

# The use of a general method for calculating the resistance of strengthened concrete columns

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**Abstract.** Columns are one of the basic types of vertical load-bearing elements. In buildings, they are used where there is a requirement to create free spaces. This results in an effort to minimize the cross-section dimensions of the column and to maximize the use of its resistance. The need of the increase in resistance is the reason for columns strengthening. The choice of a suitable strengthening method must take into account the way the column is loaded. Some methods are universal, the others are more effective in the case of predominant compressive, or predominant bending loading. In presented research, the attention is focused on strengthening slender reinforced concrete columns by progressive methods - FRP (fibre reinforced polymer) and FRC (fibre reinforced concrete) applications. For slender columns, second-order effects must be taken into account. The current Eurocode lists three methods of analysing these effects. The simplified methods proposed for the analysis of a reinforced concrete column cannot be applied directly to the calculation of the resistance of a column strengthened by any of the progressive methods. The article is devoted to the calculation of column resistance using a general method that allows the contribution to column resistance from strengthening to be considered.

## 1 Introduction

The requirement to increase the resistance of an existing column can result from various reasons, the required increase in resistance can be achieved by one of several methods of column strengthening. When choosing a strengthening method, it is important to know the way the column is loaded so that the optimal method can be chosen.

In general, strengthening methods can be divided into two groups. The first achieves an increase in resistance by adding a material involved in load transfer. The second group includes methods using the confinement effect, which increases the concrete strength of the existing column. Strengthening by confinement is suitable for short columns stressed mainly by an axial compressive force. We can include e.g. jacketing with FRP sheets, wrapping with mortars reinforced with steel mesh or strengthening with steel caging.

For columns loaded mainly by bending moments, as well as for slender columns, strengthening by confinement is less effective. Methods based on the addition of material are

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more suitable. With the classic method of strengthening by adding concrete reinforcement and layer of concrete, both flexural and compressive resistance increases, but there is a significant increase in the cross-section dimensions. Progressive methods include the application of CFRP (Carbon Fibre Reinforced Polymers) strips both by the NSM (Near Surface Mounted) and EBR (Externally Bonded Reinforcement) method, which enables an increase in bending resistance with a minimal increase in cross-section. Another option is to jacket the column with a layer of FRC (Fibre Reinforced Concrete) or UHPFRC (Ultra-High Performance FRC). The jacketing can actively participate in load transfer and at the same time it acts as a confinement on the column core. Unlike jacketing with traditional concrete with the addition of reinforcement, strengthening with thinner layers of 3-4 cm is possible with FRC, or UHPFRC.

For slender columns, it is necessary to consider second-order effects. In the currently valid Eurocode (EC2), three methods for the analysis of second-order effects are given [1]. Two simplified methods: 1. Method based on nominal stiffness, which is based on the estimation of bending stiffness by a model taking into account the effect of cracks, physical nonlinearities and creep. 2. The method based on the nominal curvature, which is based on the calculation of the deflection based on the buckling length and the estimation of the maximum curvature. None of the simplified methods directly allow to consider the strengthening by progressive materials. The third method is the general method, which is the only one that allows the strengthening with the progressive materials to be considered when calculating the resistance of the column.

The presented study is focused on increasing the resistance of slender concrete columns using progressive methods, application of NSM CFRP and jacketing with FRC/UHPFRC. The aim of the paper is to verify the calculation of the resistance of strengthened columns by the general method on already realized experiments. In the future, the calculation by the general method will be used in the design of the own experiment.

## 2 General Method

The general method makes it possible to consider geometrical nonlinearities in addition to material nonlinearities. It allows to analyse any cross-section shape, boundary conditions, any stress-strain diagram, uniaxial, or biaxial bending moment. The general method is based on following assumptions [2]:

- Linear strain distribution in a cross-section;
- Equal strains in materials at the same level of the cross-section;
- Given stress-strain relationships for materials.

