Calculations of emergency response capabilities as parameters of the queueing system

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Abstract. The article defines as a scientific problem that requires research the development of a sufficiently simple (for use "in field conditions") methodology for calculating the capabilities of forces and means of response to man-made, natural and military emergency situations. It is noted that many researchers used the mathematical apparatus of the queueing theory, as well as process modelling with the help of appropriate software, for the theoretical analysis of similar tasks. It is shown that to solve the problem, it is possible to use a mathematical model of the M/M/1 type queueing system (QS), which takes into account the probabilistic nature of the processes of occurrence of emergency situations and response to them, while it was supplemented with an analytical method of calculating the parameters of the QS, which ensure the necessary capability of the forces and means of emergency response. The results of the study are suitable for practical use by emergency services to quickly determine the required number of rescue or repair crews, sanitary teams, vehicles, etc., their equipment and capabilities in responding to emergency situations and overcoming their consequences.

1 Problem statement

One of the branches of modern logistics is humanitarian logistics, which specializes in the organization of delivery and storage of humanitarian aid items during natural disasters or emergency situations for the affected area and population. The field of humanitarian logistics also includes the priority evacuation of the population, which is threatened by various dangers, victims and wounded during emergency situations and military actions. It is clear that none of these tasks can be performed without reliable transport service.

Emergency situations of man-made and natural origin require quick response to them and elimination of their negative consequences for people, environment and infrastructure facilities as soon as possible. The state of constant preparedness for this requires all necessary measures to prevent them. But, in the event of their occurrence, the availability of forces and means of response in sufficient quantity, including appropriate vehicles both for the evacuation of victims from the disaster area and the delivery of everything necessary for providing humanitarian aid and overcoming its consequences. In this connection, there is a problem of reasonable methods of determining the type, number and capabilities of forces and means of response to emergencies. Given the stochastic nature of both the occurrence of emergency situations and the processes of responding to them, queueing theory methods are traditionally used as scientific tools for solving this problem, supplemented by mathematical modelling tools using appropriate applied software. Optimization methods are widely used, where both natural (time, distance) and economic indicators are used as criteria.

Russia's large-scale war against Ukraine led to an avalanche-like increase in the intensity of flows and the geography of emergency situations. Combat operations, evacuation of the wounded, shelling, rocket and bomb attacks on civilian infrastructure joined the accidents in transport and industry, floods, earthquakes and snowstorms were. These tragic events are emergencies in their extreme form, to which it is necessary to respond and make decisions in conditions of extremely limited resources, lack of time, which in itself is often a criterion for decision-making. For example, when evacuating wounded from the battlefield, or injured civilians from of a destroyed house when the second wave of attack is possible.

Therefore, an urgent scientific problem that requires research is the development of a sufficiently simple (for use "in the field") methodology for calculating the capabilities of responding to emergency situations as parameters of a queueing system, which is characterized by extreme unevenness of both the arrival flow of customers and service time. This will the proper determination of the adequate number and type of vehicles and the rational organization of their work to ensure maximum emergency response capabilities.

2 Analysis of the state of research

The latest studies on the modelling of various mass service (queuing) systems are increasingly common in scientific works. In particular, the article [1] presents a

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simulation model of a queuing system (QS) with a queue and relative priority, which can be used to manage the reliability of transport systems under resource constraints. The developed simulation model combines agent and discrete-event simulation principles and allows studying queuing systems in terms of establishing regularities: probabilities (service, failure, push-out), time delays (waiting in a queue, under service), queue sizes, order of queue formation upon arrival of consumers of different priority. An approach to determining the parameters of physical security units for effective response to attacks on critical infrastructure facilities is discussed in the paper [2], where conclusion was drawn that the use of methods of queuing theory of Markov and non-Markov types for modeling the counteraction of security personnel by a malicious group with a random number of criminals in a group and different ways of organizing the actions of such personnel is quite adequate.

Among the publications, it is appropriate to note the work [3], based on which research is conducted to evaluate the effectiveness of data analysis in determining the feasibility of routes, effective planning and distribution of vehicles among passengers. This study was used to determine the geographic locations of accidents and their characteristics, in road management to develop effective accident prevention measures, to determine estimated travel times and in analysis. The results of the study provide the number of passengers on each route and how vehicles can be assigned and scheduled. On the basis of the Weibull distribution studies, trains were introduced at the time of arrival at the station, signaling the arrival delay, and an effective system of providing transport services was developed, taking into account the viability of the routes.

