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Mathematical modelling and mechanics approaches in investigation of structural failure causes

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Abstract. An approach has been proposed for analyzing the causes of failure of parts, which is based on the use of mathematical modeling in calculating mechanical loads and strength parameters by finite-element analysis, followed by an assessment of the durability of the structure using methods of mechanics of damaged media that take into account the accumulation of structural defects in the material of products. The developed approach was used to calculate critical loads that led to the destruction of the crankshaft of a passenger car engine, and to establish the reasons for their appearance, and also allowed to propose technical solutions to eliminate such emergency situations.

1 Criticality of investigation of failure causes in expert studies

The provision of performance during the pre-set time interval at certain loads is essential at designing and functioning of mechanisms as well as assemblies of transport systems.

In case when parts, assemblies or mechanisms fail, engineering investigations are carried out in order to determine causes that led to their premature destruction [1].

The other vital task of expert investigations is the prevention of accidents i.e. offering recommendations on prevention of subsequent equipment emergency stop.

To overcome such problems, an increase in strength parameters and durability of individual parts and assemblies is required. Moreover, it is necessary to enhance safety and efficiency of machines and mechanisms through a more complete consideration of influence of all operational process factors.

The most promising ways to establish cause-and-effect relations leading to the breakdown and failure of parts is an integrated use of methods of computer modelling, mechanics of deformable solids and computational material science. Such approach is urgent in connection with the impossibility of reconstructing an emergency situation in the reality [2].

2 Generalized formulation of the problem of part failure analysis

Most often in the course of analysing failures of mechanical parts the following questions arise [3]:

1) whether the workmanship (repair) of a failed or damaged piece is in accordance with the regulations;
2) whether there are any defects in the material of a part indicating a violation of specifications for its manufacture (if any, what is their origin and impact on the failure);
3) whether there are mechanical damages on the part surface which are related to its failure (if yes, what is their origin (causes);
4) whether the technical condition of an assembly, in which a part operated, affected the occurrence of failure to a part;
5) the manner a part is destroyed (momentarily or during time interval);
6) the cause of part failure;
7) whether a part failed in connection with the accident (before, during or after the accident).

From the point of view of deformable solid mechanics, these problems pertain to the class of inverse ones [4].

Problems arising in the search for answers to questions 1, 2, 3 refer to inverse coefficient problems; to question 4 – to inverse boundary problems; to questions 5, 6, 7 – to inverse retrospective problems.

It should be noted that in general these problems can be nonlinear, non-correct and have no single solution.

3 Mathematical problematic relations regarding the evaluation of defect-free structure durability

In most cases, failure is understood as the breakdown of a part into pieces due to local critical values of stress-strain state [5].

This definition is quite suitable for the analysis of a part failure under prevailing static loads when the

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structure of material is considered as homogeneous and stable in time.

When intensive cyclic and dynamic loads come to the fore, failure is understood as a multi-stage process characterized by the following phases [6, 7]:

1) accumulation of individual local submicro- and micro-failures (submicro-failure sized approximately 0.0005-0.2 μm).
2) stages of irreversible damage accumulation when micro-defects sized from 0.2 to 10 μm occur, which leads to deterioration of serviceability and end-of-life of material resource;
3) assemblage of micro-structurally short fractures into one or several physically short fractures;
4) breakdown of a part due to further development of several macro-fractures (one macro-fracture), which are described by fracture mechanics methods.

Most often in computational mechanics the first two phases are simulated by introducing an additional variable, i.e. material damageability, which is associated with structural transformations in the material.

It is accepted that changing of the said value is subject to the following model relation [8]:

$$\frac{d\omega(x,\tau)}{d\tau} = f(\omega, \hat{\sigma}, T, \tau), \qquad (1)$$

where $\omega(x,\tau)$ − the damageability, $f(...)$ − the damageability control functional, T − the temperature, τ − the time, $\hat{\sigma}$ − the stress tensor, x − the investigated region point.

When the damage reaches the critical value ω^* in any region, it is assumed that the failure processes took place in that region:

$$\{x : \omega \geq \omega^*\}, x \in X, \qquad (2)$$

where X is a spatial region occupied by the part.

An intermediate (indirect) relations consideration (1) is also possible by entering the number of cycles (piece operation) under the constant loading conditions [6]:

$$N = F(\hat{\sigma}, T, M), \qquad (3)$$

where $F(...)$ − a given functional, M − the set specifying body material properties.

