Searching for optimal routes for mixed road-rail freight transportation

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Abstract The main direction for improving the efficiency of cargo transportation is the implementation of combined road-rail transportation. The difficulty in of solving this problem arises from the various conditions that occur when transporting goods by different modes of transport. There are several options for organizing transportation, namely: Option 1 - transporting goods to the railroad, transshipment and further transportation by rail, Option 2 - transportation by road in parallel with rail. The aim of this research is to develop a mathematical model for combined transportation on the transport network, using both road and rail transport with transshipment from one type of transport to another. The research was conducted using the methodology for dual problems of linearly constrained optimization, makes it possible to establish the existence and finding a solution to the stated problem. The results of this work can be used in information systems for decision-making by cargo shippers regarding route selection in the field of cargo transportation under conditions of competition or interaction between different modes of transport.

1 Introduction

Before the war began, Ukraine's export volume totaled 118 million tons, with over 50 million tons were grain cargo. Since the outbreak of the war, Ukraine faced a series of problems related to the export of agricultural products abroad due to the redirection of cargo from seaports to the western borders of Ukraine. Certain types of cargo (especially those transported in bulk) can be moved both by road and rail transport. These are multimodal road-rail transportations, where the change from one mode of transport to another with cargo transshipment is envisaged to continue their transportation. These transportations are characterized by the combination of the advantages of rail and road transport, with rail being used for long distances and large volumes and road transport for collection and distribution over short or medium distances.

Before the war, the containerization of Ukraine's transport accounted for less than 1%, but today there is an increase in intermodal transport, allowing the export and import of goods. In this regard, research aimed at improving the efficiency of cargo transportation through the implementation of combined road-rail transport is relevant.

The topic of interaction and interchangeability between road and rail transport for freight transportation has been the subject of a significant number of scientific works [1-4]. These works analyze and explore the main advantages of using both types of transport together, establish decision criteria, and discuss methods for solving relevant problems. Much attention is given to the optimization of multimodal transportation using both rail and road transport, including the analysis of costs and their structure (fixed costs, infrastructure costs, and transshipment costs), as well as the selection of performance indicators [5, 6]. Review of studies related to the optimization of multimodal transportation is provided in [7].

However, there are relatively few studies that focus on the distribution of cargo volumes between different modes of transport in conditions of competition between them [3, 8, 9]. Various models are used to select the mode of transport, including demand forecasting models [9], which consider various elements of the logistics chain, and models that distribute cargo flows based on the TEN-T network in the context of competition between railway carriers, formulated as a non-linear integer optimization problem [3].

Mixed road-rail transportation can be used for both domestic and international transport. For instance, in the work [10], a model of the regional intermodal transport system in the Stockholm region is constructed, determining the costs of rail operations, road transport, and terminal handling.

International transport raises questions related to competition for cargo flows and their distribution under the conditions of differences in transport laws and traffic organization in different countries [11-14]. Researchers examine problems in finding rational delivery routes for various modes of transport, including direct or transshipment at ports and in the course of goods movement, with criteria such as the shortest time or minimal costs [15]. The methodology for determining the equivalent distance of transportation and recommendations for choosing the mode of transport

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during long-haul cargo transportation is presented in the work [16].

In conclusion, a wide range of issues related to the distribution of cargo volumes between different modes of transport, the optimal placement of terminals, cost minimization for participants in the transportation process, and the improvement of cargo delivery speed, as well as the reduction of pollutant emissions into the environment, have been covered in scientific publications [7]. It's important to note that there is a network of road and railroads used for such transportation. Under certain conditions, considering distances, cargo volumes, the available fleet, technical and technological specifics of cargo handling, it is reasonable to use one mode of transport or another. Nevertheless, there are possibilities where transportation occurs concurrently by road and rail or sequentially with transshipment from one mode of transport to another. Therefore, the addressed problem requires further study. Even for just two modes - road and rail transport, each with their own transportation networks that connect the same transportation hubs, this problem remains relevant.

