Effect of soil-foundation interaction on the modulus of subgrade reaction

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Abstract. For design of raft foundations, the soil is commonly modelled by using elastic springs, which is based on the Winkler's theory. In the static analyses, these springs are considered as a constant value k called the modulus of subgrade reaction. However, no foundation is infinitely flexible due to the interaction between the soil and foundation, which is investigated in the present work. For this purpose, the effect of foundation dimensions and foundation material on the subgrade reaction modulus is examined for square-shaped foundations with uniformly distributed load, and the results obtained from these analyses are compared with those in infinitely flexible foundations. It is shown that the effect of Young's modulus of foundation material on the k-value can be ignored while the foundation dimensions have a great effect on the k-value. The ratio of the subgrade reaction modulus obtained from the analyses taking into account the soil-foundation interaction to the subgrade reaction modulus in infinitely flexible cases varies between 0.36 and 0.45 in the middle area of foundations as well as 0.45 and 0.76 on the edges of foundations with a width between 5 m and 20 m as well as a thickness between 0.25 m and 1.0 m.

1 Introduction

Winkler model is very oft used for the design of raft foundations. In this model, the soil is represented by number of linear elastic springs with a spring constant called the modulus of subgrade reaction k, as shown in Fig. 1a [1, 2]. Therefore, the accurate estimation of the subgrade reaction modulus is essential for geotechnical and structural design.
One of the most common methods to estimate the modulus of subgrade reaction is the plate load test [3]. Here, the modulus of subgrade reaction is described as the ratio of contact pressure at any given point to the settlement caused by the pressure in that point, $k = q/s$ (see Fig. 1b). However, no foundation is infinitely flexible due to the interaction between the soil and foundation so that the contact pressures developing between the foundations and soils do not correspond to the column and wall loads on the foundations. This has been investigated by several researchers, and various methods have been developed for the accurate estimation of the k-value, as listed in Table 1 [3-8].

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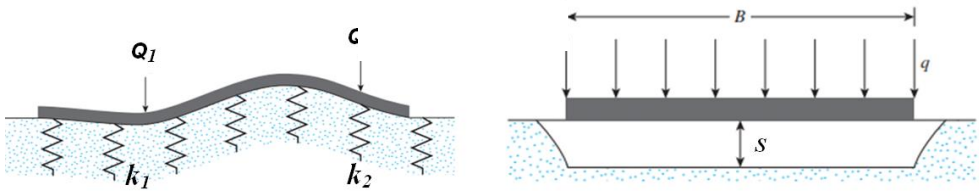


(a) (b)

Fig. 1. Design of flexible foundations [2]: a) Winkler's model, b) Plate load test

In recent studies, Poulos [9] stated that the modulus of subgrade reaction is not a fundamental soil property, and its value varies with the type and dimensions of foundation as well as with the type of loading. Avci and Gürbüz [10] indicated that the k-value varies with soil deformation, foundation depth, foundation dimension and internal friction angle of soil. Roy and Deb [11] carried out a series of model-scale plate load tests on layered soil (sand over clay) with or without geogrid reinforcement. The results of these tests indicated that the interface effect leads to a reduction on the k-value, and it appears when the spacing between the plates is less than 1.33 times the plate-width. Hamza et al. [12] investigated the interference effect of foundations on the k-value by using 3D-finite element analyses. It has been shown that the interface effect can be ignored when the spacing between the foundations becomes larger than three times the foundation-width.

Table 1. Equations recommended by various researchers to estimate the subgrade reaction modulus [3-7]

Reference	Equation		Explanations
Terzaghi 1955 [3]	$k = k_{PLT} \cdot (\frac{B + B_{PLT}}{2B})^2$	for sands	k_{PLT} : Subgrade reaction modulus from the plate load test
	$k = k_{PLT} \cdot (\frac{B_{PLT}}{B})$	for clays	$B_{PLT} = \sqrt[2]{\frac{\pi \cdot D^4}{4}}$
			D : Diameter of plate
			B : Foundation width
Biot 1941 [4]	$k = \frac{0.95 \cdot E}{B \cdot (1-\nu^2)} \cdot \left[\frac{B^4 \cdot E}{(E_f \cdot I_f) \cdot (1-\nu^2)} \right]^{0.108}$		E : Elasticity modulus of soil
			ν : Poisson's ratio of soil
			E_f : Elasticity modulus of foundation material
			I_f : Moment of inertia of foundation
Vesic 1961 [5]	$k = \frac{0.65 \cdot E}{B \cdot (1-\nu^2)} \cdot \left[\frac{B^4 \cdot E}{(E_f \cdot I_f)} \right]^{0.083}$		
Meyerhof and Baikie 1963 [6]	$k = \frac{E}{B \cdot (1-\nu^2)}$		
Bowles 1996 [7]	$k = \frac{q_u}{s}$		q_u : Ultimate bearing capacity
			s : Allowable settlement

