

# Technological aspects of grinding the side surfaces (relieving) of the teeth gear-cutting worm hobs

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**Abstract.** Contact line geometry at relieving of tooth flanks of gear cutting hobs and functional relations for calculation of relieving machine setting parameters for achieving the required accuracy of the tooth profile of the generating rack are determined. They include the setting angle of the grinding wheel axis and profiles of grinding wheel active surfaces for each of the tooth flanks. A method of calculating the single setting angle of the grinding wheel axis for both tooth flanks of the hob is developed.

## 1 Introduction

The basis of the generating worm of a hob at cutting spur and helical gears of the involute meshing is the generating rack of the hob which is a set of cutting edges of teeth in one groove of the hob. When cutting the teeth of spur and helical gears, the generating worm of a cutting tool on a gear hobbing machine is formed by positioning the axis of the helical surface of tooth cutting edges in such a way that the generating rack in the machining area is parallel to the normal cross section (or perpendicular to the direction) of the gearwheel teeth to be cut. The helical surface of the hob on which the cutting edges of the hob teeth are located is the generating surface of the hob. The relieved surfaces of the hob flanks which form rear angles during cutting are analogs of helical surfaces with helix angles one of which is increased for one flank relative to the helix angle of the hob generating surface thread, and decreased for the other flank.

The nomenclature of hobs and deviations from the straightness of the cutting edges profile of the hob teeth are specified by the valid standards ISO 2490:2007 "Solid (monobloc) gear hobs with tenon drive or axial keyway, 0.5 to 40 module – Nominal dimensions" and ISO 4468:2020 – "Gear hobs - Accuracy requirements". These standards only apply to single-thread precision hobs.

In the technological process of hob production, relieving of flanks with a grinding wheel is still the most time-consuming operation. In the practice of tool production of hobs, a method of machining each of the flanks with the setting angle of the grinding wheel axis equal to the helix angle of the generating surface of the hob on the pitch cylinder has been established. At the same time, the geometry of generation of contact lines of the grinding wheel and the relieved surfaces has not been functionally determined until now. In order to obtain the required accuracy of tooth cutting edge profile, a long adjustment of the relieving machine by the setting and profiling parameters of the grinding wheel is carried out; trial grinding passes and intermediate control operations are performed. Thus, the problems of relieving the tooth flanks of precision hobs for spur and helical gears remain

relevant up to the present time [1-6].

**The aim of the present work** is, first of all, to clarify the geometry of formation of contact lines of grinding wheel surfaces and relieved surfaces during the operation of relieving the flanks. On the basis of this research for the operation of radial (the most widespread type) relieving we define functional relations for the calculation of the basic parameters of relieving machine setting to achieve the required accuracy of the tooth profile of the generating rack: the inclination angle of the grinding wheel axis relative to the horizontal plane and profiles of the grinding wheel active surfaces for machining each of the flanks. In particular, we define the method of calculating a single angle of the grinding wheel axis, so that the machine resetting for machining the second flank consisted practically only in replacing the grinding wheel (on the faceplate) with the appropriate active surface on the grinding spindle of the machine.

## 2 Basic parameters of hob flanks

Parameters of the ground surface are determined on the basis of the initial data: module  $m_0$  of the generating rack (teeth of the cut gearwheel), pressure angle  $\alpha_0=20^\circ$  of the generating rack profile, helix angle  $\gamma_{m0}$  on its pitch cylinder and axial angle  $\alpha_{0x}$  of the profile of the generating surface of the hob:

$$\tan \alpha_{0x} = \tan \alpha_0 / \cos \gamma_{m0} \quad (1)$$

The value of the angle  $\gamma_{m0}$  is determined from the known relation:

$$\sin \gamma_{m0} = Z m_0 / d_{m0} \quad (2)$$

where  $d_{m0}$  is the pitch cylinder diameter. For a new hob you can accept  $d_{m0} = d_{a0} - 2,5 m_0$  where  $d_{a0}$  is the standardized value of the hob external diameter.