For this purpose, this work proposes the development of a mathematical model for mixed transportation on the transportation network using both road and rail transport.

2 The problem of mixed transportation on the transport network

When modeling freight transportation on a network, undirected or directed graphs are commonly used. The points between which transportation is carried out correspond to the vertices of the graph, and the roads between adjacent points, considering the possibility of two-way traffic, are represented by edges connecting the vertices. This approach allows you to create graph that represents the transportation network G_A road or graph G_R railway networks. The sets of vertices of these graphs will be denoted $V(G_A)$ and $V(G_R)$ respectively, and their sets of edges as $E(G_A)$ and $E(G_R)$. Here, it is assumed that each edge e_{ij} connects two vertices of the graph v_i, v_j and in a simple graph, a pair of vertices can be connected by no more than one edge. In graph theory [17], two vertices are considered adjacent if there exists an edge connecting them, i.e., an incident edge. If all vertices of the graph are labeled or numbered, it is possible to associate the adjacency matrix of the graph, which can be used to describe the graph alongside its graphical representation.

If it is necessary to describe the transportation using two types of transport, graphs G_A and G_R are combined, so that their sets of vertices $V(G_A) \cup V(G_R)$ and edges $E(G_A) \cup E(G_R)$. The connectivity of the combined graph is ensured by introducing additional edges that connect individual vertices from $V(G_A)$ to vertices in $V(G_R)$. These edges represent the transfer of cargo between the two types of transport and do not reflect the possibility of transportation between points on the road network. They are drawn if the vertices of graphs G_A and G_R correspond to transfer points, railway stations with cargo operations, and so on. In the information system containing data about road networks and the points located there, they refer to the same point. For the road and railway network, these points usually have different names, and there is a system for encoding stations in the information system. Therefore, we will consider them as different vertices of the graphs with their designations, connected by edges, and with other properties such as resource requirements for loading and unloading operations. Thus, we obtain a connected complex graph G with an additional subset of edges

$$E(AR), \text{ i.e. } V(G) = V(G_A) \cup V(G_R),$$
$$E(G) = E(G_A) \cup E(G_R) \cup E(AR).$$

With graph G we can also associate the matrix of vertex incidence. In this case, the edges that belong to the set $E(AR)$ are incident to vertices of different graphs G_A and G_R.

Let's pose the problem of transporting goods using interchangeable transportation on a network described by the graph G. The accumulated goods in quantities S_i at various points s_i $(i = \overline{1;m})$, need to be transported across the network q_j to the planned destinations in the corresponding quantities Q_j $(j = \overline{1;n})$. We will also label the corresponding vertices of the graph as s_i i q_j. The total transportation in the system cannot exceed the quantity of available goods, and for the sake of simplifying the problem statement, we will assume that there are enough goods to meet the demand, i.e

$$\sum_{i=1}^{m} S_i \geq \sum_{j=1}^{n} Q_j .$$

The other vertices of the graph G correspond to intermediate points on the roads, transit stations, or transfer points. For convenience, we will renumber all the vertices in the graph but retain the label to which type of network the vertex belongs $v_k^a \in V(G_A)$, $v_k^r \in V(G_R)$ $(k = \overline{1;K})$, where K where K is the total number of vertices in the graph G. The vertices s_i and q_j also include.

The movement of cargo occurs between points of the transportation network, corresponding to the vertices of a graph. Therefore, we need to specify the direction from which node to which node it passes. Let's denote by x_{ij}^a the number of units of cargo being transported by road from v_i^a to v_j^a node, through x_{ij}^r - by rail from the station v_i^r to v_j^r and through x_{ij}^{ar} - The number of cargoes being transshipped from one type of transport to