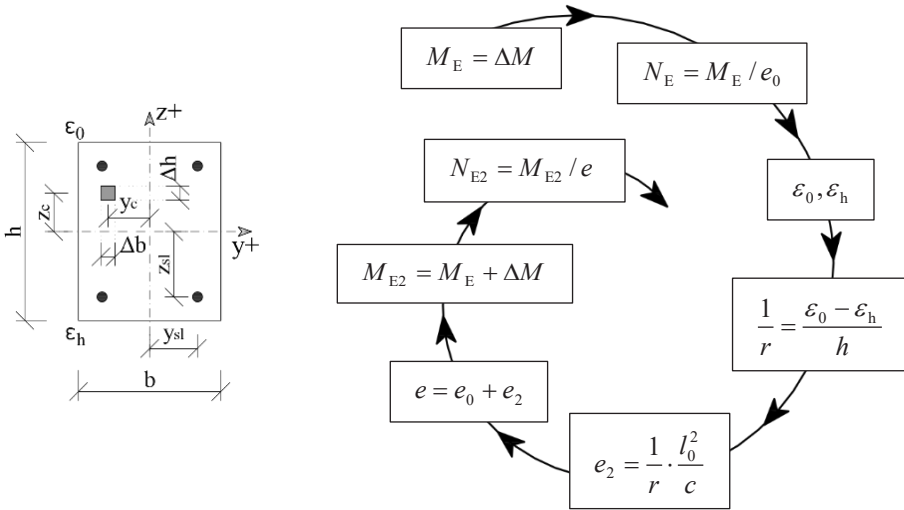
### 2.1 Calculation algorithm

For the analysis of slender columns by the general method, the calculation method A was chosen, described in dissertation of Čuhák (2013) [3]. The result of the calculation is the N-M load curve of the column for the specified eccentricity of the first order.

Based on the assumptions of the calculation that the strain in the cross-section is linear, it is possible to define the strain at any level of cross-section based on the values of strains at its edges ( $\varepsilon_0$ ,  $\varepsilon_h$ ). The column cross-section is divided into parts, for which the dimension and the position in the cross section are determined (Fig. 1). The stresses in the individual parts are determined based on the defined stress-strain diagrams of materials and the position in the cross-section.

In the cross-section loaded by the external forces  $N_E$ ,  $M_{EY}$  the equilibrium of forces must apply:  $\sum N=0$ ,  $\sum M=0$ . The strains  $\varepsilon_0$ ,  $\varepsilon_h$  are unknown. The non-linear character of stress-strain

diagrams requires the use of one of the optimization methods for the solution. Newton's method is used for calculation of strains corresponding to the applied external load.



**Fig. 1** Cross-section division scheme and principle of cyclic calculation.

Calculation method A itself is a cyclic calculation, where in each round of the cycle the acting moment is increased by the value  $\Delta M$ . The result is the N-M loading curve of the column. The calculation starts by  $\varepsilon_0=0, \varepsilon_h=0, N_E=0, M_E=0$ . The first order eccentricity of force  $e_0$  is defined. An increment of the moment  $\Delta M$  is added to the acting moment  $M_E$ . Subsequently, through the eccentricity of the first order, the corresponding axial force  $N_E$  is calculated. The deformations  $\varepsilon_0, \varepsilon_h$  are calculated from the equilibrium condition of forces in the cross-section using Newton's method. Based on the deformation of the cross-section, the curvature is calculated using the equation (1). Based on the curvature, it is possible to calculate the eccentricity of the second order according to (2) [1]. The total eccentricity is determined by the sum of the eccentricities of the first and second order. Subsequently, the next round of calculation is entered. An increment is added to the bending moment. Using the resulting eccentricity from the previous cycle, the axial force is calculated, then strains are determined, etc. The calculation ends at the moment when the ultimate strain is reached in one of the materials and its failure occurs.