Queues in mass service systems are one of the problems that are often encountered in everyday life. A long waiting line in a service system usually results in customers leaving the system, resulting in losses for the service provider. In [4], research was conducted using a mass service system by collecting data on customer arrival times, service times, and cancellation times while waiting for service. According to the results of the calculations, a service model was developed, with the help of which it is possible to determine the expediency (or impracticality) of creating additional service objects.

Queueing systems are important in modelling transport processes. Increasing demands from the beneficiaries of logistics services led to the expansion of offers. The presence of a queue significantly disciplines and affects the performance of the queue system. In [5], the authors developed a model and found a solution to reduce the waiting time, the total time in the system and, in general, the cost associated with queue delays. Their research demonstrates how priority service can improve system performance by reducing system time for high-spending customers. In addition, the total time in the system is reduced for all customers, which leads to the saving of transport infrastructure resources.

The application of the queueing theory for the analysis of transport processes in relation to railway transportation is proposed in the article [6]. Certain aspects of the application of the queueing theory for the analysis and evaluation of railway flows are presented. The Java Modeling Tools-JSIMgraph application was used to study railway flows. The process of train movement is analyzed taking into account traffic flows.

The paper [7] presents technological processes in logistics centers as a tool for computer simulation. The proposed approach to the modeling of technological processes of the logistics center allows to study the influence of numerical parameters of production resources and organizational influences on the efficiency of the operation of the logistics center. The development of software models allows researchers to obtain an effective tool for solving a wide range of tasks in the field of logistics center management.

Mathematical modeling is one of the most important tools for solving various transport problems. As an example, for applying the popular systemic methods in railway accident analysis. Accident analysis models and methods explain and even predict accident mechanisms, which help to choose and implement effective and efficient countermeasures. The article [8] provides literature review that identifies research evolution, trends, and gaps in studies of railway accidents and compares the methods regarding systems theory criteria (i.e., theoretical basis, system structure, system components, and system component relationships) and application characteristics (i.e., reliability, usability, and graphical representation).

Most traffic flow modeling concepts were originally developed for passenger transport. Safety and passenger issues are especially relevant for air transport. Major civil aviation accidents can lead to multiple deaths and injuries, environmental and financial damage, and serious family, media and political impacts and concerns. Antecedents and 'causes' of such accidents are typically multi-factorial, and complexity in socio-technical systems can increase the risk and unpredictability of such outcomes [9].

As about road transport, paper [10] gives a brief description of a selection of some important, recognized, and commonly used methods for investigation of accidents. In conclusion, it is decided, that the aim of accident investigations should be to identify the event sequences and all (causal) factors influencing the accident scenario in order to be able to suggest risk reducing measures suitable for prevention of future accidents.

It is of great importance in road transport to alleviate the impact of incidents by reducing incident duration. Although there have been a certain number of achievements on incident clearance time modeling, limited effort is made to investigate the relative role of incident response time and its self-selection in influencing the clearance time. To fill this gap, the study [11] uses the endogenous switching model to explore the influential factors in incident clearance time.

As we can see from the studies cited here and in other areas, the application of the methods of queueing theory in transport and other areas to analyse the processes of emergency situations and justify the methods, forces and means of responding to them is a generally accepted scientific approach, which is also followed by the authors of this article. At the same time, it should be noted that computer simulation tools are almost always used together with theoretical models of queuing systems, and we are also supporters of this approach. However, not all emergencies that occur, even in peacetime, have simulation software available, and in times of war, it is not always possible to apply them. Therefore, the scientific problem of developing relatively simple methods of calculation of emergency response capabilities as parameters of the queueing system is very relevant for practical application, especially in extreme conditions.

3 The main results of the study

The purpose and task of the research is to calculate the capabilities of responding to emergency situations as parameters of the queueing system (QS), first of all, the required number of service channels at the customers' arrival flow rate and service time that fluctuate with maximum variation. This will make it possible to predict the need for the number of forces and means of response to such situations given other system parameters.

When modelling various QS, the exponential distribution law [12] is widely used, in which the random variable T – the time interval between two consecutive events – is distributed according to the exponential law, that is, its distribution density has the form:

$$f(t) = \begin{cases} 0, & t < 0; \\ \lambda e^{-\lambda t}, & t \geq 0 \end{cases}, \quad (1)$$

where λ is the intensity of events flow, that is, in queueing systems, the number of customers' arrivals per unit time.