another at a point split into two vertices v_i^a and v_j^r. Furthermore, if the notation of vertices and transport flows pertains to a complex network involving two types of transport on it, we will omit the upper indices *a* and *r* in these notations unless there is a specific need to distinguish the type of transport. So, when we talk about a connected graph, the movement through v_k vertex implies the presence of at least one preceding vertex v_i (if v_k is not a cargo accumulation point for dispatch) and at least one subsequent vertex v_j (if v_k is not a cargo arrival point). Let's denote E_k^+ the set of edges incident to vertex v_k, connecting the preceding vertices to v_i from v_k and E_k^- the set of edges incident to, connecting v_k to the subsequent vertices v_j. We express the balance equations for the movement of cargo through the points of the transport network in the following form

$$\sum_{E_k^+} x_{ik} - \sum_{E_k^-} x_{kj} = 0, \text{ if } v_k \text{ - a transit point;} \quad (1)$$

$$S_k + \sum_{E_k^+} x_{ik} - \sum_{E_k^-} x_{kj} \geq 0, \text{ if } v_k \text{ - the point of previous cargo accumulation;} \quad (2)$$

$$\sum_{E_k^+} x_{ik} - \sum_{E_k^-} x_{kj} - Q_k = 0, \text{ if } v_k \text{ - the point of cargo destination.} \quad (3)$$

There is $k \in (1, 2, \ldots, K)$, so, for each node, we will have one linear condition. Additionally, since the direction is specified, the quantities of cargo moved from v_i to v_j have to satisfy the non-negativity condition

$$x_{ij} \geq 0. \quad (4)$$

In the stated conditions, for the sake of simplification, it is not specified to which subnetwork relate v_k, whether it is road or rail, as the form of conditions (1)-(4) will not differ in this case. When needed to emphasize which type of transport is carrying the cargo, we will use the notation x_{ij}^a or x_{ij}^r for the quantity of transport and x_{ij}^{ar} for indicating the number of transshipments. Transshipment points here are related to transit.

The transportation of goods from points $v_i = s_i$ in the transport network occurs until they reach their final destinations $v_j = q_j$. This corresponds to the sequential traversal of vertices $v_i, \ldots, v_k, \ldots, v_j$ And the formation of routes from the starting points to the final destinations.

To make decisions regarding the selection of rational routes, we need to establish a criterion for the optimality of choice. Typically, economic indicators [18] and their aggregate value are considered $\sum_{E(G)} c_{ij} x_{ij}$. Here, the coefficients $c_{ij} = c_{ji}$ depend on the type of transport used, the cargo handling activities, transport volumes, and so on. To take the values c_{ij} as the cost of transporting one unit of cargo from v_i to v_j. If this criterion is chosen for decision-making, we need to minimize the linear function

$$L(x) = \sum_{E(G)} c_{ij} x_{ij} \to \min \quad (5)$$

The sets E_k^+, E_k^- in equations (1)-(3) for each vertex v_k of the network are taken using the most efficient routes from each vertex of departure s_i $(i = \overline{1;m})$ to each vertex of cargo destination q_j $(j = \overline{1;n})$. Then we will get linear programming problems for determining the optimal quantities x_{ij} of transportation and routes $v_i, \ldots, v_k, \ldots, v_j$. If the solution to the problem (1)-(5) is not unique, additional analysis of the solution set is conducted. They can be assessed in terms of the rational use of the existing rolling stock fleet. If it turns out that the transport quantities are quite significant on certain segments, it is necessary to consider the turnover of the vehicle fleet and introduce additional constraints into the problem's conditions.

For example, let's write an additional constraint if the fleet of road transport is limited to a quantity d^a. For example, one vehicle transport one unit of cargo volume. Then we cannot transport more than d^a units of cargo simultaneously by vehicles. This results in a constraint on the total road transport in the form of

$$\sum_{E(G_A)} x_{ij}^a \leq d^a. \quad (6)$$

It is possible to impose constraints on specific segments if the solution to the problem without considering the fleet size leads to an excessive number of vehicles. For example, we can write constraints on the transport quantities for edges e_{ij}^a connecting v_i^a to v_j^a in the form of inequalities

$$x_{ij}^a \leq d_{ij}^a, \text{ if } e_{ij} \in E(G_A). \quad (7)$$

Then the total number of vehicles on the resulting transportation routes will be sufficient.

