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Semi-Markov model for the Kaplan preventive repair system for water turbines

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Abstract. The article presents a semi-Markovian model of a system of preventive repairs according to age, carried out in water power plants with the use of Kaplan turbines. In the case of the analyzed technical objects, the profit per unit of time is considered as a criterion for the considered function. On the basis of the adopted assumptions, formulas describing the criterion function were presented for the developed mathematical model, and then the conditions for the existence of the maximum of this function were formulated. The proposed method makes it possible to determine the optimal time for preventive repair of the water power plant under consideration. Theoretical considerations presented in the article are presented on the basis of a calculation example developed on the basis of estimated data obtained from a real Kaplan water turbine operation system.

1 Introduction

The demand for electricity is growing all the time, and as a result, over the past few years there has been a rapid increase in interest related to energy derived from renewable sources. One example is hydroelectric, wind or solar energy. Renewable energy reduces emissions associated with the combustion of energy fuels. Wind energy is characterized by the independence of fossil fuel prices. The very process of generating this energy is close to the target consumers, causing minimization of costs associated with the expansion of the power grid, as well as the consequences of the failure of large power plants. The competitiveness of wind energy is dynamically increasing, as it is distinguished by its fixed cost. However, hydroelectric power plants as well as wind power plants can be subject to random damage, so an important factor is to ensure the continuity of electricity production and minimize their damage to a minimum [1].

Hydropower plants are designed on a case-by-case basis to match local conditions. Therefore, some of the most important criteria are the use of river flows, a small construction time as well as reduced construction costs [1, 2].

Hydroelectric turbines are characterized by the need to model complex flows of the working medium. Modeling complications are related to turbine rotors and flow. Tidal mechanics computational tools available in the literature fulfill their role in modeling flows in water turbines. The accuracy of the calculations ensures their use already at the level of

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preliminary design of the technical facility. This results in the minimization of design costs, but still there is a lack of computational models associated with experimental studies [2]. Cost minimization can also be achieved by introducing preventive diagnosis and repair, the cost of which is less than diagnosis and repair after damage has occurred. In order to be able to implement an optimal repair schedule, it is necessary to assess the technical condition of water turbines. Preventive repairs can be implemented in two variants: as thorough repairs, after which the technical object is "good as new," and minimal repairs, after which the technical object is "bad as old."

In contrast, the purpose of this work is to develop and study a 6-state semi-markov model of preventive repairs using periodic evaluation of the technical condition of technical objects in service. In the work, the object of study is the operating system of Kaplan hydroelectric turbines and the criterion function considered is the profit of the system per unit time, determined over an infinite time horizon. In the developed model, the basis for the construction of the criterion function is the limit theorem for semi-Markov processes [14]. The results of the model, developed on the basis of real data, can provide the basis for decision-making in the analyzed operation system.

1.1 States of the Kaplan water turbine operation process model

The technical object considered in this work is a Kaplan hydroelectric power plant. This object can reside in one of the six considered states of the operation process model:

state 1 - the state of task execution - the state in which the technical object produces electricity,

state 2 - the state of interruption of task execution related to weather conditions,

state 3 - the state of control after the interruption of the task implementation due to weather conditions,

state 4 - state of interruption of task implementation associated with damage to the technical object,

state 5 - state of diagnosis and corrective maintenance after the occurrence of damage to the object,

state 6 - the state of diagnosis and preventive maintenance, when a fit technical object is subject to diagnosis and preventive maintenance after a certain hourly mileage and in accordance with the previously adopted operational strategy.

Figure 1 shows a directed graph of the reflection of changes in the states of the model of the operation process of the Kaplan water power plant under consideration.