The exponential distribution law has only one parameter λ (the value inverse of the mathematical expectation of the random variable *t*, which simplifies its use for theoretical analysis and practical calculations.

In addition, the exponential (exponential) distribution has the property of "lack of memory". For example, if T – is the time to wait before a certain event occurs, then the probability that the event will occur during a time interval of t does not depend on how much time (τ) has already passed without the event, but only on the length of the interval t: $P(T \leq \tau + t \mid T > \tau) = P(T \leq t)$.

This distribution is often used to provide probabilistic answers to questions such as:

a) How long will it take before an earthquake occurs in a certain region?

b) How long do we have to wait for a customer to come to our store?

c) How long will the machine work without breaking down?

All of these questions refer to the amount of time we have to wait before a certain event occurs [13].

Another property of the exponential distribution, important for solving the problem we are studying, is that more frequent occurrence of events (with a shorter time interval between them) is more likely than less frequent (with a longer interval). With such a distribution, the probability of a random amount of time that has passed between consecutive events in the interval from 0 to 5 minutes is always higher than its occurrence in the interval from 5 to 10 minutes, although the duration of the interval itself (5 minutes) remains unchanged. That is, such a distribution can simulate the most severe (extreme) conditions of the system's functioning, when the events to which it must respond are more likely to occur more often than less often.

Events are considered to be the entry into the system of customers that require service. In the modern realities of Ukraine, Israel and other countries living in conditions or under the threat of war, it is more than relevant to provide probabilistic answers to questions other than those formulated above (a, b, c), namely:

– how much time will pass before the next missile attack and which area of the city will it hit?

– how many rescue teams and means to respond to the consequences of sabotage actions or enemy attacks on civil and transport infrastructure should be there?

– how many vehicles, items of humanitarian aid and personnel should be available when evacuating the population from areas affected by emergency situations to safe places?

Here are examples of events that occur during war and emergencies. Obviously, when calculating the forces and means of response to such events, it should be assumed that they can happen more often (than we would like) than less often, and the duration of the elimination of the consequences of these events will significantly deviate from a certain calculated time. In the theoretical aspect, this is the basis for using a mathematical apparatus for queueing systems (QSs) of the M/M/1 type to calculate the parameters of such systems, and in the practical sense, it provides greater stability to the forces and means of responding to such events. However, in this article, this mathematical apparatus will be modified to take into account the possible extreme conditions of the QS operation, when one service channel (service device, server) is clearly not enough, and it is necessary to determine their number X, based on the fact that the service time of one customer by one server significantly exceeds the interval between customers' arrivals in the system.

From the queueing theory, it is known that for a QS of the M/M/1 type, the time of customer's stay in the system (waiting plus service) W is

$$W = \frac{\rho^2}{(1-\rho)\lambda} + \frac{1}{\mu}, \quad (2)$$

where λ – is the arrival flow rate (the value inverse of the average time interval between customers' arrivals);

μ – is the customer service rate (the inverse of the average customer service time);

ρ – is the separate service channel load factor.

$$0 \leq \rho = \frac{\lambda}{\mu} < 1 \quad (3)$$

Enter the value α, ($0 \leq \alpha = \frac{\lambda}{\mu}$), which, in the general case, can be $\alpha \geq 1$, and this means that one service channel (server) is not enough, so it is necessary to increase their number to X. This number can be determined as

$$X = \lambda W = \lambda \left[\frac{\rho^2}{(1-\rho)\lambda} + \frac{1}{\mu}\right]. \quad (4)$$

Then, taking into account the condition $0 \le \rho = \frac{\lambda}{\mu} < 1$, it is necessary to determine the load factor of a separate channel as

$$\rho = \frac{\lambda}{\mu X}. \quad (5)$$

Substituting this value into the right-hand side of the equation $X = \lambda\left[\frac{\rho^2}{(1-\rho)\lambda} + \frac{1}{\mu}\right]$ and performing the transformation, we obtain the following cubic equation with respect to X:

$$X^3 - 2\frac{\lambda}{\mu}X^2 + \left(\frac{\lambda}{\mu}\right)^2 X - \left(\frac{\lambda}{\mu}\right)^2 = 0. \quad (6)$$

After substituting $\alpha = \frac{\lambda}{\mu}$, we have:

$$X^3 - 2\alpha X^2 + \alpha^2 X - \alpha^2 = 0. \quad (7)$$

This equation will be solved later by a well-known exact analytical method (Cardano's formula, etc.), but first we will do it by a graphical method, the results of which are more visible and understandable for analysis.

