

Decision analysis within the small company for specialized equipment destined for monitoring the safety mining parameters

Marioara Tulpan^{1*}, Cristina Suci¹, Marius Cucăilă², Simona Cucăilă², and Mircea Saracu¹

¹Doctoral School - University of Petrosani, Universității no.20 Street, Petrosani, 332006, Romania

²University of Petrosani, Universității no.20 Street, Petrosani, 332006, Romania

Abstract. This paper offers a review of fundamentals in decision analysis and the construction of evidence-based probabilities for use in decision-making. The term quantitative risk analysis generally connotes reliance on probability and statistics. However, select quantitative risk-based decision-making methodologies, such as game theory, do not require knowledge of probabilities. Maximizing the minimum (maximin) gain, minimizing the maximum (minimax) loss, or maximizing the maximum (maximax) gain are but a few examples of decision-making criteria for handling risk and uncertainty without adhering to probabilities. Quantitative risk assessment builds on the existence of probabilities that describe the likelihood of outcomes, such as consequences. In general, probabilities are derived on the basis of historical records, statistical analysis, and/or systemic observations and experimentation. We commonly refer to probabilities that are derived from this process as "*objective probabilities*". Often, however, situations arise where the database is so sparse and experimentation is so impractical that "*objective probabilities*" must be supplemented with "*subjective probabilities*" or probabilities that are based on expert evidence, often referred to as "expert judgment". This paper focus on generating probabilities on the basis of expert evidence, using by the decision rules under uncertainty.

1 Introduction

Risk-based decision-making approaches in decision-making are terms frequently used to indicate that some systematic process that deals with uncertainties is being used to formulate policy options and assess their various distributional impacts and ramifications. Starting from theory to practice, this paper objectively highlights, using dedicated mathematical tools, the important aspects in the field of assisting the decision regarding the valorization and securing of mineral resources, as well as the implications of these actions on the economic and social environment, providing viable solutions to protect them [1].

* Corresponding author: mari@gerom.com

Today an ever-increasing number of professionals and managers in industry, government and academia are devoting a large portion of their time and resources to the task of improving their understanding and approach to risk-based decision-making. Brooks offers the following succinct definition of knowledge management, which is adapted from the American Productivity and Quality Centre, "*Knowledge management: Strategies and processes to create, identify, capture, organize and leverage vital skills, information and knowledge to enable people to best accomplish the organization mission*". Explicit in this orientation is the holistic vision that the goals of a system or a decision-maker can be achieved by addressing and managing them as integral parts of the larger system. A central tenet of the vision of successful organisations is building and codifying trust that transcends institutions, organizations, decision-makers, professionals and the public at large. Numerous studies have attempted to develop criteria for what might be considered "*good*" risk analyses, an early and the most prominent of which is the Oak Ridge Study, according to Fischhoff. Good risk studies may be judged against the following list of ten criteria. The study must be: *Comprehensive, Adherent to evidence, Logically sound, Practical, Open to evaluation, Best on explicit assumptions and premises, Compatible with institutions, Conductive to learning, Attend to risk communication, Innovative, Repeatable*. Risk analysis has emerged as an effective and comprehensive procedure that supplements and complements the overall management of almost all aspects of our lives. Risk, a measure of the probability and severity of address effects, is a concept that many find difficult to comprehend, and its quantification has challenged and confused laypersons and professionals alike. There are myriad fundamental reasons for this state of affairs. One is that risk is a complex composition and amalgamation of two components – one real (the potential damage, or unfavourable adverse effects and consequences), the other an imagined mathematical human construct termed *probability*. Probability per se is intangible, yet is omnipresence in risk-based decision-making is indisputable. At various levels of the decision-making process, managers often encounter situations where sparse statistical data do not lend themselves to the construction of probabilities. Through case study showed in the dedicate section, this paper will make it possible for such managers to augment evidence gained through their professional experience with evidence collected through other means [2-5].