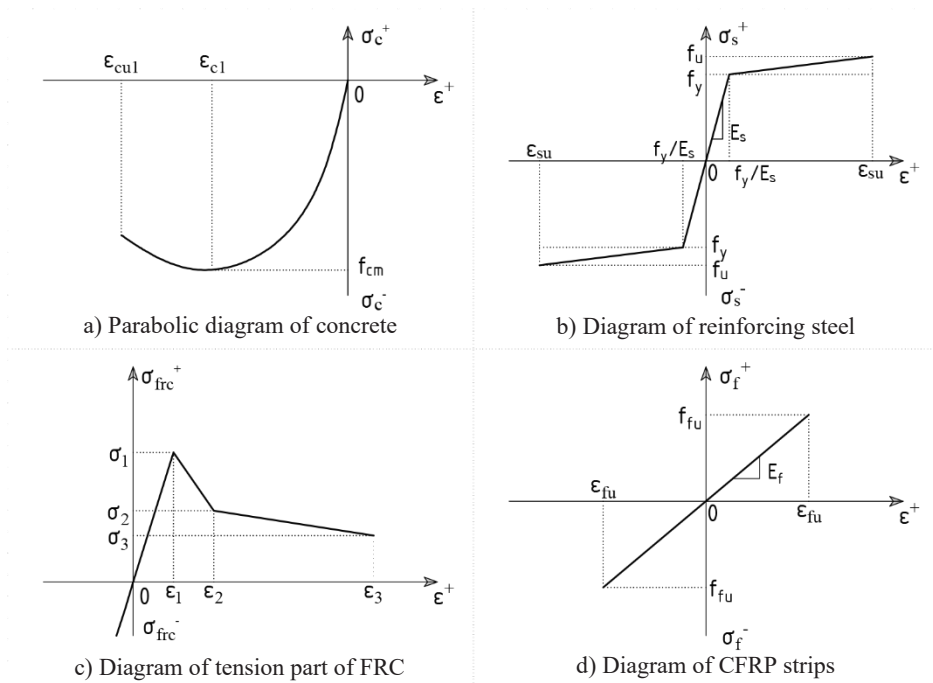
$$\frac{1}{r} = \frac{\varepsilon_0 - \varepsilon_h}{h} \tag{1}$$

$$e_2 = \frac{1}{r} \cdot \frac{l_0^2}{c} \tag{2}$$

In addition to the N-M curve calculated using the general method, an interaction diagram of the critical cross-section (ID) is also constructed for the purpose of presenting the resistance of the column cross-section. The conditions for failure of the cross-section are the same as the conditions for ending the cyclic calculation of the N-M curve. On the compressed side, it is the achievement of the ultimate strain of the material in the most stressed fibre of the cross-section. On the tensile side, it is the achievement of the ultimate strain of the reinforcement or the CFRP strips, depending on condition which occurs first.

## 2.2 Stress-strains diagrams

Material nonlinearities are taken into account by defined stress-strain diagrams. For concrete, a parabolic, alternatively block parabolic diagram defined according to EC2 [1] was used. The values of the material characteristics were taken from the literature. A bilinear stress-strain diagram with an ascending branch is considered for the reinforcement. For CFRP strips, the stress-strain diagram with linear elastic behaviour until failure is chosen. The compressive strength of the CFRP is not neglected. For FRC, a parabolic stress-strain diagram is used in the pressure area. In the area of tensile stresses, the diagram used is based on RILEM TC 162-TDF. The stress-strain diagrams are shown in Fig. 2.



**Fig. 2** Stress-strains diagrams of materials.

## 3 Verification of the calculation method

The calculation of the resistance of strengthened columns by the analytical general method was chosen for the higher availability of software in which the calculation can be written.

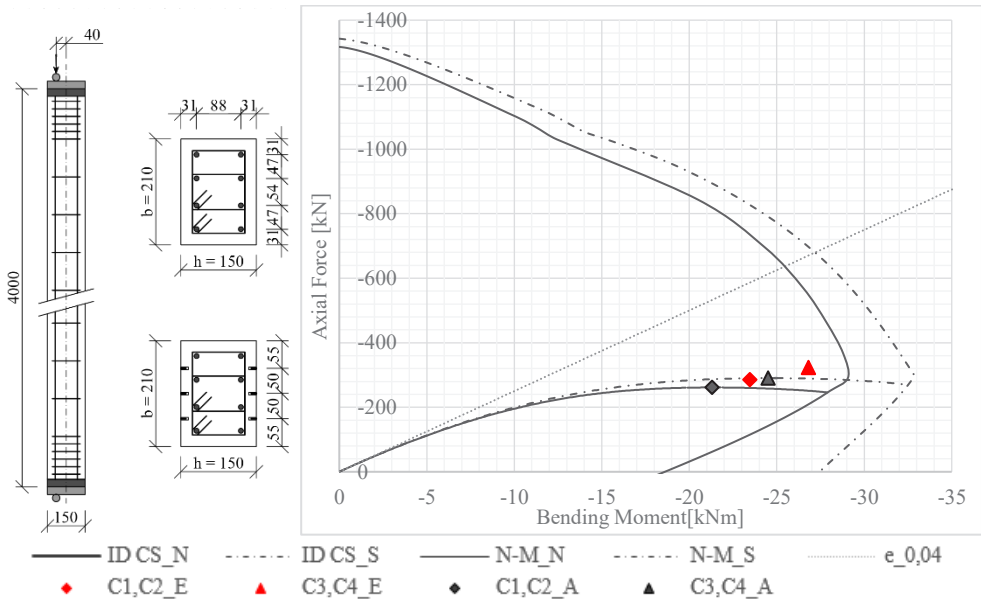
For the purposes of verification of the calculation method, experimentally tested specimens of columns with a rectangular cross-section with hinged supports at both ends were chosen from literature review. The specimens are loaded with a combination of axial force and uniaxial bending moment. The following chapter is devoted to the comparison of the calculated results with the results obtained from the experimental measurements. The calculated results are graphically presented using N-M column curves along with the ID of the critical section. For comparison, the points corresponding to the experimentally determined resistance are plotted graphically - marked *E* (Experimental) and the calculated resistance marked *A* (Analytical). The mode of failure of the specimens is also compared. The designation of the specimens is taken from the literature. Other designations used: *N* (Non-strengthened), *S* (Strengthened), *CS* (Cross-Section).

