \frac{p}{m} \right) + \sum \frac{P}{m} * w_P^2, \]  \hspace{1cm} (5)

Where \( N_0 \) is a parameter dependent on boundary conditions of slab, that is also provided in Table 1, \( p \) is load posed on a surface of slab, \( P \) is point load and \( w_P \) is displacement of slab caused by point load \( P \).

**Table 1. Parameters for Jänich’s approximation [10].**

<table>
<thead>
<tr>
<th>Boundary Conditions</th>
<th>K₁</th>
<th>K₂</th>
<th>K₃</th>
<th>N₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free edge</td>
<td>0.25</td>
<td>0.50</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Hinged edge</td>
<td>12.00</td>
<td>8.00</td>
<td>12.00</td>
<td>2.25</td>
</tr>
<tr>
<td>Clamped edge</td>
<td>0.1667</td>
<td>0.0760</td>
<td>0</td>
<td>0.1667</td>
</tr>
</tbody>
</table>

**4.3 Bachmann’s Formulas**

Hugo Bachmann in cooperation with Walter Amman, came up with formulas for calculation of first two natural frequencies of slabs with various boundary conditions. These calculation procedures are based on their vast experience in the field of structural dynamics. The formula for calculation of first two natural frequencies of a two-way slab is:

\[ f_n = \frac{\varphi_n}{l_x^2} \sqrt{\frac{E * h_d^3}{12 * (1 - \nu^2) * m'}}, \]  \hspace{1cm} (6)

The parameters based on boundary conditions of investigated structure are in Table 2.
Table 2. Parameters for Bachmann’s formulas [11].

<table>
<thead>
<tr>
<th>Boundary Conditions</th>
<th>φ - factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \varphi_{1,1} = 1.57 \times (1 + \alpha^2) )</td>
</tr>
<tr>
<td></td>
<td>( \varphi_{2,1} = 6.28 \times (1 + 0.25 \times \alpha^2) )</td>
</tr>
<tr>
<td></td>
<td>( \varphi_{1,2} = 1.57 \times (1 + 4 \times \alpha^2) )</td>
</tr>
<tr>
<td>Hinged edge</td>
<td>( \varphi_{1,1} = 1.57 \times \sqrt{1 + 2.5 \times \alpha^2 + 5.14 \times \alpha^4} )</td>
</tr>
<tr>
<td></td>
<td>( \varphi_{2,1} = 6.28 \times \sqrt{1 + 0.625 \times \alpha^2 + 0.321 \times \alpha^4} )</td>
</tr>
<tr>
<td></td>
<td>( \varphi_{1,2} = 1.57 \times \sqrt{1 + 9.32 \times \alpha^2 + 39.06 \times \alpha^4} )</td>
</tr>
<tr>
<td>Clamped edge</td>
<td>( \varphi_{1,1} = 1.57 \times \sqrt{5.14 + 3.13 \times \alpha^2 + 5.14 \times \alpha^4} )</td>
</tr>
<tr>
<td></td>
<td>( \varphi_{2,1} = 9.82 \times \sqrt{1 + 0.298 \times \alpha^2 + 0.132 \times \alpha^4} )</td>
</tr>
<tr>
<td></td>
<td>( \varphi_{1,2} = 1.57 \times \sqrt{5.14 + 11.65 \times \alpha^2 + 39.06 \times \alpha^4} )</td>
</tr>
</tbody>
</table>

4.4 Other Methods for Approximate Calculations

Other methods, that may be used for approximate calculations of natural frequencies [12] are listed below in order to create an overview, however, they would not be described further in this paper:
- Equivalent beam method,
- Concrete society method,
- Static deflection method,
- Modified deflection method,
- Approximation by Hearmon,
- Approximation by Blevins.

4.5 Voided slabs

When calculating first natural frequency of a voided slab, various methods may be used. The most common one is to use the Finite Element Method (FEM). In FEM, the slab may be modelled by using various modelling approaches. The slab is either modelled as a full cross-section slab, but its material properties are changed according to the recommendations of manufacturer of given lightening system, or, it is modelled with hollows, to match reality.

With full cross-section approach in general, volumetric mass of the concrete is lowered in order to account for lightening and elastic modulus is lowered to account for lower flexural stiffness of the slab. The main advantage of this modelling approach is its ease of use in engineering practice.

In models with hollows, material properties remain unchanged, and the slab is modelled as is. This type of modelling is deemed like the most accurate one, however, it is rather labour-intensive, thus, it is not best suitable to be used in practice.
It is also possible to use aforementioned approximate methods with various degrees of success, however, their accuracy is dependent on the parameters of investigated slab, such as its thickness, or the shape of void formers [6].

5 In situ measurements of dynamic parameters of slabs

Calculated dynamic parameters of a structure in the field of civil engineering may generally not be regarded as certainly accurate, whether they are obtained by approximate hand calculations or highly precise FEM analysis. This is not only due to the fact, that the construction process of standard buildings is rather inaccurate, and industry standard tolerances are relatively benign, but also the assumptions and approximations in calculations play a non-negligible role. Therefore, especially in the cases, when some precise machinery, susceptible to vibration damage is to be placed on the slab, additional verification of dynamic behaviour is recommended.

This verification may be done by either direct or indirect measurement with the use of accelerometers or sensors measuring velocity of the oscillation. These measurements are based on their nature, measurements of Experimental Modal Analysis (EMA). This is due to the fact, that the excitation is generally well known, defined, and controlled [13]. Standard requirement for these measurements is, that their results should be reproducible [7].

Indirect measurements are recommended to be used in cases when it is not possible to achieve relevant excitation mechanism. This is the case for most human induced vibrations, as they are highly dependent on the weight of person, frequency of their pace, sole of their shoes, walking path etc. Therefore, during indirect measurements transfer functions are obtained as a result. These functions may be then combined with standard loading, in order to determine the dynamic behaviour of slab in real conditions [7].

Direct measurements, on the other hand, are used in cases where the source of vibrations is known and constant. These sources are for example machines [7]. In this case, the excitation during the measurements may be replied by using exciter with same frequency as the machinery, that is to be placed on the slab. These measurements may also be done for human induced vibrations; however, this type of measurement is rather complicated.

Another option is to perform so-called Operational Modal Analysis (OMA). In this case, measurement data obtained from the operational responses are used to estimate the parameters of models that describe the system behaviour [13]. These measurements do not require artificial excitation, that is well defined and controlled to a high extent. The sources of excitation are generally wind, traffic, or unknown signals random in time and space with some spatial correction [13].

For more information on measurements, general requirements, and limitations, reference relevant literature and/or applicable standards, for example [7, 13].

6 Conclusion

Voided slabs seem like a viable way of reducing material consumption and related production of emissions in monolithic concrete buildings. However, the dynamic behaviour of this type of slabs is generally not as easy to describe as it is in the case of full cross-section slabs.

This paper provided a brief overview of main principles and background of dynamic behaviour and excitation, as well as applicable requirements, methods of calculation, as well as outlines for in situ measurements.

In following work, we would like to build up on the basis provided in this paper and perform a measurement of dynamic behaviour of full scale two-way voided monolithic slab. The evaluation of results would be then done according to the background provided in this article.
Acknowledgements

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References

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