Hopf Bifurcation for Detritus Food Chain in Mangrove Ecosystem

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Abstract. We proposed a prey-predator model portraying interaction in Detritus food chains. The primary view of this paper is to describe the interaction in the Detritus food chain due to the presence of a predator in mangrove ecosystems. Predation by assuming that the predator takes more time to eat its prey. We analyze the local stability of the equilibria point. Numerical simulations are achieved to clarify the theoretical result and display the change in the equilibria point solution through a bifurcation diagram. The numerical continuity of carrying capacity parameters in the system solution indicated the presence of the Hopf Bifurcation.

Keywords. Food chains, Hopf-Bifurcation, Stability, numeric-simulation

1 Introduction

In the natural environment, food chains describe phenomena and interactions between prey-predator species [1]. The development of mathematical models of ecological systems has been studied by many authors [2-5]. From a mathematical and biological nook, the predator-prey model can be stated as a system of differential equations [6]. Models [7] are based on ecology that follows the form of Lotka-Volterra equations and equations that include interaction on detritus food chains. Neutel explains that the two-species detritus-based food chains are asymptotically stable [8]. Model [9] discusses how dynamics of the detritus environment and the stability of food webs.

Different ecological systems also give rise to other interactions between other predators and prey. Thus, the pattern of predation of predators on prey can change, in this case, called functional responses [10-13]. The Holling–Tanner model has been used widely to model many real-world species interactions and views a unique role in the exciting dynamics [14-15]. We examine the Holling–Tanner functional response for the predation of our proposed interaction model. This current article analyzes the local stability of all existing fixed points and possible bifurcation.

The comity of this paper is as follows: the mathematical model is constructed in section 2, the existence of the equilibria of the model and the local stability properties in section 3, and in the next section some numerical simulations to clarify our analytical findings, and then the conclusion in the last section.

2 The Mathematics Model

2.1 The Assumption

Researchers are interested in the study assumption [16] where a single population grows logistically with carrying capacity no longer a parameter that has a constant value. In this assumption, environmental carrying capacity in the Mangrove ecosystem becomes a variable whose magnitude is also influenced by time. Different assumptions with [17-19], this study assumes that the environment carrying capacity grows naturally and there is less interaction between the two populations. For a more natural functional response and the Leslie-Gower predator's growth rate model was used [20-22].

2.2 The Formulation

The proposed model is the Holling Taner prey-predator model with the growth rate of prey depending on environmental carrying capacity defined as follows:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \frac{\alpha xy}{m + x},$$
$$\frac{dy}{dt} = \beta y\left(1 - \frac{y}{x}\right)$$

$$\frac{dk}{dt} = k(\rho - \delta x - \mu y) \tag{1}$$

With $x(t)$ and $y(t)$ represent the population of prey and predator at time t. The function $k(t)$ means the carrying capacity of Detritus in the Mangrove Ecosystems. The parameters r and β denote the natural growth rate of prey and predator. The parameters $\alpha, m, \rho, \delta, \mu,$ and ρ are the parameters that half capture saturation constant. System (1) satisfies the initial state $x(0) > 0, y(0) > 0,$ dan $k(0) > 0.$

3 The Stability Analysis

3.1 The Fixed Points

The fixed points of the system (1) is obtained. When all population growth rates are zero. The fixed points illustrated a constant solution of the system.

$$\begin{aligned} &rx\left(1-\frac{x}{k}\right)-\frac{\alpha xy}{m+x}=0,\\ &\beta y\left(1-\frac{y}{x}\right)=0,\\ &k(\rho-\delta x-\mu y)=0. \end{aligned} \quad (2)$$

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System (2) has three proper equilibria points.

Case 1: The equilibria of predator extinction

$$T_1=\left(\frac{\rho}{\delta},0,\frac{\rho}{\delta}\right)$$

Prey can survive without predation by predators and depend on the carrying capacity of Detritus in the Mangrove Ecosystems. As the sign of the elements is positive, therefore T_1 always exists.

