Robust Attitude Control of Helicopters with Three-Degree Freedom Based on Intermittent Event-Triggered Mechanism

Chaoyang Zhang

School of Mechanical and Material Engineering, North China University of Technology, China

Abstract. In the field of the widely used helicopter, this paper studies whether the helicopters with three-degree freedom (3DOF helicopters) have a new control method of stable attitude. Therefore, two research methods are adopted. One is to use intermittent control, so as to achieve control stability in a short time without its inputting. The other is to find and update the time needed to meet the next control through the event-triggered mechanism. A theorem is given to prove the feasibility of these methods, and Lyapunov’s second law is also applied in this paper to analyze the stability of the model designed for robust attitude control. Besides, we use the linear matrix inequality solver to solve the feedback gain of the controller in the theorem. Finally, through MATLAB simulation, the system error state of helicopters tends to zero under intermittent control, which highlights the advantages of short communication time and few times of simulation, thus achieving the simulation goal.

Key words: Intermittent Control, Event-Triggered Mechanism, Helicopter Homeostatic Control, LMI Toolbox

1. Introduction

Helicopter has been a very popular flight equipment recently [1] because it can be operated remotely with a wide range of applications in aerial photography, agriculture, reconnaissance, surveying, mapping, disaster relief, etc [8]. Thus, it is necessary to study helicopters. However, we have observed that the influence of various factors during aerial work prevents helicopters from keeping a relatively stable attitude for tasks at all times [2], which makes it much more difficult to perform tasks. Therefore, it is also imperative to study its attitude stability control.

Nowadays, there are many methods of helicopter attitude control. For example, the robust attitude control with wind resistance disturbance and information transmission delay is introduced [3] by comparing its efficiency and stability with the traditional H-infinity control model; an adaptive model based on fault-tolerant control is designed for attitude control so that helicopters can work in a larger controllable range [4] [5]. The helicopter attitude based on neural network technology is studied [6] [9], from which the uncertainty and external disturbance of the reconstructed model is approached to realize the attitude tracking control of 3DOF helicopters. A modified adaptive law is proposed to approximate the upper bound of disturbance by using the attitude tracking control model so that the tracking error converges to a preset region with a small overshoot in the user-defined time [7]. Rigatos, G. who discussed the optimal model of nonlinear control realized the fast and accurate tracking of all state variables of three-dimensional helicopters under the moderate change of control input [10]. A robust unknown input estimation scheme based on a second-order sliding mode observer is put forward to ensure the finite-time convergence of the boundary layer [11] [19]. A control model based on quaternion multi-body coupling modeling is studied to improve the workspace and application range of micro four-axis aircraft [12]. Besides, the robust control model of multi-mode switching control is discussed [13] [14]. By introducing the concept of PI control, the pitch and stroke angles can be tracked with less error. The possibility of helicopter control by mechanism search method is applied [15]. After ESC is applied to PID controller tuning, it is obtained that the pitch angle and its tracking error converge to the vicinity of the expected working point, which has high robustness. Step-by-step control of three-dimensional helicopters model with input and output constraints adopts enhanced state observer to estimate an unknown state, unexpected external disturbance, and modeling uncertainty [16]. The application of a dual-loop control system of a laboratory helicopter with 3DOF pitch, elevation, and flight is studied [18]. Similar examples can also be found in structural control, machine learning, and so on. In order to ensure the control performance of helicopter homeostatic systems in practical application, the widely used robust control as a technology is used to deal with system uncertainties and external disturbances. In this paper, the methods of intermittent control and event-
triggered control are applied, making the following main contributions.

- Compared with the traditional control method, the actuator is intermittent control, which saves control time and cost, realizing intermittent communication.
- After the event trigger condition is adopted, the data transmitted by the signal in the network will be reduced. Only when the signal meets the trigger condition can it be output to the control component, which further reduces the communication burden in the network and realizes the stability of control with less data flow.