To get the required profile axial angle  $\alpha_{0x}$  of the generating surface of the hob with helical chip grooves (tooth faces), the relieved surface of teeth should have the profile angle  $\alpha_x$  defined taking into account the lead  $P_z$  of the chip groove, the number of threads  $Z$  of the generating surface, the decline  $K$  of the relieved surface on one angular pitch of the hob teeth. For right side hobs, the values of angles  $\alpha_x$  for the right (R) and left (L) flanks are determined by the formulas [7-10]:

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$$\alpha_{xR} = \alpha_{0x} + 57,296 k \sin^2 \alpha_{0x} / [P + p + k \tan \alpha_{0x}];$$

$$\alpha_{xL} = \alpha_{0x} - 57,296 k \sin^2 \alpha_{0x} / [P + p - k \tan \alpha_{0x}] \quad (3)$$

where:  $P = P_z / 2\pi$  - is the helical parameter of the face surface,  $P_z$  is the lead of the chip groove (helical face surface);

$p = 0,5 m_0 Z / \cos \gamma_{m0}$  - is the helical parameter of the generating surface of the hob,

$k = K z_0 / 2\pi$  - is the parameter of relieving,

$K$  - is the decline of the tooth back assigned in the direction of relieving movement (for radial relieving - perpendicular to the hob axis);

$z_0$  - is the number of chip grooves of the hob (or teeth in its cross section);

57,296 - is the coefficient of transition from the radian to degree value of the angular correction.

Thus, for a right-handed hob the axial angle  $\alpha_{xR}$  of the right flank profile is greater than the axial angle  $\alpha_{0x}$  of the generating surface profile of the hob, the angle  $\alpha_{xL}$  of the left flank profile of teeth is less than the angle  $\alpha_{0x}$ . For left-hand hobs, it is the reverse order:

$$\alpha_{xL} > \alpha_{0x} > \alpha_{xR}.$$

Helix angles  $\gamma_{R,L}$  of both relieved flanks on the pitch cylinder of the right-hand hob are determined by the formula:

$$\tan \gamma_R = (p + k \tan \alpha_{xR}) / r_{m0}; \quad \tan \gamma_L = (p - k \tan \alpha_{xL}) / r_{m0} \quad (4)$$

$r_{m0} = d_{m0} / 2$  - is the radius of the hob pitch cylinder.

On an arbitrary radius  $r_i$  of lateral surfaces, the helix angles  $\gamma_i$  in each point of the profile are determined from expressions:

$$\begin{aligned} \tan \gamma_{Ri} &= (p + k \tan \alpha_{xR}) / r_i; \\ \tan \gamma_{Li} &= (p - k \tan \alpha_{xL}) / r_i \end{aligned} \quad (5)$$

For left-handed hobs, the indices  $R,L$  in formulas (3)...(5) are reversed.

### 3 Contact lines when grinding helical and relieved surfaces

There is the possibility of grinding of helical surfaces of worm threads and relieving of hob flanks by the grinding wheel as initially involute ones at parallel axes  $O_1-O_1$  of the workpiece and  $O_w-O_w$  of the grinding wheel with the rectilinear axial profile. The contact line of the wheel and the ground surface touches the base cylinder and intersects the axis of the grinding wheel.

When grinding helical and relieved surfaces with a rectilinear axial profile by a disk grinding wheel, each of the points of the axial profile on the radius  $r_i$  of the ground surface with the assigned profile angle  $\alpha_x$  and helix angle  $\gamma_i$  can be considered as an element of the involute helical surface. For each point of the axial profile, its base cylinder with radius  $r_{bi}$ , can be defined. The angle  $\alpha_{bi}$  of the ground surface profile in the plane tangent to the base cylinder is equal to helix angle  $\gamma_{bi}$  of the generatrix and profile angle  $\alpha_{wi}$  of the grinding wheel in the same plane. The values of these angles are calculated by a known formula based on the axial angle  $\alpha_x$  of the ground surface profile and the helix angle  $\gamma_i$  of the surface in the calculated point of the profile:

$$\tan \alpha_{bi} = \tan \gamma_{bi} = \tan \alpha_{wi} = (\tan^2 \alpha_x + \tan^2 \gamma_i)^{0,5} \quad (6)$$