The German Institute for Standardization DIN 4018:1974 [13] suggests the value of relative stiffness K to estimate the behaviour of soil-foundation systems. For K-values ≥ 0.1, it can be considered rigid system while it behaves flexible for K-values ≤ 0.001.

$$K = \frac{1}{12} \cdot \frac{E_f}{E_s} \cdot (\frac{d}{L})^3 \tag{1}$$

where E_f is the Young's modulus of foundation material, E_s is the constrained modulus of soil, d is the thickness of foundation and L is the foundation length.

In this study, the effect of foundation dimensions and foundation material on the subgrade reaction modulus is investigated depending on the constrained modulus of soil for square-shaped foundations with uniformly distributed load. The analyses are performed by using the Software GGU-Slab [14], and the results of these analyses are compared with those obtained from the infinitely flexible cases by using the Software GGU-Settle [15], in which the vertical stress increase due to the contact pressure is calculated based on the theory of Boussinesq, and then the settlements are calculated using the conventional analytical method based on the theory of elasticity.

2 Numerical analyses

In the present work, the constrained modulus method is used to represent the interaction between the soil and foundation. The constrained modulus E_s corresponds to the stiffness modulus from the one-dimensional consolidation test, which is calculated by using Eq. (2):

$$E_S = \frac{\Delta\sigma_i}{\Delta\varepsilon_i} \tag{2}$$

where $\Delta\sigma_i$ is stress interval, $\Delta\varepsilon_i$ is strain interval corresponding to $\Delta\sigma_i$ assuming a linear behavior of soil within this interval.

The relationship between the constrained modulus and Young's modulus of the soil can be obtained by using Hooke's law and the boundary conditions in one-dimensional consolidation test as follow:

$$E_S = \frac{E \cdot (1-\nu)}{(1+\nu)\cdot(1-2\nu)} \tag{3}$$

where E and ν are the Young's modulus and Poisson's ratio of soil.

In the present study, the soil-foundation interaction is considered using the software GGU-Slab [14], which requires an iteration process for the numerical solution. In the first step of the analyses, the settlements in an elastic-isotropic half-space at all nodes of the finite element mesh resulting from a contact pressure of 1 kN/m² are calculated. For this purpose, the Boussinesq's equation is numerically integrated, and the contact pressure (1 kN/m² in the first step) is divided by the calculated settlements in order to determine the distribution of subgrade reaction modulus for each node. Subsequently, the foundation bending is calculated based on Eq. (4) that is solved by using the finite element method. Finally, the subgrade reaction modulus is varied in further iteration steps until the bending of the foundation corresponds to the settlement of soil in an elastic-isotropic half-space.

$$\frac{d^4 \cdot s}{dx^4} + 2 \cdot \frac{d^4 \cdot s}{dx^2 \cdot dy^2} + \frac{d^4 \cdot s}{dy^4} + (-q + k \cdot s) \cdot \frac{12 \cdot (1-\nu_f)^2}{E_f \cdot d^3} = 0 \tag{4}$$

where s is settlement, d is foundation thickness, E_f and ν_f the Young's modulus and Poisson's ratio of foundation material, q is contact pressure and k is subgrade reaction modulus.

Table 2 shows the parameters varied in this study. The square-shaped foundations with a uniformly distributed load of 50 kN/m^2 are considered on an elastic half-space with unit weight of γ = 19 kN/m^3. The key design parameters are foundation width B = 10 m, foundation thickness d = 0.5 m, Young's modulus E_f= 35000 MPa and Poisson's ratio ν_f = 0.2 as well as the unit weight of foundation material γ_f = 25 kN/m^3.