Let's consider, for example, the problem of minimizing the linear function (5) subject under conditions (1)-(4), (7). It is equivalent to a transportation problem with limited capacity for segments of the transport network.

Methods for solving linear programming problems generally allow obtaining solutions to two problems: the primal and the dual. In other words, we simultaneously solve two problems. Each condition of the primal problem corresponds to a variable in the dual problem. It should be noted that conditions (1)-(3) are formulated for

nodes v_k graph *G*. Let's denote the corresponding variables in the dual problem as u_k. We associate variables with conditions (7), which we'll denote as γ_{ij}. Then, the dual linear programming problem for the problem (1)-(5), (7) in these variables will have the following form

$$L^*(u) = \sum_{i=1}^{m} S_i u_i - \sum_{j=1}^{n} Q_j u_j \to \max \tag{8}$$

$$\gamma_{ik} + u_k - u_i \le c_{ik}, \text{ if } e_{ik} \in E(G_A) \tag{9}$$

$$u_k - u_i \le c_{ik}, \text{ if } e_{ik} \in E(G_R) \cup E(AR) \tag{10}$$

$u_k \ge 0$, if the variables correspond to the conditions (2) for the sending points, (11)

$$\gamma_{ij} \ge 0. \tag{12}$$

According to the duality theorems for linear programming problems, the optimal values of functions (5) and (8), i.e $L_{opt}(x)$ and $L^*_{opt}(u)$ should be equal, which allows for checking the solutions of cargo transportation problems.

3 Numerical calculations

To illustrate, let's consider a model example of solving a cargo transportation problem involving two modes of transportation: road and rail. We represent the road network and the railway network as their respective graphs. The composite graph, in addition to these two transportation network graphs, includes additional edges connecting them, corresponding to cargo transshipment points. Figure 1 shows such a graph for a transportation network with cargo transshipments between the two types of transportation. At the cargo destination points s_1 and s_2 (where $m = 2$) both roadways and railway tracks connect, and there is no cargo transshipment. Therefore, in the graph, we combine the railway and road points, and denote the corresponding vertices as v_1 and v_2. For other railway stations, we have vertices v_3, v_5, v_7, v_9, v_{11}, which are disconnected from the vertices of road points v_4, v_6, v_8, v_{10}, v_{12}, where cargo handling may occur. Hence, the vertices of the road network are connected to the vertices of the railway network by edges (dashed lines) to represent cargo handling. Additionally, there are cargo sending points q_1, q_2 and q_3 ($n = 3$), on the road network, corresponding to vertices v_{13}, v_{14} and v_{15}.

With these vertex notations and their connections through edges, the compatibility matrix has the form of a symmetric matrix. Here, 1 denotes the presence of an edge between vertices v_k, v_l, while 0 indicates that the vertices are not directly connected. Each "1" in the matrix for the transportation problem corresponds to the cost c_{kl} of traversing the path from v_k to v_l. It is assumed that $c_{kl} = c_{lk}$, meaning that the costs are the same in both directions. Therefore, the cost matrix corresponding to the compatibility matrix, which contains values c_{kl} instead of "1," will also be symmetric.

$$\begin{array}{c} \begin{array}{ccccccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \end{array}$$

For numerical calculations, let's consider the following data. The costs c_{kl} for transportation and transshipment of one unit of cargo shown on the edges of the graph have values as provided in matrix *C*:

$$C = \begin{array}{c} \begin{array}{ccccccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} \end{array} \\ \begin{pmatrix} 0 & 0 & 15 & 30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 15 & 0 & 0 & 5 & 10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 30 & 0 & 5 & 0 & 0 & 0 & 0 & 20 & 0 & 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15 & 20 & 0 & 0 & 5 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 10 & 0 & 0 & 0 & 0 & 5 & 17 & 0 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 & 5 & 0 & 0 & 20 & 0 & 25 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 17 & 0 & 0 & 5 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 10 & 0 & 20 & 5 & 0 & 0 & 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 15 & 0 & 10 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25 & 0 & 0 & 5 & 0 & 10 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \end{array}$$