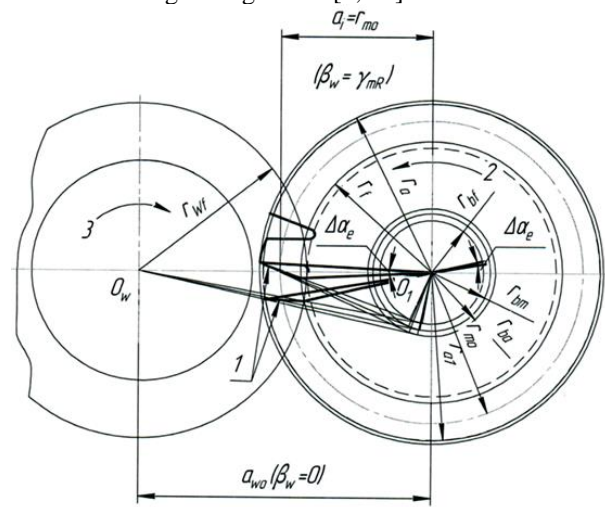
We determine the radius  $r_{bi}$  of the base cylinder for each

point of the axial profile as:

$$\begin{aligned} r_{bRi} &= (p + k \tan \alpha_{xR}) / \tan \alpha_{bRi}; \\ r_{bLi} &= (p - k \tan \alpha_{xL}) / \tan \alpha_{bLi} \end{aligned} \quad (7)$$

We set the axial angle  $\alpha_x$  of the profile of the ground helical or relieved surface to be constant. At the section of the teeth adjacent to the hob tooth heads, the helix angle of the ground surface is the smallest for the profile due to the larger diameter, therefore, the angle  $\alpha_{ba} = \gamma_{ba} = \alpha_{wa}$  of the profile and inclination of the generatrix of involute surface is the smallest, so here the radius  $r_{ba}$  of the base cylinder will be the largest for the ground profile. For the section of tooth roots the similar angle  $\alpha_{bf} = \gamma_{bf} = \alpha_{wf}$  is the largest, respectively, the radius  $r_{bf}$  of the base cylinder will be the smallest.

Let us refer to Fig. 1. When the axes  $O_w-O_w$  of the wheel and  $O_1-O_1$  are parallel, all the tangents to the base cylinders of the points of the tooth profile converge on the axis of the grinding wheel [9, 10].



**Fig. 1.** Generation of contact lines when grinding the surface with a rectilinear profile.

- 1 - contact lines of the straight section of the profile,
- 2 - direction of rotation of the cutter,
- 3 - direction of rotation of the grinding wheel.

The assigned point of the axial profile of the ground surface on the contact line of the wheel surfaces and the workpiece is obtained at the intersection point of a tangent to the base cylinder and the cylinder, on which the assigned point is located. All tangents to base cylinders corresponding to the axial profile points of the ground surface converge in one point of the interaxial perpendicular, and the enveloping line of contact points obtained in this way is the locus of points of common normal lines to these surfaces and it determines the contact line of the surfaces of the wheel and the workpiece.

When the inclination angle  $\beta_w$  of the grinding wheel axis relative to the workpiece axis is changed, tangents to the base cylinders of the ground surface also change their position. Thus, at the value  $\beta_w$  equal to the helix angle of the ground surface on the pitch cylinder, the contact line of the wheel and the ground tooth surface crosses the interaxial perpendicular at the point on the pitch cylinder, and, thus, all the tangents to base cylinders of the ground surface profile points cross the interaxial perpendicular in the same point.