When using the graphical method, the required number of service channels is determined from equation (8), derived from (7):

$$X = \frac{\alpha^2}{\left(1-\frac{\alpha}{X}\right)X^2} + \alpha, \quad (8)$$

the right part of which depends on α and X, and the left part is equal to X.

Let's calculate the value of the function of two arguments $Y = f(\alpha, X)$, which is on the right side of the equation $Y = \frac{\alpha^2}{\left(1-\frac{\alpha}{X}\right)X^2} + \alpha$. The results are given in the Table 1.

Table 1. Value of parameter $Y = \frac{\alpha^2}{\left(1-\frac{\alpha}{X}\right)X^2} + \alpha$

α, channel load factor	X, number of channels as a variable parameter of the model				
	1	1,5	2	2,5	3
0,25	0,333	0,283	0,268	0,261	0,258
0,5	1,000	0,667	0,583	0,550	0,533
0,75	3,000	1,250	0,975	0,879	0,833
1	#	2,333	1,500	1,267	1,167
1,25	#	5,417	2,292	1,750	1,548
1,5	#	#	3,750	2,400	2,000

Note: The result of the calculation is outside the range of permissible values.

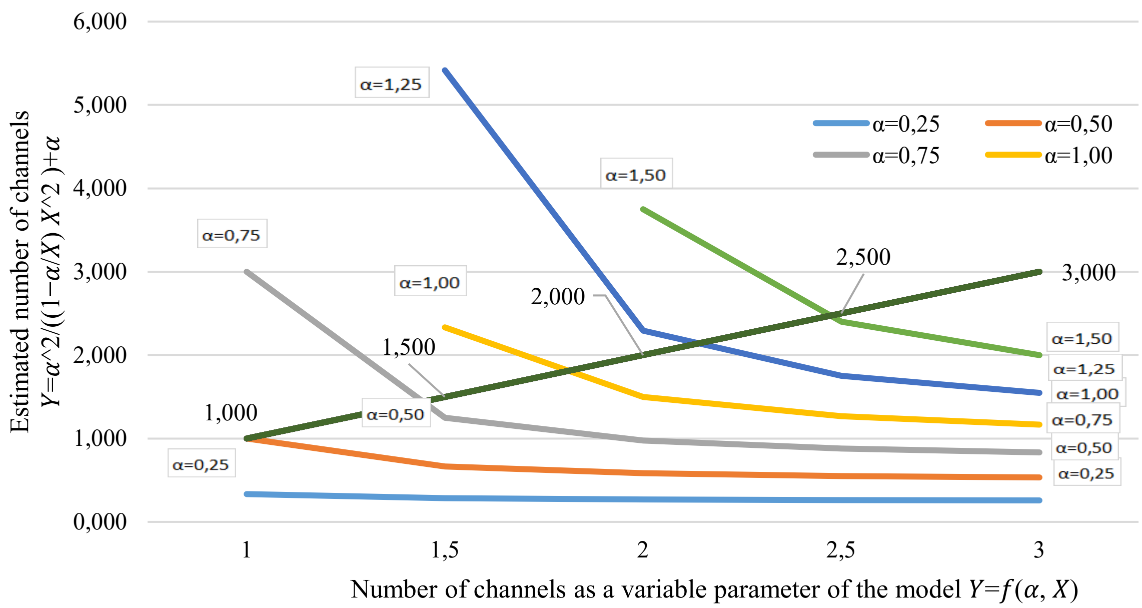

Fig. 1. Graphical solution of the equation to determine the required number of service channels

The descending six curves in Fig. 1 reflect the calculated values of the function $Y = f(\alpha, X)$ obtained by the above formula for the right side of the equation. The ascending line represents the left side of equation (8), namely $Y = X$, which is independent of α. The points of intersection of this straight line with the curves corresponding to different α (α=0. 250, …, α=1.500), in the projection on the abscissa axis X, give the search for the value of the required number of service channels.