### 3.1 Experimental work by Gajdošová

The first experimental work, chosen for comparison, was performed by Gajdošová (2010) [4]. Full-scale slender rectangular RC columns were strengthened either with longitudinal NSM CFRP strips, transverse CFRP wraps, or a combination of both strengthening methods. The cross-section of the columns was 150 x 210 mm with a length of 4.00 m. The specimens were longitudinally reinforced with 8 steel bars of 10 mm in diameter, with transverse reinforcement with diameter of 6 mm with 20 mm cover, all reinforcement was of B500B class. During test, the specimens were hinged supported on both ends with an eccentricity of 40 mm. For comparison, only reference columns and columns strengthened with NSM CFRP strips were selected. When strengthening by confining, taking into account the great slenderness of the column and significant eccentricity, the increase in resistance was insignificant [4]. The material characteristics were considered according to the literature. The results are compared in Table 1. and graphically shown in Fig. 3.

**Table 1.** Comparison of experimental results from Gajdošová and analytical solution.

Experimental Specimen	Experimental			Analytical			Comparison of $N_R$ [%]
	$N_{R,E}$ [kN]	$M_{R,E}$ [kNm]	Failure mode	$N_{R,A}$ [kN]	$M_{R,A}$ [kNm]	Failure mode	
C1, C2	-286.0	-23.5	Loss of stability	-261.4	-21.3	Loss of stability	-9.4
C3, C4	-323.0	-26.8	Loss of stability	-290.4	-24.5	Loss of stability	-11.2



**Fig. 3** The graphic comparison of experimental results from Gajdošová and analytical solution.

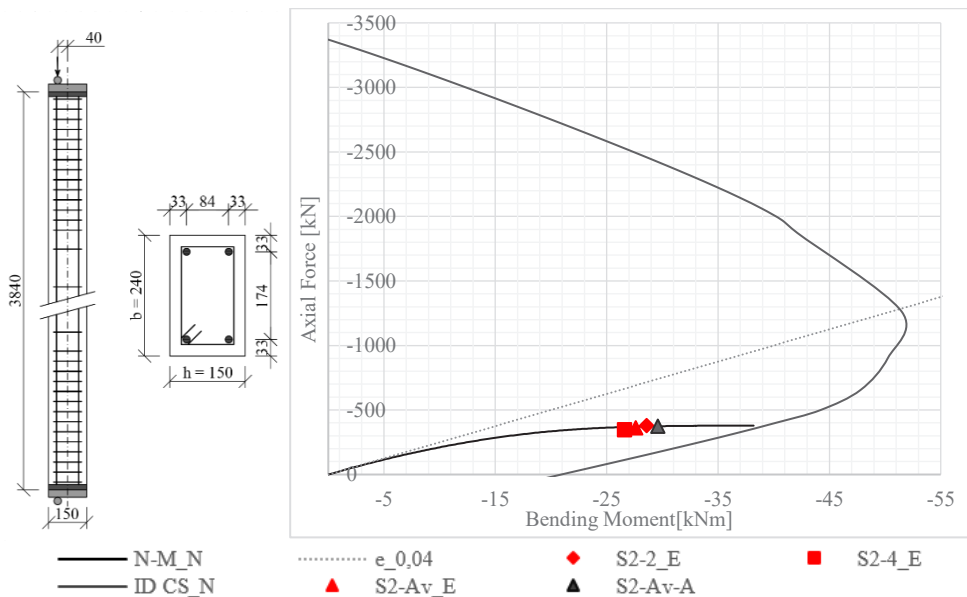
The results of the analytical calculation show the same failure mode of the specimens as in the experimental verification, namely by loss of stability. The calculated maximum compressive force at the failure of the specimens is lower by 9.4% for the unstrengthened columns and by 11.2% for the strengthened columns. The calculated bending moments are approximately 10% lower than experimentally determined. We consider the given results to be a good match.