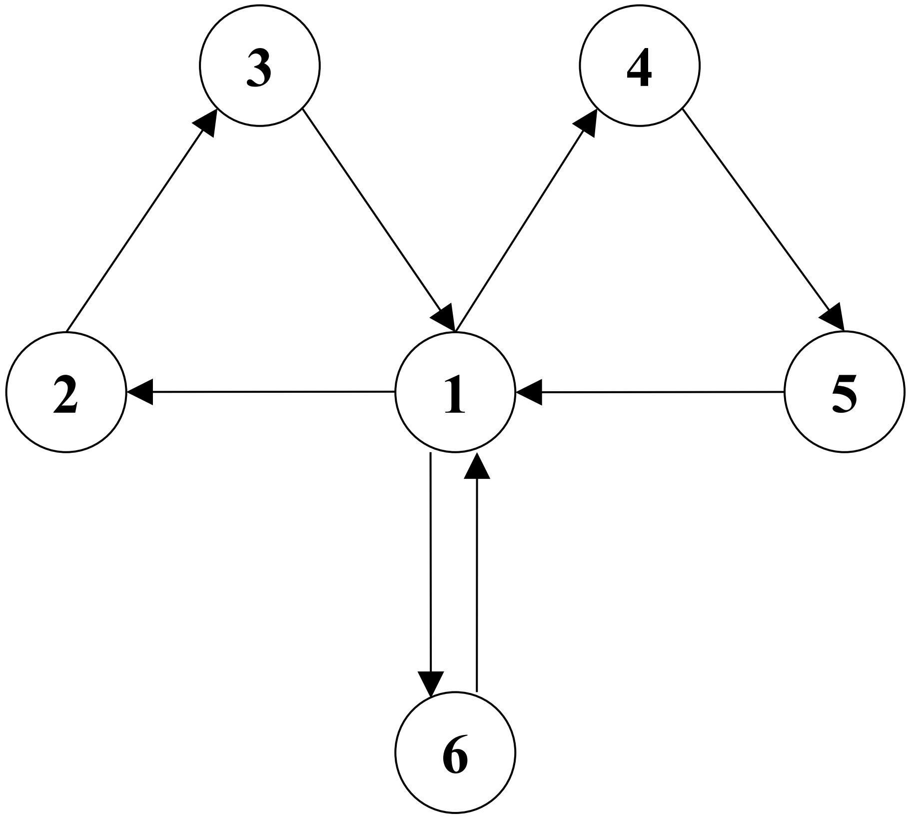

Fig. 1. Directed graph reflecting state changes of the Kaplan hydroelectric power plant operation process model with state space S = {1, 2, 3, 4, 5, 6}

1.2 Mathematical model of the Kaplan water turbine operation process

A mathematical model was built for the directed graph shown in Figure 1. At the same time, it was assumed to be a stochastic process X(t). The following mathematical model was developed on the basis of the theory of semi-Markov processes [6, 7].
In this paper, a 6-state semi-Markov process model of the Kaplan hydroelectric power plant operation process with state space S = {1, 2, 3, 4, 5, 6} is considered. If X(t) = i, then the Kaplan hydroelectric turbine shown is in state i at time t. The matrix of transitions inserted in the semi-Markov chain process for the considered model has the following form:

$$P = \begin{bmatrix} 0 & p_{12} & 0 & p_{14} & 0 & p_{16} \\ 0 & 0 & p_{23} & 0 & 0 & 0 \\ p_{31} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{45} & 0 \\ p_{51} & 0 & 0 & 0 & 0 & 0 \\ p_{61} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (1)$$

where
p_{ij}, i, j = 1, 2, 3, 4, 5, 6 - probability of transition from state i to state j.

To determine the boundary probabilities for a Markov chain, solve the following matrix system:

$$\begin{bmatrix} 0 & p_{12} & 0 & p_{14} & 0 & p_{16} \\ 0 & 0 & p_{23} & 0 & 0 & 0 \\ p_{31} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{45} & 0 \\ p_{51} & 0 & 0 & 0 & 0 & 0 \\ p_{61} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \\ \pi_6 \end{bmatrix} = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \\ \pi_6 \end{bmatrix} \qquad (2)$$

where

π_i, i = 1, 2, 3, 4, 5, 6 - the boundary probability of the Markov chain inserted in the semi-Markov process.

The article considers a model in which a Kaplan water turbine undergoes diagnosis and preventive repair at age T or diagnosis and corrective repair, when the technical object is damaged. By $T_1(x)$ is defined the time to preventive repair or damage and subsequent corrective repair of the technical object. The variable $T_1(x)$ is defined as follows.

$$T_1(x) = \begin{cases} T_1, & \text{gdy} \quad T_1 < x \\ x, & \text{gdy} \quad T_1 \geq x \end{cases} \qquad (3)$$