Obviously, the tangents to base cylinders of the

ground surface and the contact line of the latter with the grinding wheel surface may also occupy other positions relative to the interaxial perpendicular, depending on the value of the angle  $\beta_w$  of the grinding wheel axis setting. And the position of the point of the interaxial perpendicular, where all tangents to base cylinders corresponding to profile points converge, determines the value of the inclination angle  $\beta_w$  of the grinding wheel axis relative to the workpiece axis. If the inclination angle  $\beta_w$  of the wheel axis is equal to the helix angle  $\gamma_i$  of the ground surface within the height of the ground profile, then the intersection of the contact line of the wheel and the ground surface with the interaxial perpendicular takes place within the height of the ground profile at the point on the radius  $r_i$  where these angles are equal.

If the inclination angle of the grinding wheel axis differs from the values of angles of elevation of the ground surface, then the tangents to the base cylinders of points of the ground profile intersect with the interaxial perpendicular outside the profile of the ground surface, at a distance  $a_i$  from the hob axis. And for a given value of the angle  $\beta_w$ , it should be determined (for example, for the right flank of the teeth of a right-handed hob) from the expression:

$$a_i = (p + k \tan \alpha_{xR}) / \operatorname{tg} \beta_w \quad (8)$$

If the angle  $\beta_w$  is set equal to the pitch helix angle  $\gamma_{m0}$  of the generating surface of the hob, which is defined as  $\tan \gamma_{m0} = p / r_{m0}$ , then formula (8) will be as:

$$a_m = r_{m0} (1 + k \tan \alpha_{xR} / p) \quad (9)$$

The length and configuration of the contact line inside the extreme points of the ground profile with a change in the angle  $\beta_w$  are practically not changed, since the central angle  $\Delta\alpha_c$  covering the contact line of the grinding wheel surface and the ground surface is unchanged. Its slope and distance are changed relative to the interaxial perpendicular (Fig. 1). For the left side of the teeth of the right-handed hob (on the right side of the root, if you look at the front surface of the teeth) in formulas (8), (9) we should replace the sign (+) with the sign (-) and the index (R) with (L) in the notation of the angle  $\alpha_x$ .

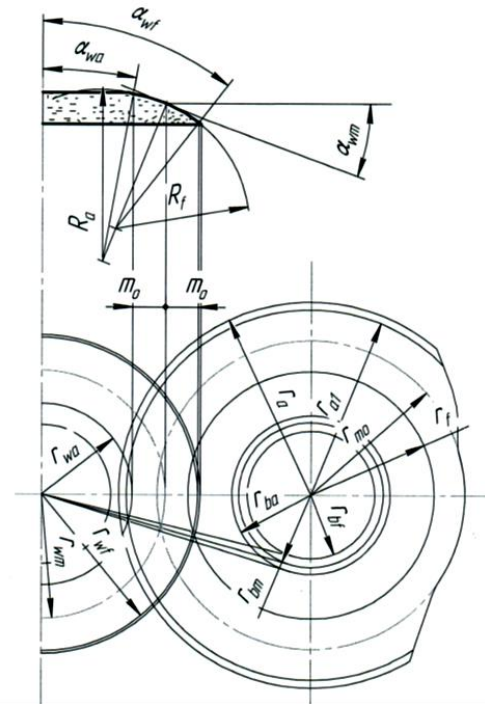
#### 4 Calculation of the grinding wheel profile along the contact line

Let us refer to Fig. 2, which shows the axial section of the grinding wheel. At parallel axes of the grinding wheel and hob, knowing from (6) angles  $\alpha_{wi} = \alpha_{bi} = \gamma_{bi}$  of the grinding wheel profile in the key points of the grinding surface profile - on cylinders of heads  $\alpha_{wa}$ , pitch  $\alpha_{wm}$  and roots  $\alpha_{wf}$  - you can define radii  $R_f$ ,  $R_a$  of curvature of the axial profile of the grinding wheel.