Type (3) relations are more convenient for engineering evaluation (computation) of the part operation parameters since they allow performing the evaluation of a structure without experimental verification of type (1) relations.

When type (3) relations are built up, the assumption of summation of the damage at its accumulation without the possibility of healing of the structure material during its operation is a priori accepted. This assumption, normally, somewhat lowers the calculated parameters of durability, but in general, allows estimating the time of operation of a part with a precision sufficient for engineering practice.

It is also possible to formulate an inverse problem:

$$\sigma - ? : N \geq N^*, \qquad (4)$$

or

$$M - ? : N \geq N^*, \qquad (5)$$

where N^* is a given value of the number of item operating cycles.

Apparently, at expert investigation of functioning of structures both direct and inverse statements of problems can be used, and their type is defined by the purpose of investigations.

3.1. Example of a failed part investigation.

As an object of our applied investigation, let us consider the failure of a crankshaft taken from a passenger car (Fig. 1).


a

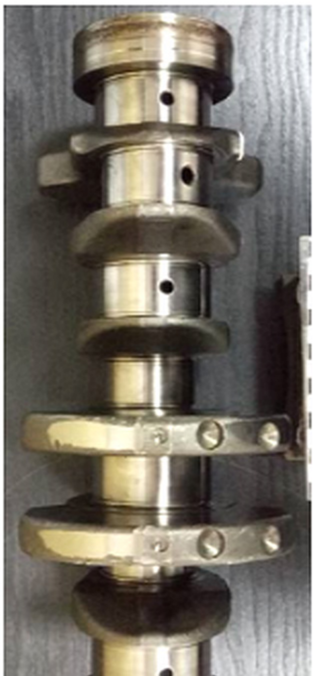
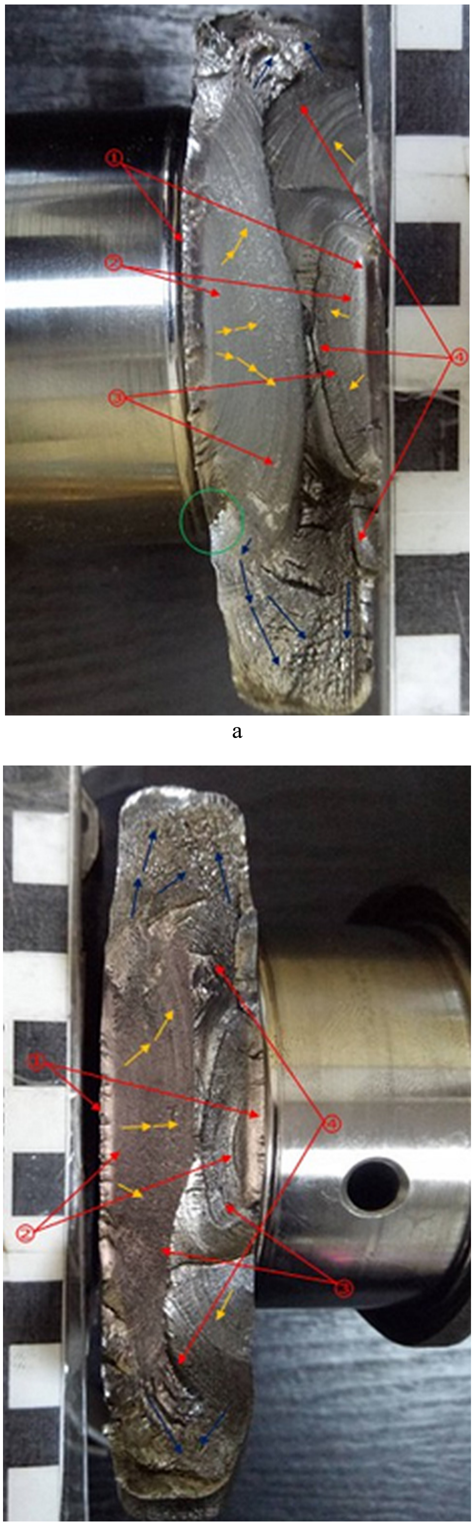

b

Fig. 1. General view of the failed crankshaft:
a – fragment 1, b – fragment 2

The detailed representation of fracture surfaces is shown in Fig. 2.



a

b

Fig. 2. General view and structure of the failed surface of fragments No. 1 (a) and No. 2 (b):
1 – smooth glossy areas; 2 – areas of fracture growth initiation; 3 – areas of accelerated fracture growth; 4 – breakdown area:
→ - direction of fracture stopping lines front distribution;
→ - direction of ridge distribution in the breakdown area.