## 2. Modeling and Controller Design

### 2.1 Modeling

As for a common 3DOF helicopter, the pitch and tilt angle can be defined by its motion attitude, so as to determine its error system model. This 3DOF dynamic error control model is discussed in an article by Xiaoyuan Zhu [5]. The tilt and pitch angles are defined as $\varepsilon_r$ and $\varphi_r$, with its error system model defined as

$$
\dot{s}(t) = Ms(t) + N\sigma(t) + Wf(t)
$$

where $s(t) = [\varepsilon_r, \ddot{\varepsilon}_r, \dddot{\varepsilon}_r, p_r, \dot{p}_r, \ddot{p}_r]^T$, $M$, $N$, $W$ is the known matrix with appropriate dimensions. Meanwhile, $s(t) \in \mathbb{R}^6$ describes the state of the system, $\sigma(t) \in \mathbb{R}^m$ represents the control signal input, $f(t)$ indicates the external disturbance, and $M$, $N$, $W$ is obtained from the known physical model.

For the set model shown in the following figure, we can get a result after listing the relationship between the tilt angle and the pitch angle.

![Figure 1. 3-DOF Helicopter Platform [5]](image)

Table 1 gives the definition of the parameters involved in the above-mentioned matrix $M$, $N$, $W$.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_f$</td>
<td>Propeller force-thrust constant</td>
</tr>
<tr>
<td>$L_h$</td>
<td>Distance from pitch axis to either motor</td>
</tr>
<tr>
<td>$L_a$</td>
<td>Distance from helicopter body to elevation axis</td>
</tr>
<tr>
<td>$J_e$</td>
<td>Moments of inertia about elevation axis</td>
</tr>
<tr>
<td>$J_p$</td>
<td>Moments of inertia about pitch axis</td>
</tr>
</tbody>
</table>

Table 1. Symbol Specification

As for this dynamic error model, after determining its basic parameters, it needs to be controlled to stabilize the error level. Therefore, the system is designed with intermittent control.

### 2.2 Controller Design

Based on the intermittent control method, the control input is defined. We assume as followed:

$$
\sigma(t) = \begin{cases}
K\sigma(t_k), & t_k \leq t < t_k + \eta_k \\
0, & t_k + \eta_k \leq t < t_{k+1}
\end{cases}
$$

where $\eta_k$ is randomly generated between $[\eta_1, \eta_2]$. This method adopts a segmented way, that is, a control input is generated to the controller when $t = [t_k, t_k + \eta_k)$, so that the helicopter does not lose balance or take control when $t = [t_k + \eta_k, t_{k+1})$. On the basis of the controller, we can determine a new system model with intermittent control method, which is

$$
\dot{s}(t) = \begin{cases}
Ms(t) + NK\sigma(t_k) + Wf(t_k), & t \in [t_k, t_k + \eta_k) \\
Ms(t) + Wf(t_k), & t \in [t_k + \eta_k, t_{k+1})
\end{cases}
$$

We continue to make the actuator intermittently control at a specific time by setting the trigger mechanism, clarifying the limit conditions of the control time, and updating the trigger conditions the next time. The trigger is set to:

$$
(t_{k+1} = \max(t > t_k + \eta_k), \quad \varphi(t) \equiv 0)
$$

$$
\varphi(t) = e^{T(t)}\Omega e(t) - \alpha(t)s^T(t)\Omega s(t)
$$

where $s(t) = \sigma(t) - s(t_k)$

As for the system after setting the controller, the control process is better expressed in the form of a flow chart. As shown in Figure 2, the sensor senses the unstable backward trigger of the attitude power system to seek control. The trigger updates the control time, with the controller and the zero-order holder passing the processing before issuing the instruction to the actuator. Finally, the actuator controls and feeds back the actual result to the attitude power system.
3. Main Results

In this paper, the following theorem is given to prove that the system error model can achieve exponential stability. Before the main results are given, the following hypothesis is necessary. Under the designed control scheme, this hypothesis is used to restrict the update time interval to fluctuate within a relatively stable interval.