Table 2. Parameters varied in this study

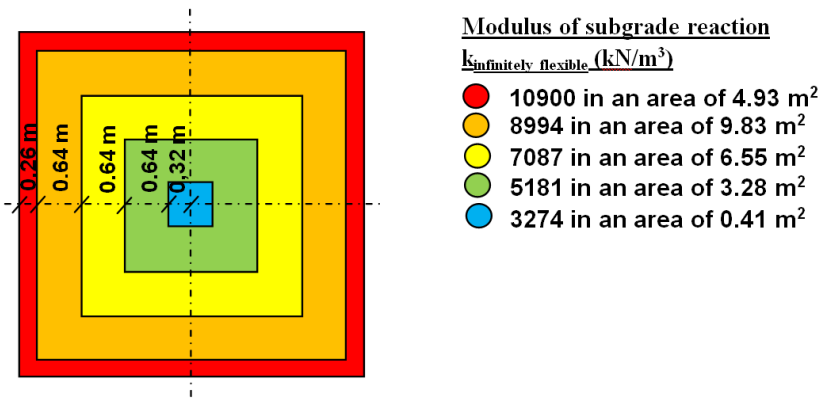
Constrained modulus of soil E_s (MPa)	Foundation width (= length) B (m)	Foundation thickness d (m)	Young's modulus of foundation material, E_f (MPa)
2.5 5.0 10 50 100	5/**10**[a]/20	0.25/**0.5**[a]/1.0	29000/**35000**[a]/41000

[a]The key design parameters

3 Results and discussion

In Fig. 2, the subgrade reaction moduli obtained from the GGU-Settle [15] are given for various foundation sizes with a uniformly distributed load of 50 kN/m^2 on a linear-elastic soil surface with E_s = 10 MPa and γ = 19 kN/m^3. Here, it should be noted that the thickness and Young's modulus of foundations are not important for infinitely cases. Furthermore, the subgrade reaction modulus for any value of E_s can be calculated by means of the values of $k_{\text{infinitely flexible}}$ given in Fig. 2 because the subgrade reaction modulus is directly proportional to E_s-values for a certain foundation size.

In Fig. 3, the subgrade reaction moduli obtained from the GGU-Slab [14] are given in the middle and corner areas of the foundations as well as in the middle of foundation edge for the soil-foundation systems in Fig. 2. In these analyses, the thickness and Young's modulus of soil are taken as d = 0.5 m and E_f = 35000 MPa. Taking into account of a uniform load of 12.5 kN/m^2 (= 0.5 m x 25 kN/m^3) resulting from the own weight of the foundations, the foundations modelled in GGU-Slab are loaded with a uniform load of 37.5 kN/m^2.



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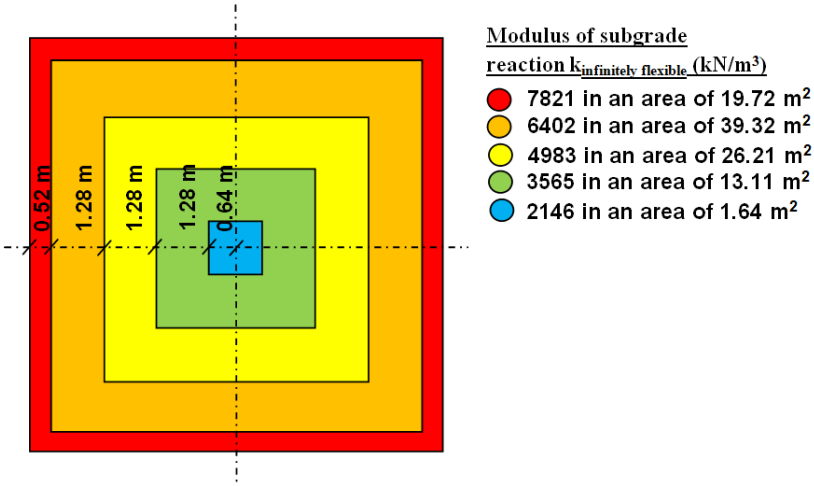

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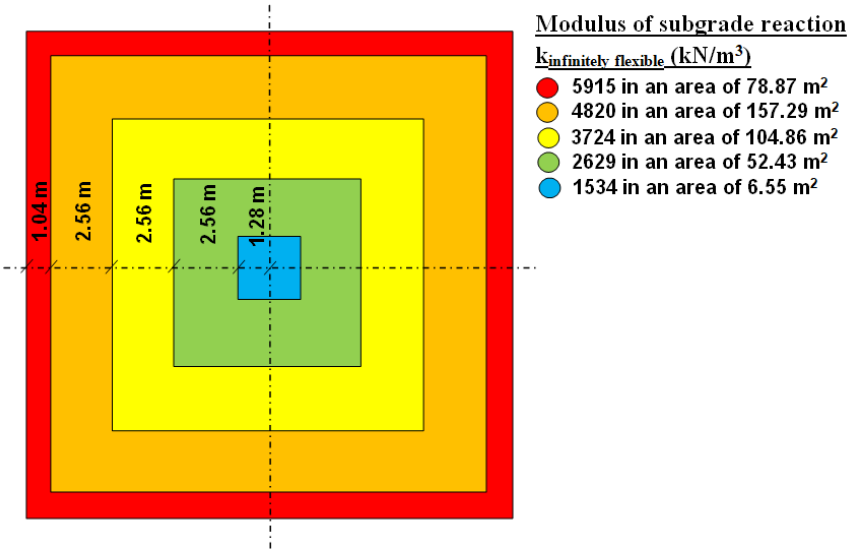

