Analysis of lateral forces at the wheel-rail contact in cases of derailment Incidents

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Abstract. This article is dedicated to a thorough exploration of dynamic interactions within the intricate "wheel-rail" system during the occurrence of rolling stock derailments – a prominent and challenging aspect in the realm of railway transportation. Through meticulous analysis, the multifaceted factors influencing rolling stock derailments are meticulously dissected, embracing an in-depth scrutiny of dynamic interaction calculations. The investigation encompasses a comprehensive examination of various methodologies for comprehending the lateral forces that travel from the wheels of rolling stock to the rail track. By applying these methodologies, the study extrapolates insights from a real-world railway accident that transpired on Ukrainian territory. The research findings shine a light on a pivotal factor amplifying lateral forces: the gradual wear and tear of wheel flanges, coupled with the broadened gaps between axle boxes and the side frames of bogies. This intricate interplay ultimately leads to an augmentation in the angles at which wheel sets approach the rail track. Consequently, the research underscores the pressing need for an ongoing commitment to further exploration and the evolution of methodologies – a necessary endeavor to bolster rail safety comprehensively.

1 Introduction

When investigating railway incidents, such as cases of rolling stock derailing from the rail track, calculations of dynamic interaction within the "wheel-rail" system take precedence. Without these calculations, in most cases, it is not possible to establish the sequence of intermediate technical causes that serve as influential factors in relation to the circumstances of the derailment of rolling stock. Additionally, searching for factors that influenced the rail track derailing process, and under which the derailing might not have occurred, is necessary to determine the necessary and sufficient condition for its onset – in other words, to answer the question of the potential prevention of a railway incident. Depending on the specific conditions and circumstances of the railway incident, we can employ mathematical and computer modeling, computer simulations, and practical experiments.

The utilization of various approaches and research methods not only enhances result accuracy but also renders the analysis more flexible and tailored to the particular case.

2 The main part

2.1 Analysis of research and publications

Many works are devoted to the investigation of the dynamics of rolling stock and cases of its motion instability. Let’s take a look at several of them.

In the work [1] the influence of longitudinal quasi-static compressive forces on the form of instability of freight wagons is investigated. The relevance of this study is associated with the need to control the magnitude of the longitudinal force. The methods proposed by the author in the work will improve the stability of freight rolling stock, justify the cause of the derailment of wheel sets, as well as develop and implement technical measures aimed at preventing the lift of the carriages, widening and transverse shear of the track.

The author of the work [2] proposes a procedure for theoretical and experimental studies on the influence of rolling stock on the railway track. The main aspects and stages are considered, as well as the mathematical support for the implementation of these studies is described.

The article [3] considers the most common cause of derailment of the rolling stock of industrial transport, establishes criteria for determining the critical states of the track in conditions of safe traffic. A method for quantitative assessment of the traffic safety of industrial railways using the apparatus of reliability theory is proposed.

In the work [4], the author conducted a study of the influence of an increase in the axial load on gondola wagons, taking into account a possible increase in the speed of movement, on their main dynamic indicators and indicators of the interaction between the rolling stock with the railway track. The study was carried out by the method of mathematical and computer modeling of the dynamic loading of a gondola wagon using a model of spatial oscillations of a five-wagon coupler and a software package developed in an industry research laboratory of rolling stock dynamics and strength.

Doi H et al. [5] conducted a comprehensive study focusing on instances of flange climb derailments
occurring in sharp curves or curves within turnouts, particularly after wheel truing processes. The research delved into the intricate interplay between the safety of vehicle operation and the conditions of the wheel surfaces.

The work [6] involved the simulating the movement of freight wagons with various bogie design configurations along curved sections of the track. The analysis of the results obtained showed that the use of devices that ensure the radial installation of wheelsets has a positive effect on the processes of interaction between wheels and rails. If we talk about works that are more focused on investigating railway accidents, then we should highlight the following studies.

The work [7] considers, compares, and analyzes various methods for determining the lateral force that arises when a locomotive wheelset rolls onto a rail in a circular curve. The work [8] proposes to use as a criterion for the derailment of a freight wagon the time during which the static load on the wheel will be less than the load at which the process of rolling of the wheel onto the rail head begins. Provided that the present time is longer than the time during which the wheel flanges can rise to the rail head, the rolling stock derails from the track. This criterion allows, in addition to the parameters of the undercarriage of freight wagons, to take into account deviations in the technical condition of the rail track from the maintenance standards.