Assumption: As for $t \in [0, +\infty)$, the positive numbers $C_1, C_2, C_3, C_4$ exist to reach the following result:

$$C_3 \geq \sup_{t \in [0, +\infty)} (t - \eta_0), \quad C(t) \leq C_1 + \frac{t}{C_2}, \quad C_4 \leq \sup_{t \in [0, +\infty)} (\eta_0) \quad (5)$$

Besides, $C(t) = \min \{k \in \mathbb{N} | t > t_k + \eta_0\}$

Theorem 1: For any parameter $\alpha_1$, $\alpha_2$, the closed-loop error system (1) can be exponentially stable and has index $\alpha$ with the performance of $\xi$, if there are any symmetric matrices $P_1, R_1, Q_1$ to make the following inequality

$$V_i + 2(1)^{-1}V_i(t) \leq 0 \text{ tenable.}$$

Proof: As for this closed-loop system, all states in the error dynamics model are selected to evaluate the performance of the robust controller.

$z = Cs, C = I_6$

Performance metric is $\|z\|_2 < \gamma \|f\|_2$ and so is $\gamma$.

According to Lyapunov’s second law, the following $V(t)$ is selected.

$$V(t) = \begin{cases} V_1(t), & t_k \leq t < t_k + \eta_0 \\ V_2(t), & t_k + \eta_0 \leq t < t_{k+1} \end{cases} \quad (8)$$

$$V_i(t) = s^T(t)P_is(t) + \int_{t_i - \eta_0}^{t} e^{(1-\alpha)(t-u)} s^T(u)Q_is(u)du$$

$$+ \eta^i \int_{t - \eta_0}^{t} e^{(1-\alpha)(t-u)} s^T(u)R_is(u)du dud\theta$$

and $i = 1, 2$

To assume $V_{i1} = s^T(t)P_is(t)$

$$V_{i2} = \int_{t_i - \eta_0}^{t} e^{(1-\alpha)(t-u)} s^T(u)Q_is(u)du$$

$$V_{i3} = \eta^i \int_{t_i - \eta_0}^{t} e^{(1-\alpha)(t-u)} s^T(u)R_is(u)du dud\theta$$

We can get the the following from (12)(9)(10)(11)(12)

$$V(t) = V_{i1}(t) + V_{i2}(t) + V_{i3}(t) \quad (13)$$

As for $V_{i1}, V_{i2}, V_{i3}$, derivatives are gained respectively:

$$\dot{V}_{i1} = s^T(t)P_i\dot{s}(t) + s^T(t)P_i\dot{s}(t) = 2s^T(t)P_i\dot{s}(t)$$

$$\dot{V}_{i2} = \int_{t_i - \eta_0}^{t} e^{(1-\alpha)(t-u)} s^T(u)Q_i\dot{s}(u)du$$

$$\dot{V}_{i3} = \eta^i \int_{t_i - \eta_0}^{t} e^{(1-\alpha)(t-u)} s^T(u)R_i\dot{s}(u)du dud\theta$$

As for the integral term in $V_{i3}$, the critical value can be estimated by scaling method because it can not be solved concretely.

We have $\Phi(\mu) = -\eta^i \int_{t_i - \eta_0}^{t} e^{(1-\alpha)(t-u)} s^T(u)R_i\dot{s}(u)du$ and assume $\mu = t + \theta$.

$$\Phi(\mu) = -\eta^i \int_{t_i - \eta_0}^{t} e^{(1-\alpha)(t-u)} s^T(u)R_i\dot{s}(u)du \mu$$

We also have $Z(\mu) = \begin{cases} e^{2(1-\alpha)\mu} & i = 1 \\ e^{2(1-\alpha)\mu} & i = 2 \end{cases}$

$$Z(t) \leq Z(t_i - \eta_0), \quad \phi_{\text{max}} = -\eta^i e^{2(1-\alpha)\eta_0} \int_{t_i - \eta_0}^{t} \dot{s}^T(u)R_i\dot{s}(u)du \quad (17)$$

It is observed that there is an integral term

$$\int_{t - \eta_0}^{t} \dot{s}^T(u)R_i\dot{s}(u)du$$

and the results as followed using jasen inequality for this term:

$$\int_{t - \eta_0}^{t} \dot{s}^T(u)R_i\dot{s}(u)du \leq \left( \int_{t - \eta_0}^{t} \dot{s}^T(u)du \right) \left( \int_{t - \eta_0}^{t} \dot{s}(u)du \right)$$