2 Theoretical considerations regarding the decision rules under uncertainty

Most of this paper is written with the assumption that the reader has the ability to generate either objective probabilities or expert evidence-based probabilities. If will use the conventional notation of $p(s_j)$ as the probability associated with the scenario s_j (or the state of nature s_j). The i th decision or action adopted by the decision-maker will be denoted by a_i , and the outcome from the combination of the scenarios and actions are the pairs (a_i, s_j) . the payoff associated with the pair (a_i, s_j) , $i = \overline{1, I}$, $j = \overline{1, J}$, will be denote by μ_{ij} . When $p(s_j)$ and μ_{ij} are known, the conventional criterion for decision-making is the expected value of gain (or loss or risk). As noted before, a supplement to the expected value of risk, is termed the conditional expected value of risk. Thus, maximizing the expected monetary value of gain can be written as [5-9]:

$$\max_{1 \leq i \leq I} \sum_{j=1}^J p(s_j) \mu_{ij} \quad (1)$$

In the absence of any knowledge of probabilities, it is not possible to use the expected value as a gain or risk index. The following decision rules are then common for this situation:

- **The Pessimistic Rule (Maximin or Minimax Criterion):** Following this criterion, the conservative decisionmaker seeks to maximize the minimum gain or, alternatively, minimize the maximum loss. If μ_{ij} represents a payoff, then we have:

$$\max_{1 \leq i \leq I} (\min_{1 \leq j \leq J} \mu_{ij}) \tag{2}$$

If μ_{ij} represents a loss or a risk, then we have:

$$\min_{1 \leq i \leq I} (\max_{1 \leq j \leq J} \mu_{ij}) \tag{3}$$

These criteria ensure that the decisionmakers will at least realize the minimum gain or avoid maximum loss.

- **The Optimistic Rule (Maximax Criterion):** Following this criterion, the decisionmaker is most optimistic and seeks to maximize the maximum gain. Mathematically, the maximax criterion can be represented as:

$$\max_{1 \leq i \leq I} (\max_{1 \leq j \leq J} \mu_{ij}) \tag{4}$$

- **The Hurwitz Rule:** The Hurwitz rule offer the compromise between two extreme criteria through the use of an α -index. The decisionmaker's degree of optimism is specified through a parameter α that range between 0 and 1 ($0 \leq \alpha \leq 1$). More specifically, to apply the Hurwitz rule, one has to form a linear combination between the maximin and the maximax criteria for each alternative α_i :

$$\max_{1 \leq i \leq I} \mu_i(\alpha) = \max_{1 \leq i \leq I} [\alpha \min_{1 \leq j \leq J} \mu_{ij} + (1 - \alpha) \max_{1 \leq j \leq J} \mu_{ij}], 0 \leq \alpha \leq 1 \tag{5}$$

Note that for $\alpha=0$, $\max_{1 \leq i \leq I} \mu_i(\alpha)$ represents the maximax criterion; and for $\alpha=1$, $\max_{1 \leq i \leq I} \mu_i(\alpha)$ represents the maximin criterion.

3 Case study - Decision analysis for specialized equipment destined for monitoring the safety mining parameters

A small company for specialized equipment destined for used in the safety parameters monitoring field, wishes to come out with a new line of these type of equipment [10,11-14]. They decide they can make a high-performance, medium-performance, or economic (low-performance) model to release it in the business. It is assumed the company knows the ales potential for each of the equipment lines as eighter excellent, good, or poor (see Table 1). The probability of excellent ales is 0.25, good is 0.6, and poor is 0.15. the company wishes to decided on the best development plan based upon minimizing risk of financial loss (expected opportunity loss) and/or maximizing the expected profit:

Table 1. Sales potential.

Equipment (performance)	Excellent (RON)	Good (RON)	Poor (RON)
Economical	750,000	250,000	-100,000
Medium	1,500,000	875,000	-500,000
High	2,250,000	750,000	-750,000

Building Blocks of the Mathematical Model:

- Objectives:
 - Minimize risk of financial loss or expected opportunity loss; or
 - Maximize the expected profit
- Assumptions:
 - The company will produce only one type of equipment
 - The net return as a function of the sales is given (see Table 1)
 - Probability of excellent sales is 0.25
 - Probability of good sales is 0.6
 - Probability of poor sales is 0.15
- Decision Variables:
 - Which equipment to developed and sell on the open market
- Input Variables:
 - National and Comunitar support for small business
 - National and Comunitar regulation of the open market to maintain prices and/or stimulate the market with incentives
- Exogenous Variables:
 - Cost of manufacture for each of the equipment
 - Financial return for each of the equipment as a function of the sale potential
 - Cost for advertisement of new equipment
 - Probability of sales potentially assumed exogenous variable:
 - Probability of excellent sales is 0.25
 - Probability of good sales is 0.6
 - Probability of poor sales is 0.15
- Random Variables:
 - Periodic fluctuation of the market
 - Operation, maintenance, replacement fees for maintaining the production facility
- State Variables:
 - Number of each type of equipment
 - Financial return
- Output Variables:
 - Total number of each type of equipment
 - Net profit
- Constraints Variables:
 - Regulatory laws regarding investment in the production and sales of equipment
 - Resources available to manufacture equipment