### 3.2 Experimental work by Kendický

The second selected experimental work was performed by Kendický (2017) [5]. In his work, he focused on the general method of calculating the resistance of slender reinforced concrete columns made of high-strength concrete. The experimental program consisted of 6 specimens S2-1 to S2-6. Cross-section dimensions were 150 x 240 mm with a length of 3.84 m. During experimental investigation, the specimens were hinged supported on both ends with an eccentricity of 40 mm. The longitudinal reinforcement consists of 4 steel bars of diameter of 14 mm, one in each corner of the column. The transverse reinforcement was of diameter of 6 mm with a 20 mm cover. The average cylindrical strength of concrete was 87 MPa and the modulus of elasticity 37 GPa. For comparison, the specimens S2-2 and S2-4 are chosen, which achieved the smallest and the largest value of the axial force at failure and the average value of all tested specimens. In the experimental investigation, all specimens failed due to loss of stability. The results are compared in Table 2. and graphically shown in Fig. 4.

**Table 2.** Comparison of experimental results from Kendický and analytical solution.

Experimental				Analytical			
Specimen	$N_{R,E}$ [kN]	$M_{R,E}$ [kNm]	$e_{2,E}$ [mm]	$N_{R,A}$ [kN]	$M_{R,A}$ [kNm]	$e_{2,A}$ [mm]	Comparison of $N_R$ [%]
S2-2	-378.8	-28.6	35.5	-374.1	-29.6	39.1	-1.3
S2-4	-346.0	-26.6	36.8				8.1
Average	-361.2	-27.6	35.7				3.6



**Fig. 4** The graphic comparison of experimental results from Kendický and analytical solution.

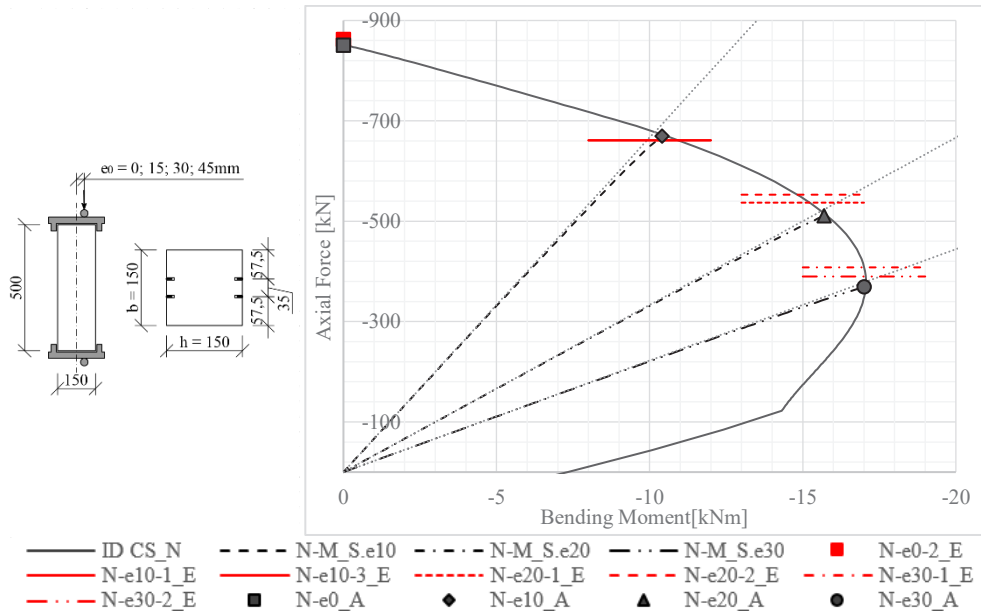
The analytically calculated N-M curve of the column reaches its maximum inside the cross-section ID. Column failure occurs due to the loss of stability, as it was the case with experimentally tested specimens. The individual experimental results are close to the analytical N-M curve, but they achieved a little bit lower bending resistance as assumed analytically. The difference in the average maximum compressive force for all specimens is of 3.6% lower than the analytically calculated value. Which can be considered as a very good agreement between experimental verification and analytical calculation.