The fundamental works devoted to investigations of real cases of railway accidents are the works of Sokol E.M. [9, 10], in which analytical expressions were derived to determine the necessary conditions for the derailment of rolling stock based on the developed mathematical model of the movement of the wheelset of a freight wagon.

Research in works [1-6] and works [7-10] are different. In works [1-6], the dynamics of rolling stock and the stability of its movement have been extensively studied, accompanied by proposed approaches for preventing railway accidents. On the other hand, works [7-10] primarily address the inverse problem, where a railway transport accident has already transpired, necessitating the determination of the circumstances and factors contributing to its occurrence.

2.2 Setting objectives

The paramount objective of addressing the safety concerns pertaining to the movement of rolling stock is to identify and establish effective methodologies that can comprehensively describe the intricate dynamic processes intrinsic to the motion of rolling stock on railway tracks. These methodologies are expected to not only provide qualitative and quantitative insights into these processes but also to replicate with precision the actual behavior of vehicles traversing real track sections. This endeavor is driven by the imperative to enhance the safety standards of railway operations and safeguard the lives of passengers and the integrity of cargo transportation.

By delving into the fundamental principles that govern the interaction between rolling stock and railway tracks, this research endeavors to bridge the gap between theoretical models and practical applications. The developed methods should not only elucidate the underlying mechanisms of rolling stock behavior but also offer a predictive framework capable of anticipating potential challenges and hazards.

Moreover, in an increasingly dynamic and evolving rail transport landscape, it is imperative that these methodologies are adaptable to a diverse range of scenarios. The multifaceted nature of railway systems necessitates a holistic perspective, encompassing variables such as track geometries, vehicle specifications, operational conditions, and environmental factors. Thus, the established objectives seek to propel the study beyond theoretical abstraction into the realm of practical applicability, effectively bridging the gap between research and real-world rail safety enhancement.

2.3 Presentation of the main material

2.3.1 Analytical expressions for determining the transverse forces acting on the rails for wheel sets of wagons

To explore the forces transferred from wagon wheel pairs to the rails, we employ the methods outlined in [10].

The lateral force, transmitted from the wheels to the rails, is determined by the formula:

\[ N_B = 2Y_f + v \cdot \sin \gamma \cdot \sqrt{C_y \cdot m}, \quad (1) \]

- \( Y_f \) – frame force, kN;
- \( v \) – train speed, m/s;
- \( C_y \) – transverse horizontal stiffness of rails, kN/m²;
- \( \gamma \) – angle of wheel flange rolling onto railhead, rad;
- \( m \) – wheelset weight, kN.

The angle of wheel flange rolling onto railhead will be determined by the following formula:

\[ \gamma = \gamma_1 + \gamma_2, \quad (2) \]

- \( \gamma_1 \) – angle of incursion arising due to the presence of a gap in the rail track, rad;
- \( \gamma_2 \) – the angle of incursion that occurs due to the non-parallelism (misalignment) of the bogie axle, rad.

The angle of incursion arising due to the presence of a gap in the rail track is determined by the formula:

\[ \gamma_1 = \frac{\delta}{L} \quad (3) \]

- \( \delta \) – gap in the rail track, m;
- \( L \) – bogie base, m.

\[ \gamma_1 = \frac{\Delta}{(b_1+b_3)} \quad (4) \]

- \( \Delta \) – gap between the letter and the slot in the side frame of the bogie, m;
\[ Y_f = m \cdot l \cdot \text{ctg} \cdot (\beta + \beta_1 \cdot t + \theta \cdot \xi), \]  
(5)

\[ l = \text{half the distance between the inner faces of the wheels of the wheelset, } mm; \]
\[ \alpha = \text{angle of inclination of the rolling surface of the wheel to the vertical, } rad; \]
\[ \theta = \text{wheelset nutation angle, } rad; \]
\[ \beta, \beta_1, \xi = \text{coefficients.} \]

Wheelset nutation angle when determining the frame force by the formula (5) is determined by the formula:

\[ \theta = \frac{0.5 \cdot \text{tg} \cdot (\Delta - \Delta_0)}{l}, \]  
(6)