Three vertices of the graph, located on the road network $q_1 \Rightarrow v_{13}$, $q_2 \Rightarrow v_{14}$ and $q_3 \Rightarrow v_{15}$ have accumulated cargos in quantities of Q_1=1000, Q_2=1000, Q_3=1000 units of cargo, respectively. These cargos need to be delivered to two destinations $s_1 \Rightarrow v_1$ and $s_2 \Rightarrow v_2$ in quantities S_1=1500, S_2=1500. Both roadways and railway tracks connect to these destinations, and each mode of transportation can deliver the cargo.

Given these input data, when the cargo quantities for sending and the total demand for delivery are equal, conditions (2) are satisfied as equations. The movement between points v_k to v_l is denoted as x_{kl} and the direction of movement is indicated from the point with index *k* to the point with index *l*. In other words, x_{kl} and x_{lk} are used for movement in opposite directions.

The following is the solution for the minimization of the function (5) subject to conditions (1)-(4) without

x_{31}=1500, x_{73}=1000, x_{87}=1000, $x_{13\,8}$=1000, x_{52}=1500, x_{53}=500, x_{65}=1000, x_{95}=1000, $x_{10\,9}$=1000, $x_{14\,10}$=1000, $x_{15\,6}$=1000. The other x_{kl}=0. Moreover $L_{opt}(x)$ = 115000.

We obtain the solution for the dual problem (8)-(10), (12). For this example, condition (11) is absent because (2) is transformed into equations. It's worth noting that the solution to this problem is also not unique. Any solution u_k ($k=\overline{1;K}$) is equivalent to the solution $u_k+\theta$, where θ - a constant factor, as they satisfy conditions (9),(10),(12) and $L^*(u)$ with the same values. Here is one of such solutions:
u_1=100, u_2=90, u_3=85, u_4=80, u_5=75, u_6=70, u_7=75, u_8=70, u_9=65, u_{10}=60, u_{11}=55, u_{12}=50, u_{13}=60, u_{14}=50, u_{15}= 60. Moreover, $L^*_{opt}(u)$=1500·100+1500·90-1000·60-1000·50-1000·60 = 115000. The optimality condition is satisfied.

In Figure 2, the transportation in the network is illustrated. The directions of cargo transportation on the edges are indicated by arrows, and the quantities of transportation are shown above them.

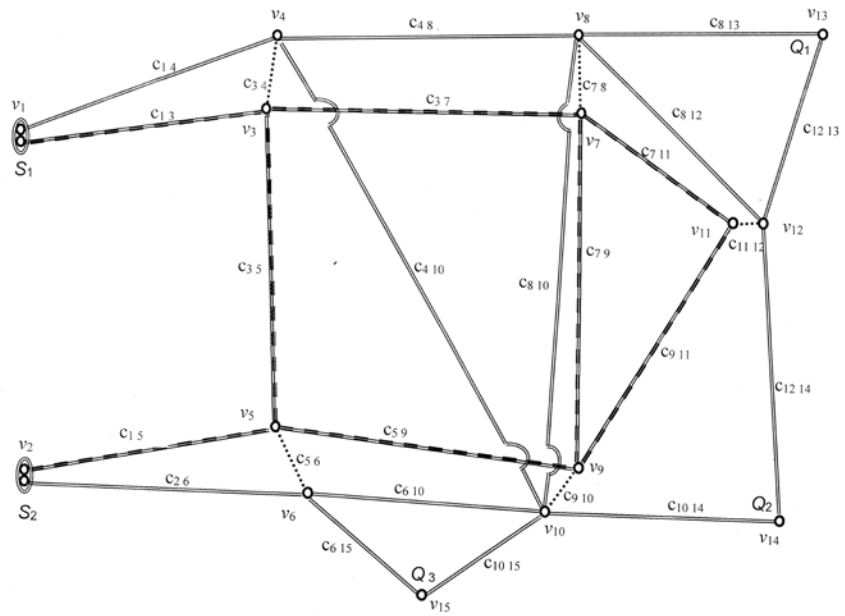

Fig. 1. The graph of the mixed cargo transportation network.