### 3.3 Experimental work by Khorramian and Sadeghian

The third selected experimental work was carried out by Khorramian and Sadeghian (2017) [6]. The specimens with square cross-section of 150×150 mm and a length of 500 mm were tested under concentric and eccentric compressive loading up to failure. Eccentricities of 0, 10, 20, and 30% of the cross-section height were used, i.e. 0, 15, 30, and 45 mm. Nine of the specimens were strengthened with 4 NSM CFRP strips of dimensions of 10 x 1.2 mm (N-ex-y). These were analytically compared, the block-parabolic diagram for concrete was used. The modes of failure were: concrete spalling in compression (CS), concrete crushing in compression by the strain 0.0035mm/mm (CC), compressive FRP crushing (CFC). The results are compared in Table 3 and graphically shown in Fig. 5.

**Table 3.** Comparison of experimental results from Khorramian and Sadeghian and analytical solution.

Experimental				Analytical			
Specimen	$N_{R,E}$ [kN]	$M_{R,E}$ [kNm]	Failure mode	$N_{R,A}$ [kN]	$M_{R,A}$ [kNm]	Failure mode	Comparison of $N_R$ [%]
N-e0-1	-686.9	-	CS	-850.4	0	CC	23.8
N-e0-2	-862.1	-	CS	-	-	-	-1.4
N-e10-1	-660.7	-	CC→CFC	-	-	-	1.3
N-e10-2	-556.0	-	CC→CFC	-669.2	-10.4	CC	20.4
N-e10-3	-661.6	-	CC→CFC	-	-	-	1.1
N-e20-1	-537.0	-	CFC→CC	-	-	-	-5.1
N-e20-2	-552.9	-	CFC→CC	-510.9	-15.7	CC	-8.2
N-e30-1	-408.0	-	CC→CFC	-	-	-	-10.5
N-e30-2	-389.8	-	CFC→CC	-369.3	-17.0	CC	-5.6



**Fig. 5** The graphic comparison of experiment from Khorramian and Sadeghian and analytic solution.

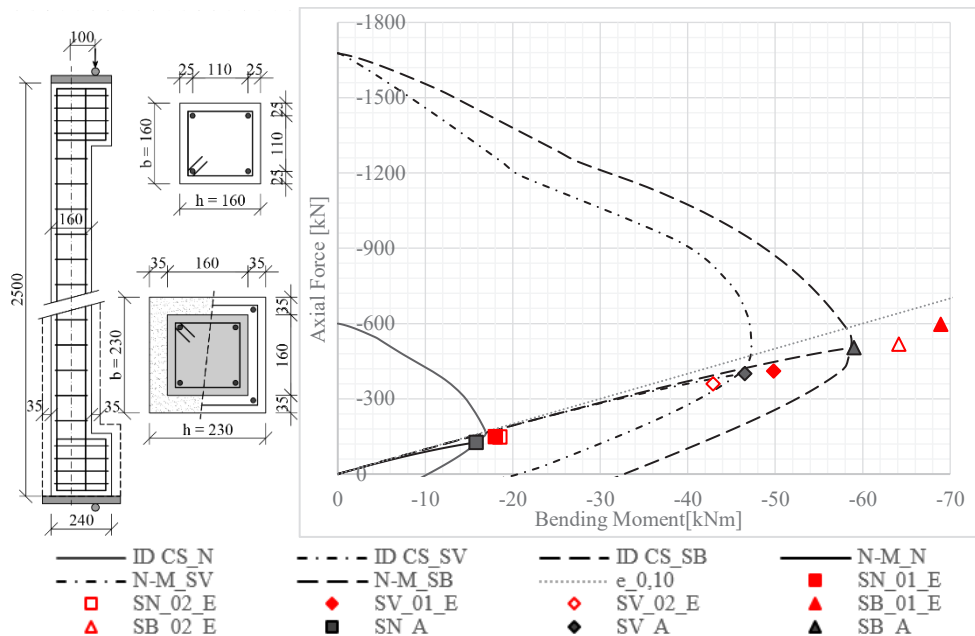
The comparison can be considered as verification of ID of cross-section. Experimental results differ from analytical results up to -10% as in the comparison with Gajdošová. Due to the large differences in results of columns with same parameters, it is not appropriate to take the results of N-e10-2 and N-e0-1 into account. All the others results are in a good match.