The Pessimistic Rule (Maximin or Minimax Criterion):Objective

$$\max_{1 \leq i \leq l} (\min_{1 \leq j \leq l} \mu_{ij}) \quad (6)$$

Applying the pessimistic rule (maximize the minimum gain), the maximin criterion for the sale of equipments yields the following:

- For a_1 (economic): $\min(750,000; 250,000; -100,000)=-100,000$
- For a_2 (medium): $\min(1,500,000; 875,000; -500,000)=-500,000$
- For a_3 (high): $\min(2,250,000; 750,000; -750,000)=-750,000$

Thus, applying the maximum criterion implies a gain of at least 100,000 following a_1 – that is, the manufacture of economic equipment.

The Optimistic Rule (Maximax Criterion):

Objective

$$\max_{1 \leq i \leq I} (\max_{1 \leq j \leq J} \mu_{ij}) \tag{7}$$

Applying the optimistic rule (maximize the maximum gain), the maximax criterion for the sale of equipments yields the following:

- For a_1 (economic): $\max(750,000; 250,000; -100,000)=750,000$
- For a_2 (medium): $\max(1,500,000; 875,000; -500,000)=1,500,000$
- For a_3 (high): $\max(2,250,000; 750,000; -750,000)=2,250,000$

Thus, the best policy following the most optimistic criterion is to manufacture high equipment (a_3), yielding a return of at most 2,250,000.

Hurwitz Model:

Applying the Hurwitz rule, which compromises between two extremes through the use of the index α , yields the following:

Objective

$$\max_{\substack{1 \leq i \leq 3 \\ a_1, a_2, a_3}} [\mu_i(\alpha) = \alpha \min_{1 \leq j \leq 3} \mu_{ij}^{\text{Pessimistic (for } \alpha=1)} + (1 - \alpha) \max_{1 \leq j \leq 3} \mu_{ij}^{\text{Optimistic (for } \alpha=0)}] \tag{8}$$

$$0 \leq \alpha \leq 1$$

Tables 2, 3 and Figure 1 present a summary of the problem’s assumptions.

Table 2. Hurwitz Data.

i	j		
	1 (s_1)	2 (s_2)	3 (s_3)
1 (a_1)	750,000	250,000	-100,000
2 (a_2)	1,500,000	875,000	-500,000
3 (a_3)	2,250,000	750,000	-750,000

Solution:

- At a_1 : $u_1(\alpha)=-100,000\alpha + 750,000(1-\alpha)=750,000 - 850,000\alpha$
- At a_2 : $u_2(\alpha)=-500,000\alpha + 1,500,000(1-\alpha)=1,500,000 - 2,000,000\alpha$ (6)
- At a_3 : $u_3(\alpha)=-750,000\alpha + 2,250,000(1-\alpha)=2,250,000 - 3,000,000\alpha$

Results:

Table 3. Optimistic and Pessimistic Outcomes.

	Excellent (s_1)	Good (s_2)	Poor (s_3)	Optimistic	Pessimistic
Economical (a_1)	750,000	250,000	-100,000	750,000	-100,000
Medium (a_2)	1,500,000	875,000	-500,000	1,500,000	-500,000
High (a_3)	2,250,000	750,000	-750,000	2,250,000	-750,000