It is assumed that after time x, when the water turbine has not damaged, it transitions to the state of preventive repair. The process of changing states i = 1, 2, 3, 4, 5, 6, taking into account preventive repair after time x is a new semi-Markov process with a matrix P(x) of transition probabilities of the Markov chain inserted into the semi-Markov process. With respect to the matrix shown above as number 1, only the first row of the matrix P changes, then the matrix P(x) takes the form of:

$$P(x) = \begin{bmatrix} 0 & p_{12}(x) & 0 & p_{14}(x) & 0 & p_{16}(x) \\ 0 & 0 & p_{23} & 0 & 0 & 0 \\ p_{31} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{45} & 0 \\ p_{51} & 0 & 0 & 0 & 0 & 0 \\ p_{61} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (4)$$

while the boundary probabilities determined for the Markov chain are shown below:

$$\begin{aligned} \pi_1(x) &= \frac{1}{m} \\ \pi_2(x) &= \frac{p_{12}(x)}{m} \\ \pi_3(x) &= \frac{p_{12}(x) \cdot p_{23}}{m} \\ \pi_4(x) &= \frac{p_{14}(x)}{m} \\ \pi_5(x) &= \frac{p_{14}(x) \cdot p_{45}}{m} \\ \pi_6(x) &= \frac{p_{16}(x)}{m} \end{aligned} \qquad (5)$$

where

$m = 1 + p_{12}(x)(1 + p_{23}) + 4(x)(1 + p_{45}) + p_{16}(x)$

2 Determination of the criterion function

This paper considers an age-based semi-Markov model of preventive repairs. A 6-state semi-markov process X(t) with state space S = {1, 2, 3, 4, 5, 6} is considered. By z_i, i = 1, 2, 3, 4, 5, 6 is denoted the unit gain per unit time for state i. In the article it is assumed that z1 > 0, $z_i < 0$ for i = 2, 3, 4, 5, 6. Therefore, if the Kaplan water power plant is in state 1, profit is generated, while if the technical object is in state i = 2, 3, 4, 5, 6 - cost.

Based on the literature [8], for the considered model of the preventive repair system, the criterion function describing the total profit per unit time is expressed by the formula:

$$g(x) = \frac{\pi_1(x) \cdot ET_1(x) \cdot z_1 + \sum_{i=2}^{6} \pi_i(x) \cdot ET_i \cdot z_i}{\pi_1(x) \cdot ET_1(x) + \sum_{i=2}^{6} \pi_i(x) \cdot ET_i} \tag{6}$$

where

$ET_1(x)$ - the average value of the dwell time in state 1, calculated based on the formula [1, 2]

$$ET_1(x) = \int_0^x R_1(t)dt \tag{7}$$

ET_2, ET_3, ET_4, ET_5, ET_6 - average values of dwell times in states 2, 3, 4, 5, 6, respectively.

In particular, based on the literature [9], it can be written:

$$\begin{gathered} p_{12}(x) = p_{12} \cdot F_{12}(x) \\ p_{14}(x) = p_{14} \cdot F_{14}(x) \\ p_{16}(x) = p_{16} \cdot F_{16}(x) + R_1(x) \end{gathered} \tag{8}$$

where:

$F_{1j}(x)$, j = 2, 4, 6 - conditional distributions of the residence time in state 1, provided that the next state will be state j;

$R_1(x) = 1 - F_1(x)$ - reliability function of the random variable T_1.

In order to simplify further considerations, it was assumed that the equations are true

$$F_{12}(x) = F_{14}(x) = F_{16}(x) = F_1(x) \tag{9}$$

Considering the above, the criterion function (6) is expressed by the formula:

$$g(x) = \frac{ET_1(x) \cdot z_1 + p_{12}(x) \cdot (ET_2 \cdot z_2 + ET_3 \cdot z_3) + p_{14}(x) \cdot (ET_4 \cdot z_4 + ET_5 \cdot z_5) + p_{16}(x) ET_6 \cdot z_6}{ET_1(x) + p_{12} \cdot (ET_2 + ET_3) + p_{14}(x) \cdot (ET_4 + ET_5) + p_{16}(x) ET_6} \tag{10}$$

2.1 Conditions for the existence of the maximum of the criterion function - profit per unit time

The conditions for the existence of the extremum of the presented criterion function (6) will be formulated depending on the parameters of the developed semi-markov model of the Kaplan hydroelectric power plant operation process, i.e. the elements of the matrix of probabilities of changes in the states of the model P = [pij], i, j = 1, 2, 3, 4, 5, 6, the average residence times in the states of the model ET_i and the profits (costs) per unit time generated in the states of the model z_i, i = 1, 2, 3, 4, 5, 6. The considered elements are the input data of the model, and the values of the parameters depend on the type and type of technical objects analyzed, the adopted operating strategy and the specific operating conditions under which corrective repairs and preventive repairs are carried out.