After the inequality (18) is brought in (17) to eliminate the integral term, the following can be obtained.

$$V_i(t) = V_{i1}(t) + V_{i2}(t) + V_{i3}(t)$$

$$\leq 2s^T(t)P_i\dot{s}(t) + s^T(t)Q_i\dot{s}(t) = 2s^T(t)P_i\dot{s}(t) + 2(1-\alpha)V_3$$

$$+ \eta^i \int_{t - \eta_0}^{t} e^{(1-\alpha)(t-u)} s^T(u)R_i\dot{s}(u)du$$

$$\leq \left[ s(t) - s(t - \eta_0) \right]^T R_i \left[ s(t) - s(t - \eta_0) \right]$$

When $i = 1, 2$, will be brought in the equation (19) about $V_{i1}$.
Because \( \dot{s} = Ms(t) + N\sigma(t) + Wf(t) \), so the following is obtained.

\[
0 = [s^T(t) \times G_1 + s^T(t) \times G_2] \times [Ms(t) + N\sigma(t) + Wf(t) - \dot{s}(t)](21)
\]

\( G_1, G_2 \) is a free matrix with known dimensions, which is used to simplify the difficulty in the solution.

\[ s(t_0) = s(t) \quad \text{is brought into (20) to construct the item of (22)} \]

The (4) is brought into (22) and get the following.

\[
0 = \left[ \dot{s}^T(t) \times G_1 + \dot{s}^T(t) \times G_2 + \phi(t) \times \left( \begin{array}{c}
\eta_1 R_1 - G_3 \\
0
\end{array} \right) \right] \times [Ms(t) + N\sigma(t) + Wf(t) - \dot{s}(t)](23)
\]

According to (4), when \( t \in [t_k, t_k + \eta_k] \), we can obtain \( \varphi(t) < 0 \). If \( \dot{V}_i + 2\lambda_i V_i(t) \leq 0 \) need to be proven, we can prove \( \dot{V}_i + 2\lambda_i V_i(t) \leq -\varphi(t) \leq 0 \). Thus, (23) can be optimized.

\[
0 = \left[ \dot{s}^T(t) \times G_1 + \dot{s}^T(t) \times G_2 + \phi(t) \times \left( \begin{array}{c}
\eta_1 R_1 - G_3 \\
0
\end{array} \right) \right] \times [Ms(t) + N\sigma(t) + Wf(t) - \dot{s}(t)](24)
\]

According to (4), \( \alpha(t) \in [\alpha_2, 2\alpha_1 - \alpha_2] \), we can take \( \alpha(t) = 2\alpha_1 - \alpha_2 \).

Assuming that \( \varepsilon_1 = [s(t), \dot{s}(t), s(t - \eta_k), \varphi(t)]^T \), demonstrating \( \dot{V}(t) + 2\lambda_1 V_1(t) \leq \varepsilon(t) T \varepsilon^T(t) \) needs to prove \( T < 0 \), so (24) will be written in the quadratic form as follows.

\[
0 = \left[ \dot{s}^T(t) \times G_1 + \dot{s}^T(t) \times G_2 + \phi(t) \times \left( \begin{array}{c}
\eta_1 R_1 - G_3 \\
0
\end{array} \right) \right] \times [Ms(t) + N\sigma(t) + Wf(t) - \dot{s}(t)](25)
\]

When \( t \in [t_k, t_k + \eta_k, t_k + \eta_k + 1] \), \( \sigma(t) = 0 \), \( \varphi(t) < 0 \), conditions then are as follows.

\[
0 = \left[ \dot{s}^T(t) \times G_1 + \dot{s}^T(t) \times G_2 + \phi(t) \times \left( \begin{array}{c}
\eta_1 R_1 - G_3 \\
0
\end{array} \right) \right] \times [Ms(t) + N\sigma(t) + Wf(t) - \dot{s}(t)](26)
\]

 Assuming \( \varepsilon_2 = [s(t), \dot{s}(t), s(t - \eta_k)]^T \), matrix \( T_2 \) can be found in the same way.