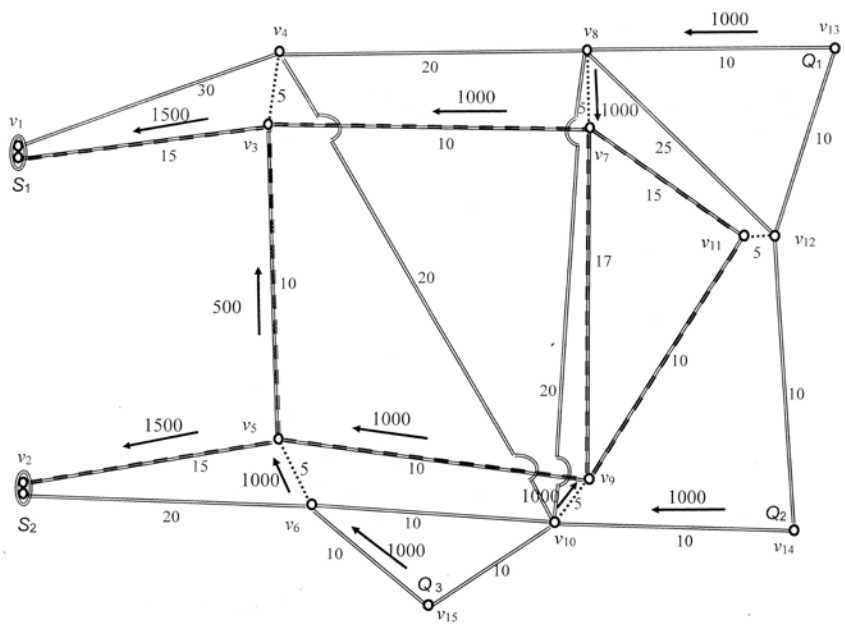

Fig. 2. Optimal transportation by rail with delivery by truck.

This solution of the problem turned out to be not unique. In Figs. 3,4 two more different solutions are shown Each of the presented variants satisfies the optimality condition for the problem. If there are no other constraints defined in the problem, they are equivalent in terms of the chosen optimality criterion.

If the solution presented in Figure 2 involves delivering goods to the railway, transshipment, and further transportation by rail, the options in Figures 3 and 4 include simultaneous transportation by road and rail. The choice between these three options can be made by considering additional criteria or constraints that arise from technical or technological reasons.

Significant volumes of transportation by road (as shown in Figures 3 and 4) with a limited fleet can be carried out by rotating the vehicles on this section. This requires additional time for all transportation, and in such a case, not all options will be equivalent.