Working out the results obtained by the graphical method shows an interesting and useful regularity. It was found that the graphical solution gives the theoretical values of X, that must satisfy equation (8), which are systematically by a certain percentage greater than the values obtained by the traditional calculation method – by determining the theoretical channel load factor $\alpha = \frac{\lambda}{\mu}$ (which can be greater than 1), with the result rounded to a larger integer. This pattern is shown in Fig. 2, from which it can be seen that in the studied range of values of α, the mentioned percentage of the number of channels compared to the value obtained by the traditional calculation method is from 100% at the minimum value of α=0.5 to 65.3% when it is increased to α=1.5.

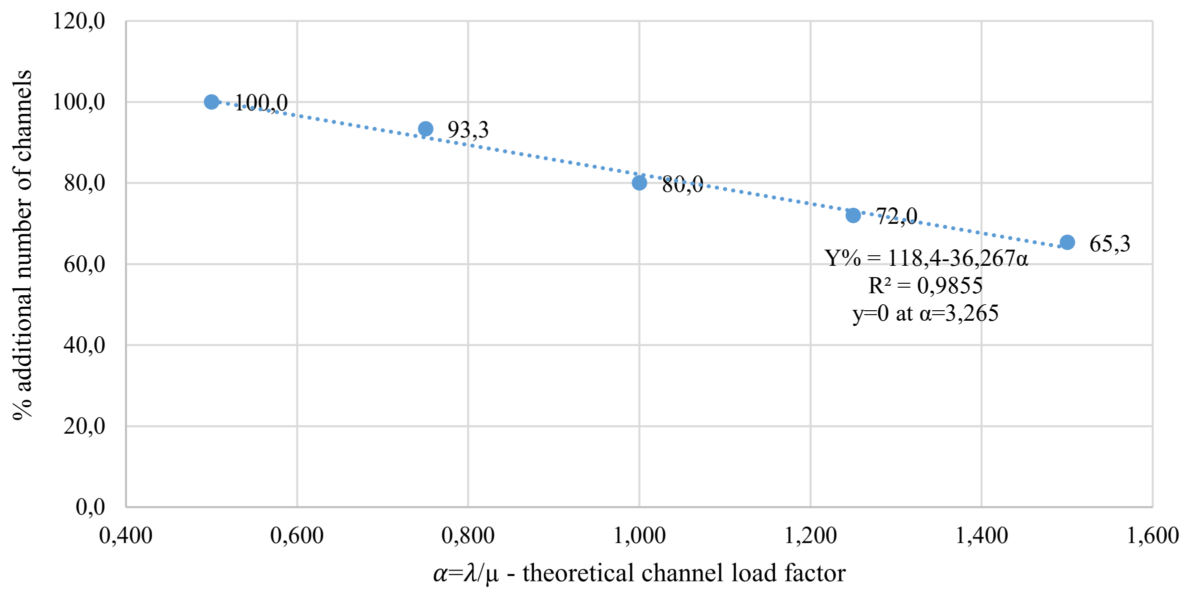

Fig. 2. Percentage Y% of additional number of channels depending on the channel load factor

For the analytical solution of the equation $X^3 - 2\alpha X^2 + \alpha^2 X - \alpha^2 = 0$, we reduce it to the canonical form

$$y^3 + py + q = 0, \quad (9)$$

for which we will first convert it into an equation of the form $aX^3 + bX^2 + cX + d = 0$ by substituting $= 1; b = -2\alpha; c = \alpha^2; d = -\alpha^2$.

The algorithm for solving the cubic equation in the canonical form is well known since the 16th century as the Cardano method. Our algebraic calculations according to this method are given below.

Coefficients of the equation are

$$q = \frac{\alpha^3}{3}\left(1 - \frac{16}{9}\right) - \alpha^2; p = -\frac{\alpha^2}{3}.$$

The discriminant of the equation is $D = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$; if $D > 0$, then the equation has one real root and two complex roots (which do not fall into the range of permissible values). The only real root is defined as

$$y = \sqrt[3]{-\left(\frac{q}{2}\right)^2 + \sqrt{D}} + \sqrt[3]{-\left(\frac{q}{2}\right)^2 - \sqrt{D}}. \quad (10)$$

In order to return to the original equation, it is necessary to make reverse substitutions, then we will get the "theoretical" value of X_{th} of the number of service channels calculated by the analytical method:

$$X_{th} = y - \frac{1}{3}\left(\frac{b}{a}\right) = y + \frac{2}{3}\alpha. \quad (11)$$

The results of calculations are given in the Table 2 and on Fig. 3

Table 2. The number of service channels determined by the traditional method X_{tr}, intermediate calculated values q, p, D, y and the theoretical X_{th} and practical X_{pr} number of service channels