One of specific features of expert problems related to softening and failure of structures is almost complete lack of possibility to reproduce running conditions in situ before the failure originates, as well as to conduct expensive experiments for stuck-at model loading.

In this paper we will try to find an answer to the applied problem of expert investigations, namely to question No. 6 "what is the cause of a part failure".

We will solve this problem only using the data provided for the investigation.

Let us consider the CAD-model of a shaft (fig. 3), constructed in the system of solid-state modelling.

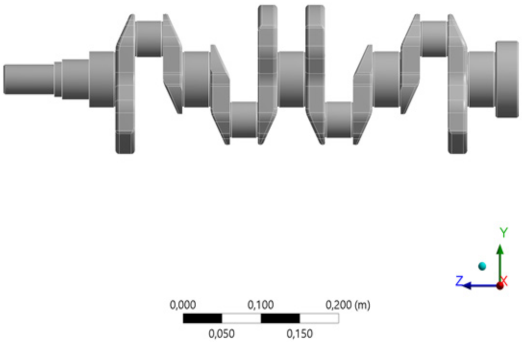


Fig. 3. CAD model of a crankshaft

Let us force load the shaft according to the pattern shown in Figure 4. Let us set the support loads in areas B, C, D, E, the values of which are equivalent to the computed operational ones.

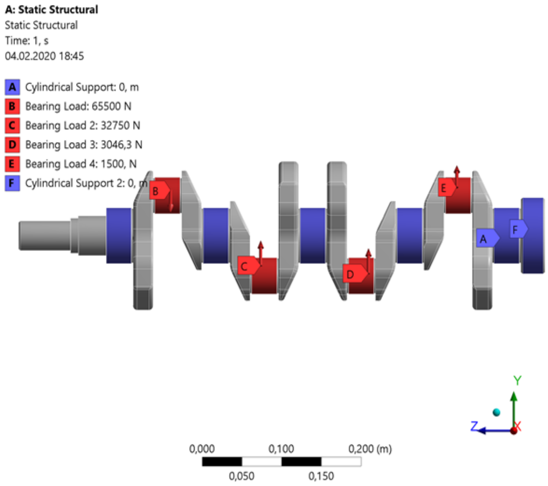


Fig. 4. Load pattern of a crankshaft

After we split the CAD-model into finite elements (Fig. 5), using the elastic media mechanics model, let us calculate the stress-strain state of the structure (Fig. 6, Fig. 7, Fig. 8).

The above calculations show that the most significant stresses and deformations occur in shaft regions where a failure occurred.

Since the shaft was running under stationary loading conditions, we calculated the number of cycles N using an experimental approximation of type (3) dependence [9]:

$$N = \left(\frac{\sigma_B}{\sigma_{\max}}\right)^{\frac{1}{n}}, \qquad (6)$$

where σ_B − the strength limit, $\sigma_{0.2}$ − the yield limit, $\sigma_{\max}$ − the maximal load, $n = 0.042 \cdot \left(1 - \frac{\sigma_{0.2}}{\sigma_B}\right)$.