The coefficient \( \beta \) is calculated using the formula:

\[ \beta = \frac{M \cdot g \cdot \text{tg} \cdot (P_1 + P_2 + G)}{m \cdot l \cdot r}, \]  
(7)

\[ M = \text{weight per wheelset, } m; \]
\[ h = \text{rail head elevation, } m; \]
\[ g = \text{acceleration of gravity, } H \cdot m/c^2; \]
\[ S = \text{the distance between the rolling circles of the wheels, } m. \]

\[ \beta_1 = \frac{v \cdot \text{tg}^2 \cdot \eta \cdot (P_1 + P_2 + G)}{m \cdot l \cdot r}, \]  
(8)

\[ \mu = \text{coefficient of friction;} \]
\[ P_1, P_2 = \text{load acting on the axle box of the wheel pair, } kN; \]
\[ G = \text{the weight of the wheel pair, } kN; \]
\[ r = \text{wheel radius, } m. \]

\[ \xi = \frac{v^2}{m \cdot l \cdot r^2} \left[ -J_k \cdot \text{tg}^2 \alpha + M \cdot r \cdot l \cdot \text{tg} \alpha \right], \]  
(9)

\[ J_k = \text{moment of inertia of the wheelset relative to the axis passing through its center of mass perpendicular to the plane of the cross-section of the rail track, } l^2 \cdot m^2. \]

Now consider an expression that takes into account the change in frame force overtime:

\[ Y_f = M \cdot \left( \frac{v^2}{r} - \theta - \frac{h}{S} \right) + r \cdot (\Delta - \Delta_0 \cdot \Delta), \]  
(10)

\[ \Delta = \frac{h_2}{k^2} + \left( \frac{h_0 - h_2}{k^2} \right) \cdot \text{cos} \cdot \left( \frac{2 \gamma_0 - h}{k^2 - \gamma^2} \cdot \frac{p}{k} \right) \cdot \text{sin} \cdot \gamma + \]  
\[ + \frac{h}{k^2 - \gamma^2} \cdot \text{sin} \cdot \gamma, \]  
(12)

\[ k^2 = \text{the frequency of free vibrations of the body, } s^{-1}; \]
\[ p = \text{the frequency of forced vibrations of the body, } s^{-1}; \]
\[ \Delta_0 = \text{the initial value of the angle that determines the position of the load in the process of rolling.} \]

\[ k = \frac{2b^2 \cdot c - \text{cos} \cdot \gamma \cdot r}{\text{cos} \cdot \gamma \cdot r + \frac{S}{f}}, \]  
(13)

\[ b = \text{half the distance from the point of application of the force acting on the axle box to the middle of the wheelset axle, } m; \]
\[ \psi = \text{the angle determined from the ratio of the rail head elevation to the distance between the rolling circles of the wheels, } rad; \]
\[ c = \text{suspension stiffness of the bogie, } kN/m^2. \]

The frequency of forced vibrations of the body is determined by the formula:

\[ p = \frac{2\pi v}{L}, \]  
(14)

\[ L = \text{the length of the irregularity of the rail track, } m. \]

In the formula (12), \( r_1 \) and \( h \) are determined by the formula:

\[ h_1 = \frac{\sin \psi \cdot g}{\text{cos} \cdot \gamma \cdot r + \frac{S}{f}}, \]  
(15)

\[ h = \frac{2b \cdot c - \eta}{\text{cos} \cdot \gamma \cdot r + \frac{S}{f}}, \]  
(16)

\[ \eta = \text{the height of irregularity of the rail track, } m. \]

Taking into account the expression (10), the side force will be determined by the formula:

\[ N_y = Y_f \cdot \left( 1 + \frac{c \cdot v \cdot m \cdot (l \cdot \text{tg} \alpha - l \cdot \text{sin} \alpha)^2}{v^2} \right), \]  
(17)

In this case, the load on the right and left wheels of the wheelset, taking into account the expression (11) can be determined by the formula:

\[ P_1 = P_{st} - \Delta, \]  
(18)

\[ P_2 = P_{st} + \Delta, \]  
(19)

\[ P_{st} = \text{static load per wheel, } kN. \]

Using formulas 18 and 19, it is possible to establish the actual value of the load that fell on each wheel until it was rolled onto the rail head when the derailment of rolling stock.