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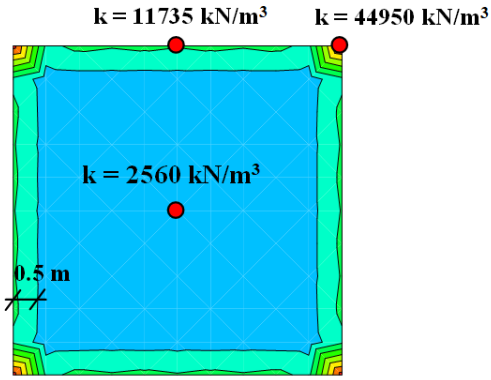
Fig. 2. Subgrade reaction modulus for infinitely flexible cases: a) B = 5 m, b) B = 10 m, c) B = 20 m



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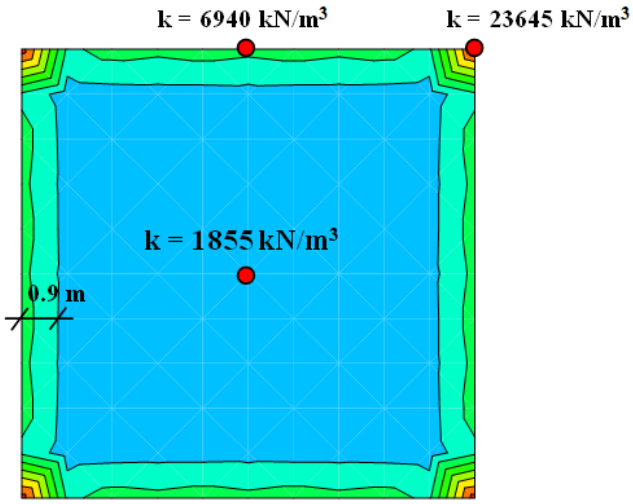

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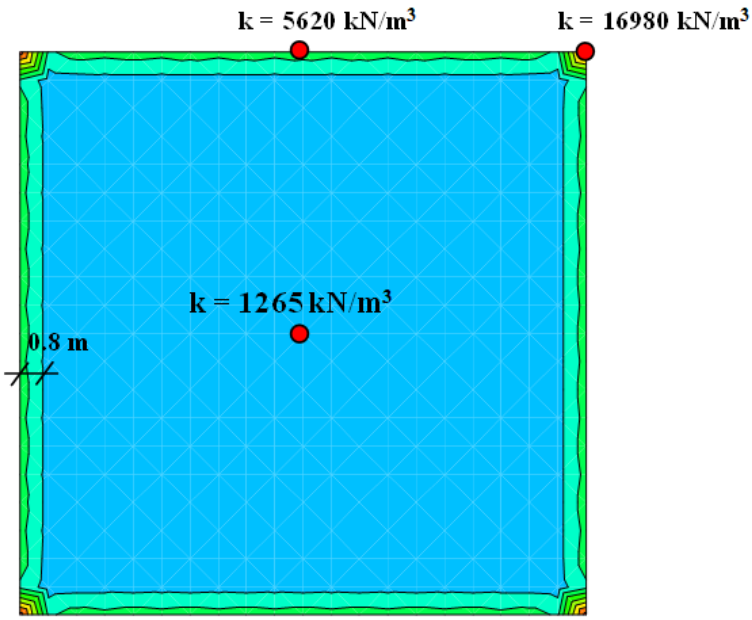