Assumptions related to the values of the parameters of the system under consideration have been defined. The assumptions made must take into account the actual relations that occur between the parameters describing the implemented corrective repairs of damaged technical objects and preventive repairs:

- Z_1: $z_1 > 0$, $z_i < 0$, for i = 2, 3, 4, 5;
- Z_2: $z_5 < z_4$;
- Z_3: $z_2 < z_5 < z_4$;
- Z_4: $z_3 < z_5 < z_4$;
- Z_5: $ET_3 + ET_4 > ET_5$;
- Z_6: $ET_3 + ET_4 > ET_2$.

In the above assumptions, however, all the relationships between unit costs and average times are not specified, such as the relationships between z2 and z3 and ET2 and ET6. In the analyzed system, it is very difficult to explicitly define the relationship between the average values of these times. In the remainder of the article, additional conditions presented by formulas (14) and (17) are defined to formulate the conditions for the existence of the maximum of the criterion function (6).

The criterion function (6) can be represented as follows:

$$g(x) = \frac{L(x)}{M(x)} = \frac{A_1 \cdot ET_1(x) + B_1 \cdot F_1(x) + C_1}{A \cdot ET_1(x) + B \cdot F_1(x) + C} \tag{11}$$

where:

$A_1 = z_1$
$B_1 = p_{12} \cdot (ET_2 \cdot z_2 + ET_3 \cdot z_3) + p_{14} \cdot (ET_4 \cdot z_4 + ET_5 \cdot z_5) - (1 - p_{16}) \cdot ET_6 \cdot z_6$
$C_1 = ET_6 \cdot z_6$
$A = 1$
$B = p_{12} \cdot (ET_2 + ET_3) + p_{14} \cdot (ET_4 + ET_5) - (1 - p_{16}) \cdot ET_6$
$C = ET_6$

In order to determine the conditions for the existence of the maximum of the criterion function, the following coefficients were introduced:

$$\begin{aligned} \alpha &= A\,B_1 - A_1\,B \\ \beta &= A_1\,C - A\,C_1 \\ \gamma &= B_1\,C - B\,C_1 \end{aligned} \tag{12}$$

The coefficients α, β and γ play an important role in formulating sufficient conditions for the existence of the extremes of the criterion function.

Taking into account the above considerations:

- the coefficient α is determined by the formula:

$$\alpha = p_{12} (ET_2 z_2 + ET_3 z_3) + p_{14} (ET_4 z_4 + ET_5 z_5) - (1-p_{16}) ET_6 z_6 - z_1 [p_{12}(ET_2+ET_3) + p_{14}(ET_4+ET_5) - (1-p_{16}) ET_6] \quad (13)$$

The inequality $\alpha < 0$ is equivalent to the inequality

$$ET_6 < \frac{p_{12} \cdot [ET_2 \cdot (z_1 - z_2) + ET_3 (z_1 - z_3)] + p_{14} \cdot [ET_4 \cdot (z_1 - z_4) + ET_5 \cdot (z_1 - z_5)]}{(1 - p_{16}) \cdot (z_1 - z_6)} \quad (14)$$

- the coefficient β is determined by the formula:

$$\beta = ET6\ (z1 - z6) \quad (15)$$

Based on the Z1 assumption, it follows that $\beta > 0$.

- the coefficient γ is determined by the formula:

$$\gamma = ET_6 \{ [p_{12} [ET_2 (z_2 - z_6) + ET_3 (z_3 - z_6) + p_{14} [ET_4 (z_4 - z_6)] + ET_5 (z_5 - z_6)] \} \quad (16)$$

the inequality $\gamma < 0$ is equivalent to the inequality

$$z_6 > \frac{p_{12} \cdot (ET_2 \cdot z_2 + ET_3 z_3) + p_{14} \cdot (ET_4 \cdot z_4 + ET_5 \cdot z_5)}{p_{12} \cdot (ET_2 + ET_3) + p_{14} \cdot (ET_4 + ET_5)} \quad (17)$$

Based on the equations and inequalities shown by formulas (13), (14), (16) and (17), the following conclusions can be made:

Conclusion 1. If the condition specified by formula (14) is satisfied, then the inequality $\alpha < 0$ is true.
Conclusion 2. If the condition specified by formula (17) is satisfied, then the inequality $\gamma < 0$ is true.