The influence of the lateral force is opposed by the resistance of the rail-sleeper grid, which is determined by the following formula:

\[ F = (A_1 \cdot y_{ms} \cdot N \cdot m_2 + A_1 \cdot y_{ms} \cdot l_{ms} + A_2 \cdot y_{ms} \cdot h_{ms}) \cdot k_2, \]  
(20)
• $A_p, A_\alpha, A_\beta, m_{1-6}$ – empirical coefficients, depending on the type of ballast and the type of sleepers;
• $l_a$ – the width of the arm of the ballast prism, $m$;
• $h_f$ – the height of the backfill of sleeper boxes, $m$;
• $k_d$ – the coefficient that takes into account the relative decrease in resistance along the base of the sleeper in the presence of a dynamic vertical load;
• $N$ – the total static load transferred to the sleeper from the wheels of the wheelset in the direction of the z axis, $kN$;
• $y$ – the movement of the sleeper under the action of applied forces, $m$.

2.3.2 The results of the calculations performed in the study of a real case of derailment of wagons from the rail track

The exploration of frame and lateral forces will be based on an actual railway accident that occurred within the territory of Ukraine. This incident involved the derailment of freight wagons from the 27th to the 31st position from the front of the train on a curved section of the track. All train wagons were fully loaded. The parameters of the running gear for these wagons were acquired through measurements taken after the incident at the wagon depot.

The calculation of frame and lateral forces transmitted from the wheels to the rails will be carried out for the wagons numbered 25th to 31st from the front of the train. Calculations will not be conducted for the wagons numbered 1st to 24th or from the 32nd to the end of the train. This decision is based on the fact that the departure of these wagons from the rails had already occurred as a result of the progression of the railway accident. We will calculate the frame and lateral forces according to the previously given formulas in the case when the frame force does not change and changes with time. The results obtained are presented in fig. 1 and 2.

![Fig. 1](image1.png)

Fig. 1. The frame force transmitted to the rails from the wheelsets of the wagons is calculated by two methods.

![Fig. 2](image2.png)

Fig. 2. The lateral force transmitted to the rails from the wheelsets of the wagons is calculated by two methods.

As evident from fig. 1 and 2, it is apparent that the lateral force on the first wheelset of the 28th carriage, numbered 28th from the front of the train, exceeded the permissible limit of 120 kN [11]. Therefore, we will focus in detail on this specific wheelset. The occurrence of the lateral force on the first wheelset of the 28th carriage can be attributed to the wear of the wheel flanges and the presence of increased angles of approach due to gaps between axle boxes and frames of the bogie.

Also, for the first wheelset, we set the value of the lateral force, which could be with different technical conditions of the running parts of the 28th wagon (fig. 3).

We conditionally provide the following conditions: unsatisfactory technical condition, characterized by wear of the running gear elements below the permissible standard value, current technical condition - in which the lateral force value of 128 kN was obtained, good technical condition, characterized by wear of the running gear at a level above the minimum allowable, near-excellent and excellent technical condition.

As can be seen from fig. 3 the deterioration of the technical condition of the running parts of the wagon leads to a sharp increase in the lateral force transmitted from its wheels to the rails.
In accordance with [10, 11], to roll a wheel onto the rail head, it is necessary that the load \( P \) on this wheel remains below a specific threshold \( P^* \) (fig. 4). This threshold is the load at which the flange transitions onto the rail head. This load is determined by a formula \( \text{(21)} \).

The ultimate confirmation of the derailment is that the time \( \Delta t \) during which the load is lower than this threshold is sufficient for the flange to move onto the rail head and roll into the inter-rail space.

\[
P^* = \frac{N_B b_2 + Y_f r - G l}{b_1 + b_2}.
\]  

\( \text{(21)} \)

Table 1 presents the loading values on the left wheels of the four wheelsets of the wagon. Upon analyzing the acquired data, it becomes apparent that during the traversal of rail track irregularities, the unloading of the wheels is insufficient for rolling of any left wheel of the wheelsets on the 28th wagon onto the rail head. In other words, within the context of this railway accident, the derailing of the wagon from the rail track due to the unloading of one of its wheels was not possible.