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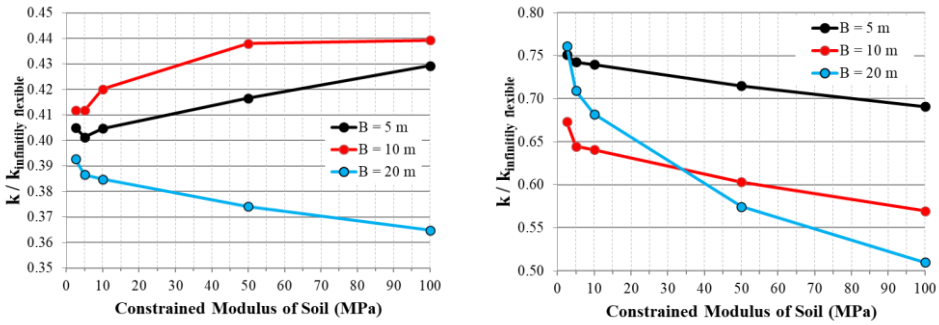
Fig. 3. Subgrade reaction modulus taking into account soil-foundation interaction: a) B = 5 m, b) B = 10 m, c) B = 20 m

In the infinitely flexible cases, the modulus of subgrade reaction increases step by step from the center to the outer edges of the foundations as shown in Fig. 2. However, in cases in which the soil-foundation interaction is considered, the values of subgrade reaction moduli are relatively constant except outer edges with a thickness 0.5 to 0.9 m.

To compare the results of both methods with each other, the average subgrade reaction moduli are calculated by using the areas giving in Fig. 2. The effect of foundation width B

on the k-value is shown in Fig. 4. In the analyses, the thickness and Young's modulus of foundations are taken as d = 0.5 m and E_f = 35000 MPa while the foundation width is varied as 5 m, 10 m and 20 m. As shown in Fig. 4, the ratio of the modulus of subgrade reaction obtained from the analyses taking into account the soil-foundation interaction to the subgrade reaction modulus in infinitely flexible cases $k/k_{infinitely\ flexible}$ varies between 0.36 and 0.44 in the middle area of foundations as well as 0.50 and 0.76 on the edges of foundations.



a) b)

Fig. 4. Effect of foundation width B on the k-value: a) in the middle of foundations, b) on the edges of foundations

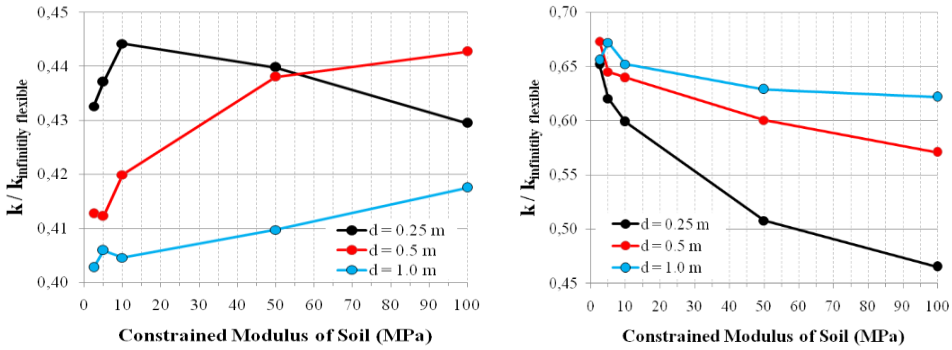
The relative stiffness K-values are calculated by using Eq. (1) and listed in Table 3. According to that, it is expected that the closest agreement between both methods, with other words the highest value of $k/k_{infinitely\ flexible}$, appears for the case with B = 20 m and E_s = 100, which was not confirmed by the results of numerical analyses above.

Table 3. Relative stiffness K-values of the soil-foundation systems with various B-values

Constrained modulus of soil E_s (MPa)	Relative stiffness K (-)		
	B = 5 m	B = 10 m	B = 20 m
2.5	**1.167**[a]	**0.146**[a]	0.018
5.0	**0.583**[a]	0.073	0.009
10	**0.292**[a]	0.037	0.005
50	0.058	0.007	**0.001**[b]
100	0.029	0.004	**0.0005**[b]

[a]Rigid system, [b]Flexible system

Fig. 5 illustrates the effect of foundation thickness d on the k-value. Here, the width and Young's modulus of foundation are taken as B = 10 m and E_f = 35000 MPa while the foundation thickness is varied as 0.25 m, 0.5 m and 1.0 m. The ratio of $k/k_{infinitely\ flexible}$ varies between 0.4 and 0.45 in the middle area as well as 0.45 and 0.67 on the edges of the foundations. The relative stiffness K-values are listed in Table 4. As in Fig. 4, there is no agreement between the numerical results shown in Fig. 5 and the relative stiffness K.



a) b)