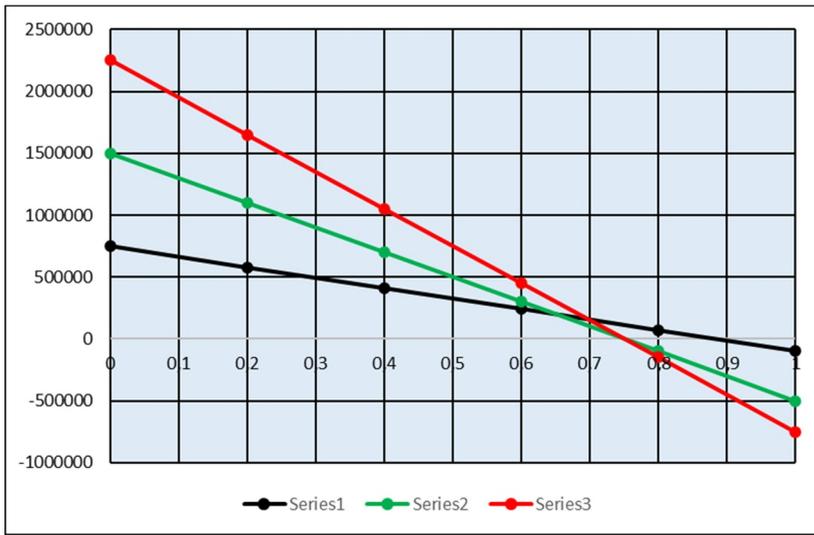


Fig. 1. Results for the Hurwitz Model: $u_1(\alpha)$ graph – black colour, $u_2(\alpha)$ graph – green colour and $u_3(\alpha)$ graph – red colour.

Intersection occurs at: $u_1(\alpha) = u_3(\alpha) \Leftrightarrow 750,000 - 850,000\alpha = 2,250,000 - 3,000,000\alpha$
 Therefore, $\alpha = 150/215 = 0.698$

Discussions:

Eq.6 represents straight-line functions of the variable α , $0 \leq \alpha \leq 1$. Plotting each of these straight lines as a function of α is depicted in Figure 1.

Thus, an analysis of the Hurwitz rule model indicates that for relatively optimistic decision levels, $\alpha \leq 0.698$, the high-performance equipment should be produced and sold.

For less optimistic decision levels of $\alpha > 0.698$, however, the economical equipment should be considered.

4 Conclusions

This paper focuses on the more philosophical, conceptual, and decision-making aspects of risk analysis. It addresses fundamental concepts of modelling and optimization of systems under conditions of risk and uncertainty, articulates the intricate processes of risk assessment and management, and presents a risk analysis methodology newly developed.

According to the Hurwitz rule model, for relatively optimistic decision levels, the high-performance equipment should be produced and sold.

For less optimistic decision levels, however, the economical equipment should be considered.

Through this case study, the results obtained will make it possible for managers to augment evidence gained through their professional experience with evidence collected through other means.

References

1. F. Nicolae, G. Vasilescu, *Modern decision support methods in the exploitation and securing of mineral resources* (UNIVERSITAS Publishing House, Romania, 2012)
2. A. M. Law, W.D. Kelton, *Simulation Modeling and Analysis* (McGraw-Hill, New York 1991)
3. W. J. Meyer, *Concepts of Mathematical Modeling* (McGraw-Hill, New York, 1984)
4. R. M. Oliver, J. Q. Smith (eds.), *Influence Diagrams, Belief Nets and Decision Analysis* (Wiley, New York, 1990)
5. K. Thearling, Data Mining and Advanced DSS Technology, an On-Line Data Mining Tutorial, 1998
6. M. Crouhy, R. Mark, D. Galai, *Risk Management* (McGraw-Hill, 2002)
7. G. Koller, *Risk Modeling for Determining Value and Decision Making* (Chapman & Hall/CRC, 2000)
8. D. Belluck, S. Benjamin, *A Practical Guide to Understanding, Managing and Reviewing Risk Assessment Reports* (CRC Press, 1999)
9. G. Koller, *Risk Assessment and Decision Making in Business and Industry: A Practical Guide* (CRC Press, 1999)
10. D. Hoffman, *Managing Operational Risk: 20 Firmwide Best Practice Strategies* (Wiley, 2002)
11. M. Van Asselt, *Perspectives on Uncertainty and Risk: The Prima Approach to Decision Support* (Kluwer Academic Publishers, 2000)
12. M. Bazerman, *Judgment in Managerial Decision Making* (Wiley, 1993)
13. T. Connolly, H. Arkes, K. Hammond (eds), *Judgment and Decision Making: An Interdisciplinary Reader* (Cambridge University Press, 2000)
14. D. Bouyssou, et al., *Evaluation and Decision Models: A Critical Perspective* (Kluwer Academic Pub, 2000)