Next, the conditions for the existence of the maximum of the criterion function (6) are formulated. A class of distributions of a random variable with a unimodal damage intensity function, namely T_1 MTFR (Mean Time to Failure or Repair), is considered. The considerations are related to classes of distributions of a random variable for which the time to failure of a technical object T1 is assumed to be a random variable with an increasing damage intensity function $\lambda 1(t)$, i.e. T_1 IFR (Increasing Failure Rate). The results of the study of the properties of the distributions of a random variable of the MTFR class are presented in detail in the works [16-18], while the sufficient conditions for the existence of the maximum of the criterion function (6) are formulated and discussed in the literature [8, 10].

Conclusion 3. If T_1 IFR, $\lambda_1(t)$ is differentiable, $\alpha < 0$, $\beta > 0$, $\gamma < 0$, $\beta + \gamma\ f_1(0+) > 0$, $\lambda_1(\infty)\ \alpha\ ET_1 + \beta - \alpha < 0$, then the criterion function g(x) reaches a maximum.

Conclusion 4. If T_1 IFR, $\lambda_1(t)$ is differentiable, $\beta + \gamma\ \lambda_1(0+) > 0$, $\lambda_1(\infty)\ \alpha\ ET_1 + \beta - \alpha < 0$, and conditions (11) and (14) are satisfied, then the criterion function g(x) reaches the maximum value.

3 Examples of test results

Example 1. Figure 2, presented below, shows the graphs of the criterion function g(x) - profit per unit time [PLN-103/month] as a function of time to preventive repair x [months]. The calculations were performed for the following data:

1) the values of the probability matrix of changes of states of the model P:

$$P = \begin{bmatrix} 0 & 0.3 & 0.38 & 0.21 & 0 & 0.11 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2) the average values of the residence times of the technical object in the states of the exploitation process model in [h]: $ET_2 = 1.5$, $ET_3 = 0.6$, $ET_4 = 11$, $ET_5 = 38$, $ET_6 = 14$; for the uptime T_1 a Weibull distribution was adopted, for which the value of the scale parameter = 8; three cases were analyzed when the value of the shape parameter of the Weibull distribution is shape {6.5, 8, 9.5, 11}, respectively;

the average values of profits (costs) per unit time in each state of the model in [PLN-103/day]: z1 = 4.1, z2 = - 2.6, z3 = - 3.8, z4 = - 2.6, z5 = - 7.1.

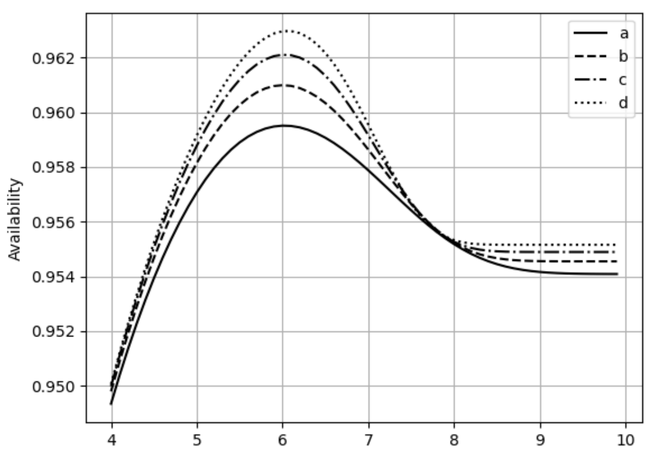

Fig. 2. Graph of the function g(x) - profit per unit time [PLN-103/month] as a function of time to preventive repair x [months], determined for a Weibull distribution with parameter values scale = 8 and shape = 6.5, 8, 9.5, 11 (curves a, b, c, d, respectively)

4 Summary

The methods presented in the article made it possible to develop a mathematical model for the preventive repairs of Kaplan's water power plant, which was built using semi-Markov processes. Based on the analysis, conditions were formulated that suffice for the existence of a maximum of this function when the time of correct operation is a random variable with an increasing function of damage intensity.

Based on the graph shown, it can be concluded that for particular values of the shape parameter of the Weibull distribution, the criterion function g(x) considered in the paper, denoting profit per unit time, reaches an extreme. In each of the three cases analyzed, there is an optimal value of time to preventive repair xmax [months], for which the criterion function g(x) [PLN-103/month] reaches its maximum value. Based on the presented graph, it can be concluded that for particular values of the parameters of the Weibull distribution, the criterion function g(x) - readiness - considered in the paper reaches an extreme (maximum) - then the optimal time to preventive repair x is about 6 months.

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