**Table 1.** Calculated values of loads on the wheelset

<table>
<thead>
<tr>
<th>( N_{\text{wh}} )</th>
<th>( P_{\text{min}}, \text{kN} )</th>
<th>( P^*, \text{kN} )</th>
<th>( \Delta, \text{kN} )</th>
<th>( P_{\text{stwh}}, \text{kN} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>57,7</td>
<td>21,9</td>
<td>44,11</td>
<td>101,81</td>
</tr>
<tr>
<td>2</td>
<td>57,73</td>
<td>22,015</td>
<td>44,08</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>57,73</td>
<td>21,99</td>
<td>44,08</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>57,74</td>
<td>21,98</td>
<td>44,07</td>
<td></td>
</tr>
</tbody>
</table>

During further research, the resistance force to the transverse shear of the rail-sleeper grid (formula 20) was explored. Figure 5 illustrates the variation in total resistance to the transverse shear of the rail-sleeper grid caused by influence of wagon wheels. Additionally, fig. 4 displays the maximum lateral force value at the contact point of each wheel from these wagons with the rail track. As depicted in fig. 5, the required condition for shifting the rail-sleeper grid beneath the 28th wagon not met. However, in our case, the lateral force from the first wheelset could contribute to the transverse shear of the track under the wheelsets of the 29th carriage.
Confirmation of the transverse shear of the rail-sleeper grid came from an investigation of the technical condition of the rail track. This investigation enabled the identification of a weakened section in the rail track where the derailment of the train wagons occurred. Through a detailed analysis the trace picture of the derailment and the mode of train's movement, it was determined that the first wagon to derail was the 29th wagon. In this wagon, the first bogie on the left derailed. The transverse shear of the rail-sleeper grid occurred due to the influence of lateral forces from the wheel pairs of the 28th wagon of the train (a transverse force exceeding 120 kN) and the transverse components of the temperature forces acting on the rail string. This happened within the boundaries of the corresponding rail section which exploited with creeping of rail. This was evidenced by the existing traces of the terminals on the sole of the rails more than 30 mm, the absence of a rail spacer, warped sleepers, transverse shear of strokes on the screed sleepers - 290 mm. The emergence of an elevated lateral force from the wheels of the bogies of the 28th wagon resulted from their technical condition, as mentioned earlier. The occurrence of increased lateral force took place in the most weakened, but still presumably able-bodied place of the railway track.

2.4 Discussion of research results

The circumstances surrounding a real railway accident that occurred on the territory of Ukraine are considered. It is important to note the key aspects identified during the study.

Examining dynamic interactions within the "wheel-rail" system helped identify factors contributing to the derailment of the rolling stock. For the 28th wagon, the lateral force from its first wheelset exceeded the allowable value of 120 kN and amounted to 125 kN. Although the condition for the transverse shear of the rail-sleeper grid was not fulfilled. This made it possible to choose the direction of further research and establish the reasons for the descent using other data.

That's why special attention was given to investigating the technical conditions of both the rolling stock and rail track.

An additional analysis of the structure and technical condition of the rolling stock made it possible to establish that the most likely source of lateral forces is the wear of the wheel flanges of the wheel pairs of bogies. Additionally, the presence of increased angles of wheel flanges rolling onto the rails, resulting from increased gaps between the axle boxes on the side frames of the bogies. The occurrence of increased lateral force occurred in the most weakened, but still presumably able-bodied place of the railway track.

However, it's important to note that the analysis of the study results highlights the importance of employing diverse methods and approaches during calculations. Such a practice facilitates attaining more precise and comprehensive outcomes. The choice of the most fitting method should be contingent on specific circumstances and incident characteristics.

3 Conclusions

On a specific example, the contact forces in a pair of wheel-rail were investigated. In the course of refined calculations, it was possible to establish the value of the frame force, which amounted to 60 kN, the lateral force - 125 kN. However, these calculation results did not allow us to say with sufficient accuracy about the causes of the accident, since the condition for rolling the wheel onto the rail head and displacing the rail-sleeper grid was not fulfilled. At the same time, the results obtained made it possible to choose the direction of further research to establish the intermediate causes that led to the direct cause of the derailment of the rolling stock.

The findings of the study highlight the intricate and multifaceted nature of the processes that contribute to the occurrence of rolling stock derailment incidents. This understanding underscores the necessity of continued research and the development of advanced methodologies to comprehensively analyze the factors leading to such events.

Bibliography