In the plane tangent to the base cylinder, at the profile point of the hob on its pitch cylinder, the contact point of the wheel profile has the profile angle  $\alpha_{wm}$ . This point of the wheel profile is the junction point of the curve sections of radii  $R_a$  and  $R_f$ . The length of working sections of the wheel profile is defined as  $m_0 / \cos \alpha_{wm}$ .

The radius  $R_a$  of the curvature of the ac surface of the grinding wheel machining the hob tooth head is defined as:

$$R_a = (m_0 / \cos \alpha_{wm}) / \sin (\alpha_{wm} - \alpha_{wa}) \quad (10)$$



**Fig. 2.** Determining the grinding wheel profile along the contact line.

Radius  $R_f$  of curvature of the active surface of the grinding wheel machining the hob tooth root is defined as:

$$R_f = (m_0 / \cos \alpha_{wm}) / \sin (\alpha_{wf} - \alpha_{wm}) \quad (11)$$

Convexity arrows  $\delta_a$  and  $\delta_f$  of the wheel profile in these sections are determined by the formulas:

$$\begin{aligned} \delta_a &= (m_0 / \cos \alpha_{wm})^2 / 2R_a = 0,5m_0 \sin (\alpha_{wm} - \alpha_{wa}) / \cos \alpha_{wm}; \\ \delta_f &= (m_0 / \cos \alpha_{wm})^2 / 2R_f = 0,5m_0 \sin (\alpha_{wf} - \alpha_{wm}) / \cos \alpha_{wm} \end{aligned} \quad (12)$$

Thus, at  $\beta_w = 0$  (parallel to the axes of the wheel and hob) the contact line defined above and the specified arrows of convexity of the grinding wheel profile compensate the deviation of the axial profile of the involute relieved surface relative to the rectilinear axial profile of the Archimedean relieved surface, which, strictly speaking, is what we aim to obtain.

The radius  $R_f$  is somewhat smaller than the radius  $R_a$  of the curvature of the section machining the tooth head. Accordingly,

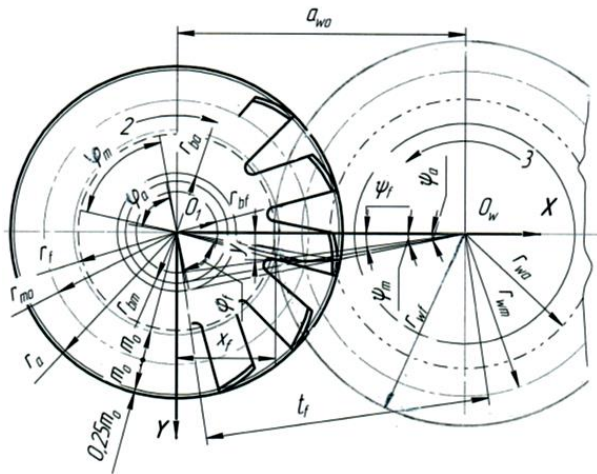
$$\delta_f > \delta_a \quad (13)$$

If the difference in values ( $\delta_f - \delta_a$ ) is within the tooth profile tolerance of ISO 4468:2020 (for a certain hob range), then the axial profile of the wheel can be dressed to a single radius of curvature, preferably to the  $R_a$  value. Since in this case the profile of the relieved tooth surface (and the cutting edge) on the hob tooth root closer to the cavity will be more complete (tooth profile deflection in +), and the gear head on the hob will have a small flange. This is in principle acceptable by the standard; and it helps to improve the characteristics of gear meshing in terms of the noise level.

The same profile of the grinding wheel can be used when setting the angle of the grinding wheel in the value  $\beta_{w1}$ :

$$\tan \beta_{w1} = (p + k \tan \alpha_{xR}) / a_{w0} \quad (14)$$

where  $a_{w0}$  is defined as follows (Fig. 3).



**Fig. 3.** Determination of coordinates of contact lines.  
 1 - contact line, 2 - direction of rotation of the cutter,  
 3 - direction of rotation of the grinding wheel.