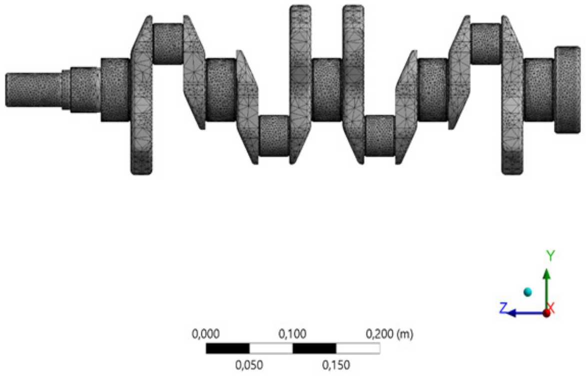


Fig. 5. Finite-element splitting of a shaft model

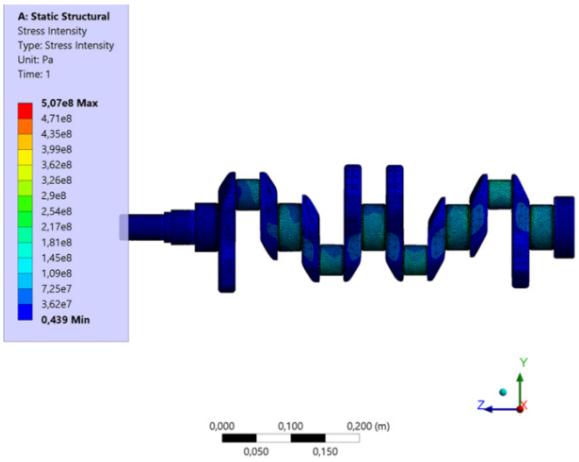
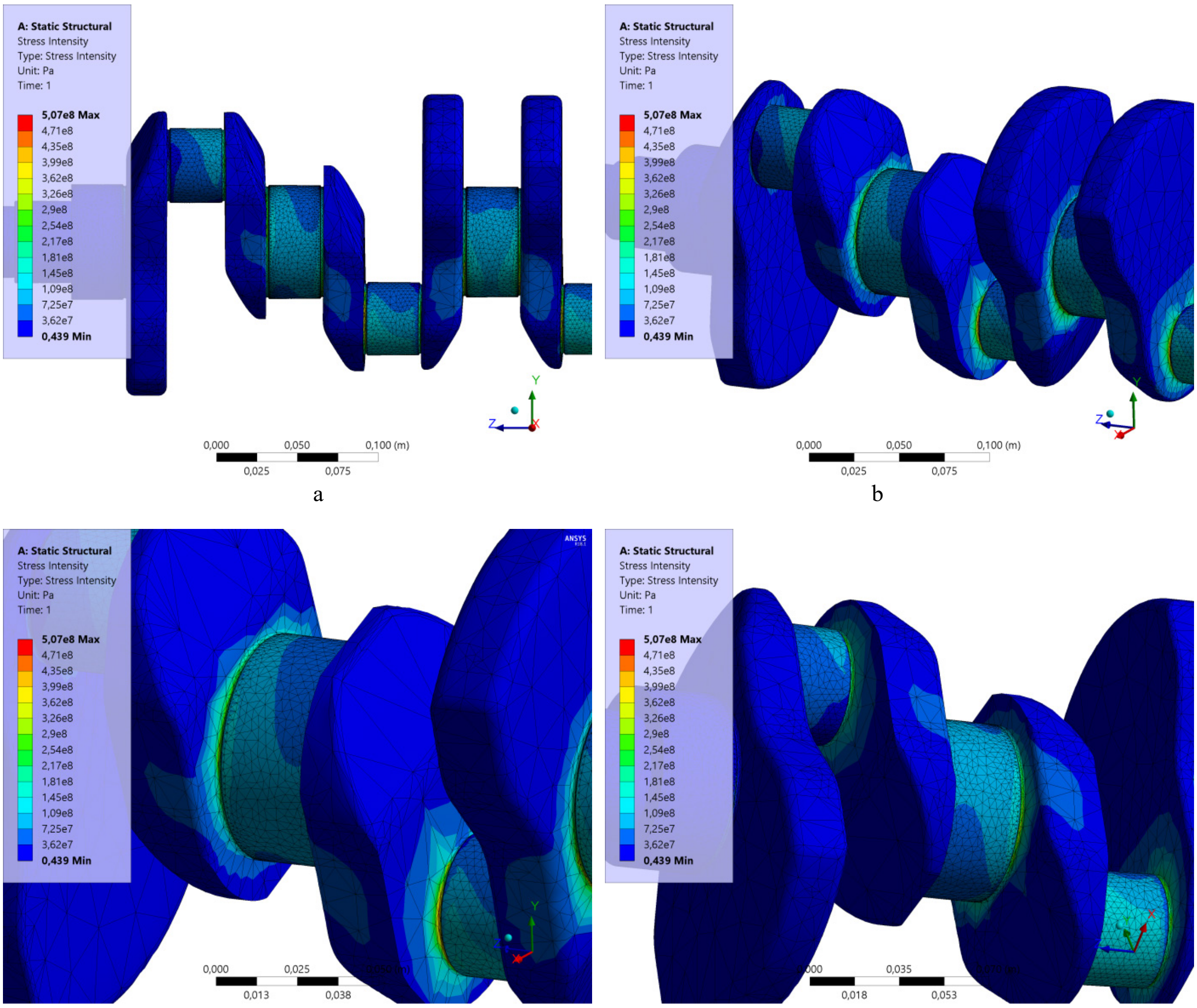


Fig. 6. Distribution of stresses in a shaft

Experimental investigations have shown that the shaft was made of steel 30HNMA (international equivalent is steel 4130) with following characteristics of mechanical properties: $\sigma_B = 780 \cdot 10^6\ Pa$, $\sigma_{0.2} = 590 \cdot 10^6\ Pa$. The maximum value of the computed load is $\sigma_{\max} = 507 \cdot 10^6\ Pa$ (Fig. 6).