α	0,25	0,5	0,75	1	1,25	1,5	1,75	2	2,25
X_{tr}	1	1	1	1	2	2	2	2	3
q	-0,067	-0,282	-0,672	-1,259	-2,069	-3,125	-4,452	-6,074	-8,016
p	-0,021	-0,083	-0,188	-0,333	-0,521	-0,750	-1,021	-1,333	-1,688
D	0,001	0,020	0,113	0,395	1,065	2,426	4,916	9,136	15,885
y	0,007	0,049	0,159	0,394	1,618	2,547	3,327	4,142	5,011
X_{th}	0,174	0,383	0,659	1,060	2,451	3,547	4,494	5,476	6,511
X_{pr}	1	1	1	2	3	4	5	6	7

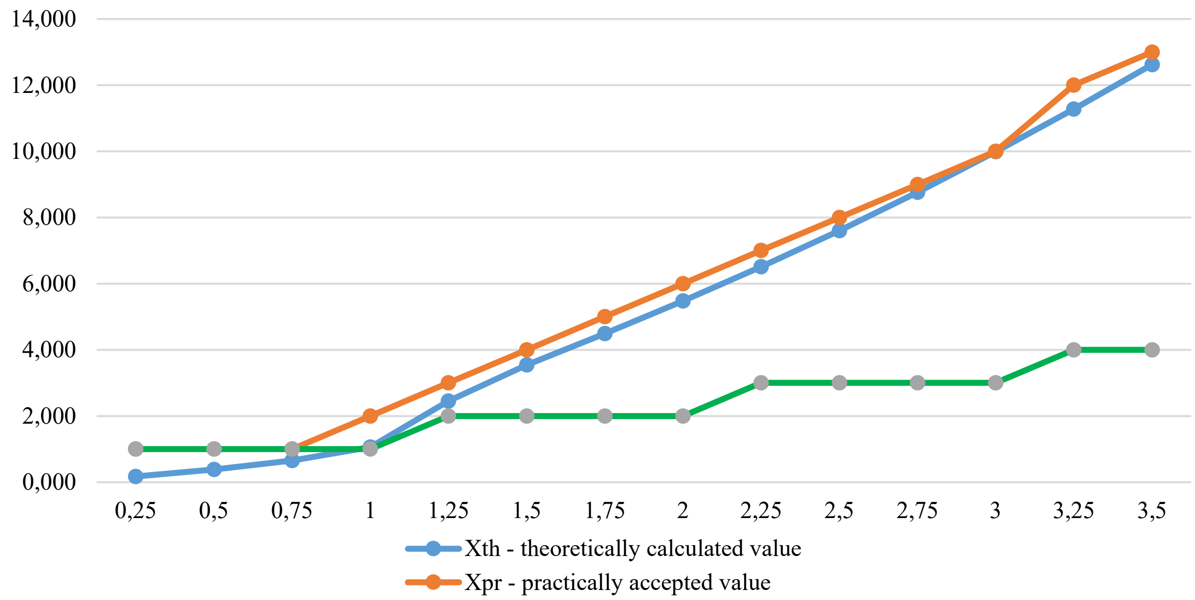

Fig. 3. Results of analytical calculations of the required number of emergency service channels

4 Conclusions

The traditional analytical method of calculating the required number of forces and means for responding to emergency situations and events (evacuation of the wounded/victims, delivery of humanitarian aid, liquidation of transport or industrial accidents involving dangerous substances, etc.) does not consider the stochastic nature of the occurrence of these events, as well as the processes of responding to them.

The conducted analysis showed that the traditional method of calculation gives significantly underestimated results, which does not provide the necessary capacity of forces and means of response to emergency situations calculated in this way and, thus, makes this response ineffective.

The authors show the possibility of using a mathematical model of the M/M/1 type queueing system (QS), which takes into account the probabilistic nature of man-made, natural, and military emergency situations, and to supplement it with an accurate analytical method of calculating the parameters of the QS, which ensure the necessary capability of forces and means for emergency response.

On the basis of the presented mathematical apparatus, for practical use by emergency services, simple methodological recommendations, regulatory documents, calculation tables, nomograms can be developed for quickly determining the required number of service channels (rescue or repair teams, sanitary units, special vehicles, etc.), their equipment and capabilities when responding to emergency situations and overcoming their consequences.

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