It is most convenient to do it with the help of a tangent to the base cylinder of radius  $r_{bf}$  of the point of the grinding surface profile on the cylinder of roots of radius  $r_f$ , as at this point there is a contact with the grinding wheel on its maximum radius  $r_{wf}$  of the active surface. From the axis of the hob, we draw a perpendicular to the tangent of the base cylinder. The tangent passes through the root profile point at radius  $r_f$ . The length of the tangent  $t_f$  is defined as:

$$t_f = r_{wf} + r_{bf} \tan \varphi_f \quad (15)$$

where  $\varphi_f$  is the angle between the radii  $r_{bf}$  and  $r_f$  determined from the expression:

$$\cos \varphi_f = r_{bf} / r_f \quad (16)$$

Similarly, we find the values of the angles  $\varphi$  for the profile points on the head of the hob tooth  $\varphi_a$  and its pitch cylinder  $\varphi_m$  from the expressions:

$$\cos \varphi_a = r_{ba} / r_a; \quad \cos \varphi_m = r_{bm} / r_{m0} \quad (17)$$

The maximum radius  $r_{wf}$  of the grinding wheel is taken either from the technical data of the relieving machine: 60 mm for hobs with a diameter  $d_a$  up to 160 mm and module up to 12 mm, or 87 mm - for hobs of greater dimensions, or by measuring the current working diameter of the grinding wheel.

Then determine the angle  $\psi_f$  between the tangent and the interaxial perpendicular from the expression:

$$\tan \psi_f = r_{bf} / t_f \quad (18)$$

and we find the value  $a_{w0}$  by formula:

$$a_{w0} = r_{bf} / \sin \psi_f \quad (19)$$

## 5 Contact of the grinding wheel and tooth surfaces when setting the grinding wheel axis to assigned angle

With an arbitrarily assigned angle  $\beta_w$  of the grinding wheel axis, the characteristic of the grinding wheel active surface contact with the relieved surface of the hob teeth is changed. The length of the contact line and its configuration within itself are constant, because the values of the angles  $\varphi_i$  at contact points depend only on the parameters of the relieved surface - the radii of base cylinders and radii of the cylinders of the location of these points.

However, when the angle  $\beta_w$  is changed, the contact line rotates relative to the hob axis and shifts relative to the grinding wheel axis. The distance  $a_i$  from the hob axis to the intersection point of the tangents to the base cylinders on the interaxial perpendicular does not coincide with the dimension  $a_{w0}$ .

As a consequence of the above, the angle  $\mu\beta$  is formed between the tangent to the base cylinder of the contact point on the pitch cylinder of the relieved surface and the axial section of the grinding wheel, passing through the same point of the axial profile of the tooth surface (Fig. 4). And to calculate the axial profile of the grinding wheel, this change in position of the contact line of the wheel and the relieved surface should be taken into account.

By formula (8) we determine the value of  $a_i$ , then for the key points of the profile ( $f, m, a$ ) we find: angles  $\psi_i$  between the tangents to the base cylinders and the interaxial perpendicular from the expression

$$\sin \psi_i = r_{bi} / a_i \quad (20)$$

lengths  $t_i$  of the tangents to the base cylinders

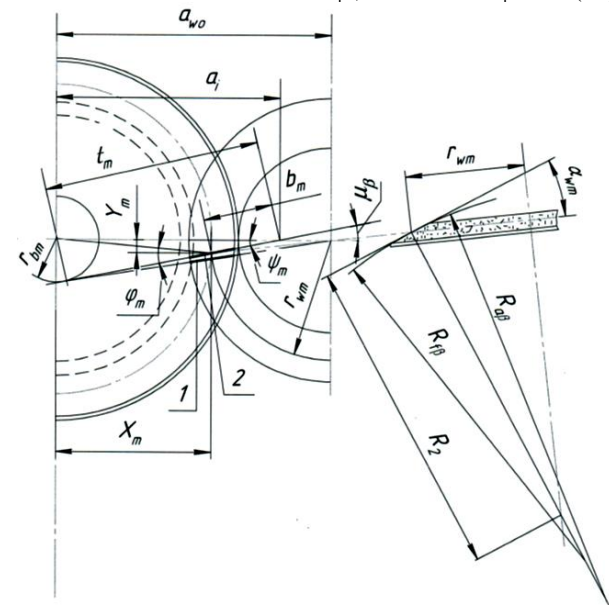
$$t_i = r_{bi} / \tan \psi_i \quad (21)$$