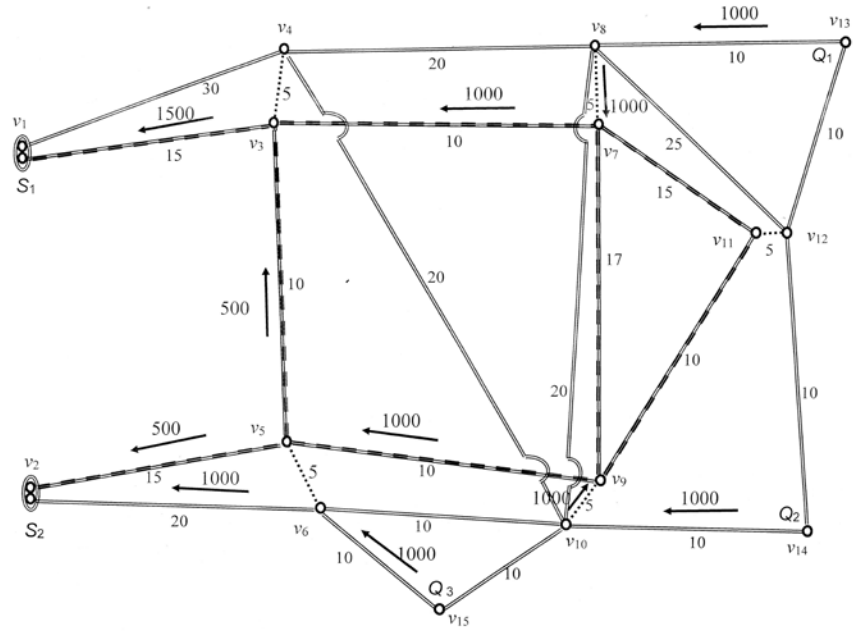

Fig. 3. The optimal transportation of goods by rail and simultaneously by road

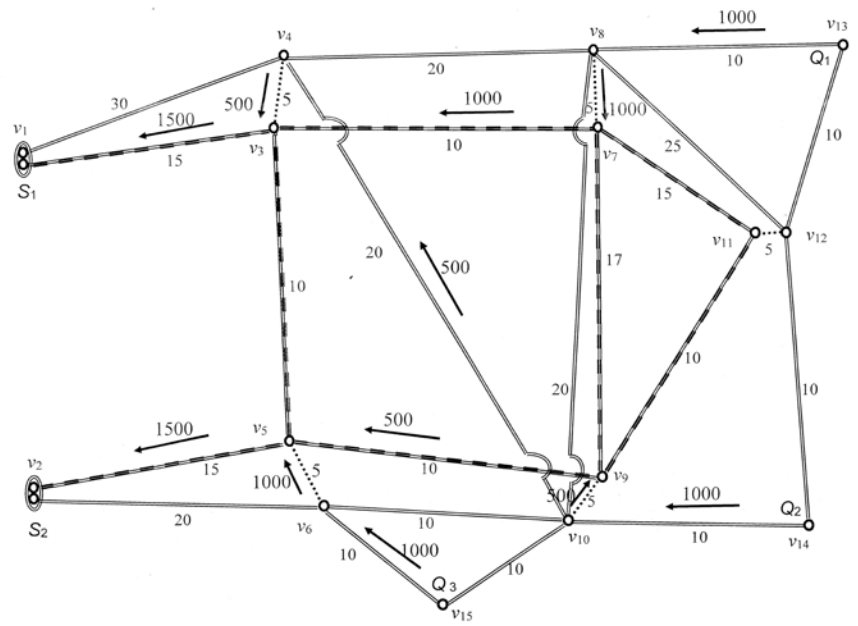

Fig. 4. Optimal transportation of goods by both rail and road

4 Conclusions

The conditions for implementing mixed transportation with both railway and road transport have significant differences due to competition for cargo flows between different modes of transport. These conditions encompass both domestic and international transportation [11,12].

When solving the problem of choosing the mode of transport and other transportation conditions, it is essential to consider the factor of interaction between various modes of transport located on both the same and parallel transportation routes. The complexity of the task arises from the various conditions that emerge when transporting goods using different modes of transport, as there are several ways to organize transportation, namely:

Bringing goods to the railway, transshipment, and further transportation by rail.

Transporting by road concurrently with railways.

Choosing one of these options may involve considering additional criteria or constraints that arise from technical or technological reasons.

The novelty of this work lies in the development of a mathematical model for mixed transportation on the transportation network, utilizing both road and rail transport with transshipment from one mode of transport to another.

The results of this work can be used for decision support systems for cargo shippers in route selection in the field of cargo transportation, considering competition or interaction between various modes of transport.

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