Thus, it can be obtained by the simultaneous equations of (33) (34) (35) (36).
The exponentially stable form of simplification construction can be obtained after simplifying (37).

$$\|s(t)\|^2 \leq V(t) \leq r_{max} e^{-\alpha t} V_0 \leq r_{max} e^{-\alpha t} \|s_0\|^2$$ (37)

Therefore, it is proven that exponential stability is satisfied.

4. Numerical Simulation and Results

Concerning the two linear matrix inequalities that need to be proved in the previous design theorem, the augmented matrix $K$ is obtained by solving the LMI toolbox in MATLAB when the conditions are met, including the values of the given coefficients $\alpha_1$, $\alpha_2$, $\alpha_3$ are 0.3000001, 0.3, 0.1, the value of $\lambda_1$, $\lambda_2$ is 0.5, $\Omega$ is a six-dimensional identity matrix, and the value of the matrix $M$, $N$ in the system error formula is given in the following formula. Finally, the augmented matrix $K$ that meets the needs is obtained by LMI linear matrix solver. Meanwhile, whether there is a solution when the external disturbance is 0 needs to be considered.

$$M = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad N = \begin{bmatrix}
0 & 0 \\
0.0758 & 0.0758 \\
0 & 0 \\
0 & 0 \\
0.04689 & -0.04689 \\
0 & 0
\end{bmatrix}$$

The following parameters related to physical pitch angle and tilt angle are shown in the following table [19, 20].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_h$</td>
<td>1.426 kg</td>
<td>$L_a$</td>
<td>0.66 m</td>
</tr>
<tr>
<td>$m_l$</td>
<td>1.87 kg</td>
<td>$L_{w}$</td>
<td>0.47 m</td>
</tr>
<tr>
<td>$g$</td>
<td>9.8 m/s$^2$</td>
<td>$J_f$</td>
<td>1.0348 kg·m$^2$</td>
</tr>
<tr>
<td>$K_f$</td>
<td>0.1188 N/V</td>
<td>$J_p$</td>
<td>0.0451 kg·m$^2$</td>
</tr>
<tr>
<td>$L_h$</td>
<td>0.178 m</td>
<td>$J_t$</td>
<td>1.0348 kg·m$^2$</td>
</tr>
</tbody>
</table>

Through LMI toolbox, the control gain under the given parameters can be obtained as follows.

$$K = \begin{bmatrix}
-49.0434 & -49.6079 & -16.7503 & 7.9255 & 8.0167 & 2.7069
\end{bmatrix}$$

After obtaining the control gain matrix $K$, we list the system equation expression on MATLAB according to it and draw the curve of error value changing with time in the error system based on the system equation. At the same time, the output voltage amplitude curve and the time-dependent band diagram of each control are given to express the experimental results more intuitively.

Figure 3 is a graph of output voltage amplitude, which shows that the voltage tends to be stable after the previous changes.

Figure 4 is about the time difference of triggering update, that is $t_{k+1} - t_k$, which means the time length of each update control within 25s in the Figure 4.1.
Figure 5 is a system error curve, and the final error tends to be 0, which indicates that the system can be stabilized at last.

5. Conclusions

This paper discusses the possibility of using intermittent controller to realize the homeostatic control of 3-DOF helicopter. Besides, according to traditional methods, the stable time of the control actuator is selected by event triggering, which has good robustness, with its correctness and effectiveness verified by simulation.

References


5. X. Zhu, D. Li, Robust fault estimation for a 3-DOF helicopter considering actuator saturation, Mechanical Systems and Signal Processing, vol 155, no.12, p 86-102, 16 June 2021


8. Besada-Portas, E.; Lopez-Orozco, J.A.; Aranda, J.; De La Cruz, J.M, Virtual and remote practices for learning control topics with a 3dof quadrotor, IFAC Proceedings Volumes (IFAC-PapersOnline), vol 10, PART 1, p78-83,2013,10th IFAC Symposium on Advances in Control Education, ACE 2013-Proceedings