lengths of the segments  $b_i$  from the point of intersection of the tangents with the interaxial perpendicular to the corresponding point of the ground surface profile:

$$b_i = t_i - r_{bi} / \tan \varphi_i \quad (22)$$

and then you can calculate the coordinates  $X_i, Y_i$  of points of the contact line by the formulas:

$$X_i = a_i - b_i \cos \psi_i; \quad Y_i = b_i \sin \psi_i \quad (23)$$



**Fig. 4.** Determination of the contact line and axial profile of the grinding wheel at  $\beta_w \neq 0$ .

1 – contact line at  $a_{w0}, \beta_w = 0$ ; 2 – contact line at  $a_i, \beta_w \neq 0$ .

In general case we refer parameters  $R_a, \delta_a, R_f, \delta_f$  of curvature of the grinding wheel profile defined by formulas (10) ... (12) to the normal section of the grinding wheel surface in the plane, tangent to the base cylinder of the profile point on the pitch cylinder, as the contact point on the pitch cylinder is a junction point of two sections of the grinding wheel profiling. The value of the angle  $\mu\beta$  of the deviation of the plane, in which the curvature parameters of the grinding wheel are defined, from the axial section of the grinding wheel will be determined

from the relation:

$$\mu_{\beta} = \arccos(Y_m / r_{wm}) - (90^{\circ} - \psi_m) \quad (24)$$

where  $r_{wm} = (r_{wf} - m_0)$  is the radius of the grinding wheel at the contact point on the pitch cylinder of the hob.

To determine the parameters of the axial profile curvature of the grinding wheel at an arbitrarily assigned angle  $\beta_w$  we will use the Euler formula, which describes the ratio of curvatures of lines on the surface: for each normal section of the surface its curvature  $1/R$  is defined as [11]:

$$1/R = (\cos^2\mu / R_1) + (\sin^2\mu / R_2) \quad (25)$$

where:  $R_1$  and  $R_2$  are the basic radii of curvature, i.e. *the largest and the smallest* values of  $R$ ; they are obtained at the basic normal sections;

$\mu$  is the angle between the planes of normal sections with the basic radius of curvature  $R_1$  and the current radius of curvature  $R$ ; in this case, it is between the axial section of the grinding wheel and the section tangent to the base cylinder, where the radii of curvature are defined by formulas (10) and (11).

$$\operatorname{tg} \mu = \tan \mu_{\beta} \cos \alpha_{bm} / \cos \beta_w \quad (26)$$

With respect to the question under consideration, in formula (25)  $R$  are the radii  $R_a$  and  $R_f$  of the curvature sections in the defined normal section to the surface of the grinding wheel along the contact line found by formulas (10) and (11);  $R_1$  are the sought radii of curvature of the axial section of the wheel, corresponding to the profiling sections for the heads and roots of teeth;  $R_2$  is the radius of curvature of the grinding wheel in the plane perpendicular to the axial profile of the grinding wheel at the contact point on the hob pitch cylinder.  $R_2$  is determined by the formula:

$$R_2 = r_{wm} / \sin \alpha_{wm} \quad (27)$$

The value of the angle  $\alpha_{wm}$  of the grinding wheel profile in its axial section at the contact point with the hob pitch cylinder is determined from the relation:

$$\operatorname{tg} \alpha_{wm} = \tan \alpha_{bm} \cos \mu \quad (28)$$

The values of sought radii  $R_{a\beta}$  and  $R_{f\beta}$  of curvature (as  $R_1$ ) in the axial section of the grinding wheel are obtained from (25):