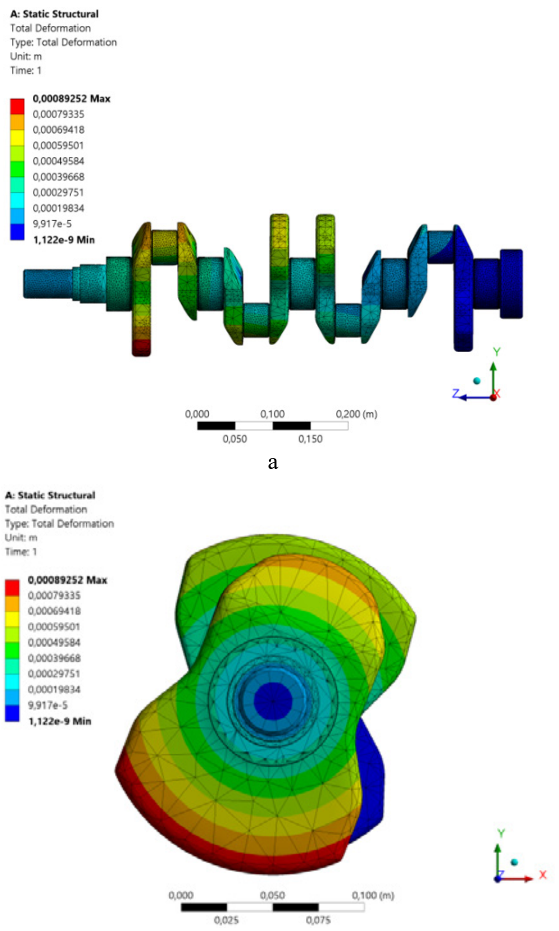
a

b

c

d

Fig. 7. Distribution of stresses in shaft fragments (a-d)



a

b

Fig. 8. Distribution of deformations in a shaft (a, b)

According to the relation (6) the total number of cycles preceding the part failure is equal to N=1.9·10^{18}.

Since according to the data provided for the investigation the actual shaft running time is significantly lower than the computed data (about 20,000 cycles), it can be concluded that the expected loads on the shaft are more severe.

Let us calculate the value of maximum stresses at which the structure durability is 20000 cycles (for this purpose we use relations being inverse to formula (6):

$$\sigma^*_{\max} = \frac{\sigma_B}{\left(N^*\right)^n}, \tag{7}$$

where $\sigma^*_{\max}$ is the computed stress value at which N^*=20000.

This results in the following:

$$\sigma^*_{\max} \approx 705 \cdot 10^6 \text{ Pa}. \tag{8}$$

As is obvious, an increase in maximum stresses by approximately 40% leads to a manifold reduction in the structure durability. This is especially apparent when maximum stresses approximate to the material strength parameters.

In this connection, it is possible to draw an important practical conclusion that there may have been following causes of premature crankshaft failure:

1. faulty condition of an assembly containing the running shaft which causes excessive loads on the shaft;
2. occurrence of surface defects that act as concentration spots causing additional stresses and deformations in shaft regions where the failure occurs.

In order to prevent premature failure of the shaft, it is necessary to carefully check the surface condition before installing the shaft, as well as to control the process of mounting of parts into assemblies.

4. Conclusions.

1. The analysis of issue of structural failure from the standpoint of computational mechanics has been considered in the paper.
2. Basic types of problems, which the investigators face in the process of establishing causes of part failures, have been described as well as mathematical statement of a problem regarding the structural durability evaluation has been presented.
3. A computational and analytical pattern of the analysis of the passenger car engine shaft long-term softening has been suggested.
4. The possible causes of premature failure of a part have been revealed and the ways of preventing such type of accidents have been suggested using a finite element method and experimental-analytical dependences of the structural design durability.

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