### 3.4 Experimental work by Vavruš

The last, fourth experimental work chosen for the verification was performed by Vavruš (2020) [7]. In his work, he dealt with the strengthening of slender columns by jacketing with concrete. A total of 6 columns with a cross-section of 160 x 160 mm and a length of 2.50 m were tested. The longitudinal reinforcement of the column was 4 Ø10 mm, the transverse reinforcement was of diameter of 8 mm. All specimens were tested in a horizontal position with an eccentricity of 100 mm. Two specimens were reference (SN\_01 and SN\_02). Two were strengthened by jacketing with concrete layer of 35 mm in width with the addition of concrete reinforcement 4 Ø10 mm (SB\_01 and SB\_02). Two specimens were strengthened by jacketing with FRC layer of 35 mm in width (SV\_01 and SV\_02). The results are compared in Table 4 and graphically shown in Fig. 6.

**Table 4.** Comparison of experimental results from Vavruš and analytical solution.

Experimental				Analytical			
Specimen	$N_{R,E}$ [kN]	$M_{R,E}$ [kNm]	$e_{2,E}$ [mm]	$N_{R,A}$ [kN]	$M_{R,A}$ [kNm]	$e_{2,A}$ [mm]	Comparison of $N_R$ [%]
SN_01	-149.5	-18.0	20.6	-126.1	15.8	25.3	-18.6
SN_02	-147.5	-18.5	25.4				-17.0
SV_01	-410.6	-49.8	21.2	-400.6	46.5	16.1	-2.5
SV_02	-359.3	-42.9	19.5				11.5
SB_01	-597.4	-68.9	15.4	-503.8	59.0	17.1	-18.6
SB_02	-517.1	-64.1	23.9				-2.6



**Fig. 6** The graphic comparison of experimental results from Vavruš and analytical solution.

The failure of the specimens in the analytical calculation occurred due to crushing of the concrete on the compressed side, as by experimental specimens. The biggest differences in calculated and experimental resistance are 17.0 and 18.6% for reference specimens. The smallest difference is 2.5% and 11.5% by strengthening with FRC jacketing. With regard to the range of experimental values, we consider the given agreement to be satisfying.

## 4 Conclusion

The issue of strengthening existing structures is becoming more and more important, whether for reasons of increasing demands on structures, their neglected maintenance or an ecological issue. When strengthening slender columns, it is necessary to consider second-order effects. The simplified methods for the analysis of second-order effects given in the EC2 do not allow directly taking into account the strengthening by progressive methods. For this reason, a general method was chosen to consider the effects of column strengthening.

As a part of the verification of the general method - method A, the resistances of the experimentally tested column specimens were calculated. The results of the comparison of the analytical solution with the experimental results are as follows:

Using a general method, it is possible to calculate the resistance of slender columns made of normal as well as high-strength concrete.

The general method makes it possible to take into account the contribution of strengthening by methods based on the application of CFRP strips, jacketing by the classic concrete layer, or jacketing by the layer of FRC to resistance.

Between the experimentally measured maximum compressive forces and the values determined by the analytical method, the difference was a maximum of 18,6 % (comparison in Table 4) with the fact that most of the values were within 10% and close to this value, which can be evaluated as a very good agreement.

The calculation takes into account whether the column will fail due to loss of stability or due to failure of the critical section. In the future, it is possible to add additional boundary conditions for failure of the critical cross-section.

The verified general method will be used in the design of the experiment aimed at strengthening slender columns. by applying CFRP strips using the NMS method and jacketing by layer of FRC alt. UHPFRC.

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