$$R_1 = R_{a,f} R_2 \cos^2\mu / (R_2 - R_{a,f} \sin^2\mu) \quad (29)$$

Our experience showed that when grinding the face of the hobs of the most widespread type Version 2, for the flank of these hobs where the helix angle of the generating surface is greater than the helix angle of the relieved surface (for example, the left flank of the hob teeth with the right direction of the generating surface), the accuracy of the profile of the generating rack is ensured with a multiple margin for all the hob types when grinding with a straight profile, and the contact line is also practically straight. For the opposite flanks (right flank of the tooth of a right-handed hob) in hobs with modules up to 2,25 mm, the accuracy of the flank profile is also technologically ensured according to accuracy class 4A without the use of radius dressing. With increasing the module, depending on the required accuracy class of the hob, radius dressing of the grinding wheel profile is required according to the above method.

## 6 Determination of a single angle of inclination of the grinding wheel axis

Setting of the grinding wheel axis inclination angle, which is uniform for grinding both flanks, provides reducing the machine readjustment to the change of grinding wheel on the machine spindle and thus limiting the time of readjustment only by achieving the required profile of the grinding wheel active surface. In addition, when using machines with a dressing mechanism that allows profiling both sides of the grinding wheel, such a setting will make it possible to produce a double-sided relieving of teeth of worm hobs.

The conditions for a single angle of the grinding wheel setting for grinding both relieved surfaces are: firstly, the intersection of the grinding wheel axis by normal lines to these surfaces; secondly, a single starting point for grinding both flanks of the teeth.

The beginning of tooth machining on each flank takes place at the apex of the tooth, since it is the apex of the tooth that initially crosses the contact line of surfaces of the hob tooth and the grinding wheel. In addition, on the side where the helix angle is greater than that of the generating surface of the hob (on right flanks of right-handed hobs), due to greater values of the radii of the base cylinders of profile points, there is a more extended area of tooth entry to the full grinding height and, accordingly, the length of the relieving stroke and the arc of hob rotation during this stroke. In this regard, a single starting point for both flanks as a necessary condition should be defined by crossing the contact lines of the right and left flanks not on the outer diameter of the straight section of the tooth profile, but at the point near the pitch cylinder, where there is the middle of the infeed section on both flanks; and where the helix angles of the generating surface of the hob and the front surface of teeth coincide.

Our research has shown that the desired value of a single angle of installation of the axis of the grinding wheel is almost very close to the angle of elevation of the producing surface on the pitch cylinder with an acceptable deviation in the direction of its decrease by  $0,1^{\circ} \dots 0,15^{\circ}$  to the value fixed by setting of the angle on the scale of the machine.

## 7 Conclusions

1. A method of constructing the contact lines between grinding wheel surfaces and ground flanks of hob teeth on the basis of considering each of the points of axial profile of the relieved surface as an involute helical surface element has been found.
2. It is determined that all the tangents to the base cylinders corresponding to the axial profile points of the ground surface, converge at one point of the interaxial perpendicular, and the enveloping line of the contact points thus obtained is the locus of the points of common normal lines to these surfaces and determines the contact line of the grinding wheel surfaces and the hob teeth.
3. The length and configuration of the contact line inside the extreme points of the ground profile with changing the inclination angle  $\beta_w$  of the grinding wheel axis is

practically not changed, as the central angle covering the contact line of the grinding wheel surface and the ground surface is unchanged. Its inclination and distance relative to the interaxial perpendicular is changed.

4. A procedure for calculating the grinding wheel profile for obtaining a rectilinear profile of the relieved surface is developed, including the case when the contact line does not coincide with the axial section of the grinding wheel.

5. It is determined that the angle of inclination of the axis of the grinding wheel for processing both sides of the teeth is  $0,1^\circ \dots 0,15^\circ$  less than the angle of elevation of the producing surface of the milling cutter on the pitch cylinder.

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