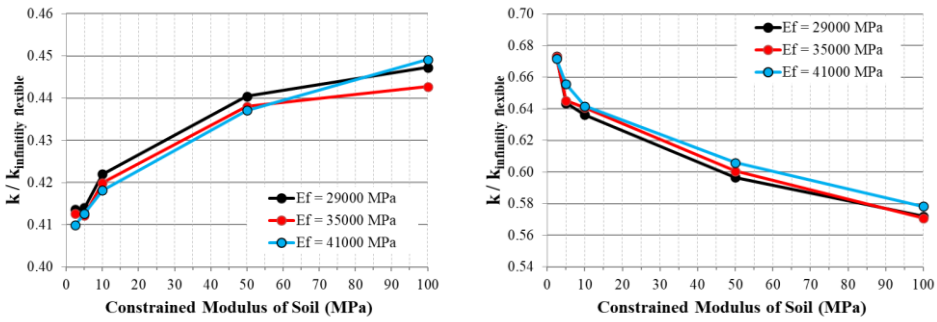
Fig. 5. Effect of foundation thickness d on the k-value: a) in the middle of foundation, b) on the edges of foundation

Table 4. Relative stiffness K-values of the soil-foundation systems with various d-values

Constrained modulus of soil E_s (MPa)	Relative stiffness K (-)		
	d = 0.25 m	**d = 0.5 m**	**d = 1.0 m**
2.5	0.018	**0.146**[a]	**1.167**[a]
5.0	0.009	0.073	**0.583**[a]
10	0.005	0.036	**0.292**[a]
50	**0.001**[b]	0.007	0.058
100	**0.0004**[b]	0.004	0.029

[a]Rigid system, [b]Flexible system

The effect of Young's modulus of foundation material E_f on the k-value is shown in Fig. 6. In the analyses, the width and thickness of foundations are taken as B = 10 m and d = 0.5 m while the Young's modulus of foundation material is varied as 29000 MPa, 31000 MPa and 41000 MPa. In Fig. 6, the ratio of $k/k_{infinitely\ flexible}$ varies between 0.41 and 0.45 in the middle area of foundation as well as 0.56 and 0.68 on the edges of foundation.



a) b)

Fig. 6. Effect of Young's modulus of foundation material E_f on the k-value: a) in the middle of foundation, b) on the edges of foundation

The relative stiffness K-values of the numerical models are given in Table 5.

Table 5. Relative stiffness K-values of the soil-foundation systems with various E_f-values

Constrained modulus of soil E_s (MPa)	Relative stiffness K (-)		
	E_f = 29000 MPa	E_f = 35000 MPa	E_f = 41000 MPa
2.5	**0.121**[a]	**0.146**[a]	**0.171**[a]
5.0	0.060	0.073	0.085
10	0.030	0.036	0.043
50	0.006	0.007	0.009
100	0.003	0.004	0.004

[a]Rigid system

4 Conclusions

In the present work, the effect of foundation dimensions and Young's modulus of foundation material on the subgrade reaction modulus is studied depending on the constrained modulus of soil for square-shaped foundations with uniformly distributed load. The results obtained from these analyses are compared with those in infinitely flexible cases. The following results can be drawn from the analyses performed in this study:

1. The Young's modulus of foundation material that varies between 29000 MPa and 41000 MPa has a relatively small effect on the k-value while the foundation thickness has a great effect on the k-value;
2. The equations that do not consider the foundation thickness, such as the equations of Terzaghi, Meyerhof & Beikie and Bowles, should be used carefully;
3. The equation suggested by DIN 4018:1974 is not sufficient to estimate the behaviour of soil-foundation systems;
4. The ratio of the subgrade reaction modulus in the middle area of foundations that is obtained from the analyses taking into account soil-foundation interaction to the subgrade reaction modulus in infinitely flexible case $k/k_{\text{infinitely flexible}}$ varies between 0.36 and 0.45 for foundations with a width between 5 m and 20 m as well as a thickness between 0.25 m and 1.0 m;
5. The higher values of k can be used on the edges of foundations. The ratio of $k/k_{\text{infinitely flexible}}$ on the edges of foundations varies between 0.45 and 0.76 for foundations with a width between 5 m and 20 m as well as a thickness between 0.25 m and 1.0 m.

In cases in which the layer thickness is relatively high compared to the foundation width, it seems to be the use of the Winkler's model is not suitable to determine the subgrade reaction modulus. Finally, it should be noted that further investigations are required with regard to the foundation shape and load distribution.

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