Case 2: The interior equilibria

$T_2=(x^*,y^*,k^*)$, with $x=\frac{\rho}{\delta+\mu}, y=\frac{\rho}{\delta+\mu},$ and $k=\frac{r\rho(\delta m+\mu m+\rho)}{\delta^2 mr+2\delta\mu mr+\mu^2 mr+\delta r\rho+\mu r\rho-\alpha\mu\rho-\alpha\delta\rho},$ if $\delta^2 mr+2\delta\mu mr+\mu^2 mr+\delta r\rho+\mu r\rho>\alpha\rho(\mu+\delta).$

T_2 is where all species can survive and live harmoniously. In its equilibria, the population of prey entirely depends on the carrying capacity of Detritus in the Mangrove Ecosystems, and the population of predators depends on the prey population.

3.2 The Local Stability

We need to evaluate the Jacobian matrix for system (1) to find the local stability of the various solutions [23].

Theorem 1. The local stability is always unstable for the equilibria point $T_1=\left(\frac{\rho}{\delta},0,\frac{\rho}{\delta}\right)$

Proof.

The exact appraisal matrix of Jacobian at T_1 offer

$$\left|J\left(\frac{\rho}{\delta},0,\frac{\rho}{\delta}\right)-\lambda I\right|=0$$

$$\begin{vmatrix} -r-\lambda & -\frac{\alpha\rho}{\delta\left(\frac{\rho}{\delta}+m\right)} & \mathrm{r}\\ 0 & \beta-\lambda & 0\\ -\rho & -\frac{\rho\mu}{\delta} & -\lambda \end{vmatrix}=0.$$

The roots are $\lambda_1=-\frac{\mathrm{r}}{2}+\frac{1}{2}\sqrt{-4r\rho+\mathrm{r}^2}$, $\lambda_2=-\frac{\mathrm{r}}{2}-\frac{1}{2}\sqrt{-4r\rho+\mathrm{r}^2}$, and $\lambda_3=\beta$. Because all parameters in (3) are all positive, then λ_1 and λ_2 should be negative. As λ_3 is always positive. Hence, the equilibria T_1 are unstable.

4 The Simulations of Numerics

In addition, we offer some numerical simulations to define the solution system by phase portraits and phenomena of bifurcation. Numerical simulations clarify the theoretical result. The bifurcation shows the change in the equilibria point solution by selecting the parameter values according to Table 1

Table 1. The Parameters value for simulation

Parameter	Example	Value
r	The intrinsic growth rate of prey	2.23
α	Maximal predator per capita consumption rate	0.8
m	Half saturation constant	1.32
β	The natural growth rate of predator	0.28
ρ	The growth rate of the environmental	0.6
δ	The natural death of the prey	0.31
μ	The natural death of the predator	0.19

4.1 Phase Potrait

First, varying the parameter growth rate environmental carrying capacity (ρ) may lead to stable equilibria. When parameter $\rho=1.6$, the solution system (1) tends to T_2 stable (see Fig 1). When $\rho=1.6<\rho^*$ the system (1) will be stable after a specific time. The locally asymptotically stable represent by the green arrow lines goes T_2. (see Fig. 1).

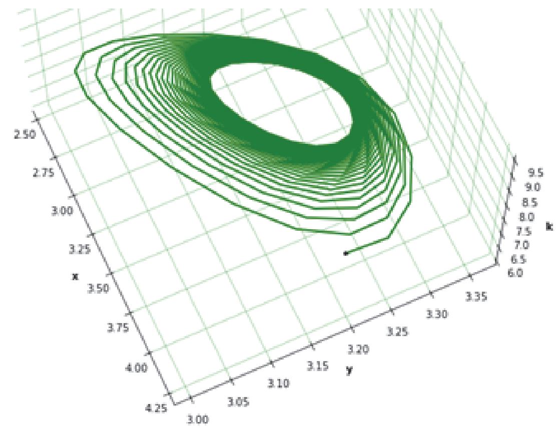

Fig. 1. Phase portrait for system (1) for case ϱ=1.6>ϱ^*.

The second simulation shows blue arrow lines representing the prey population, the magenta line representing the predator population, and the green line representing the environmental carrying capacity of the Mangrove ecosystem. (see Fig. 2).

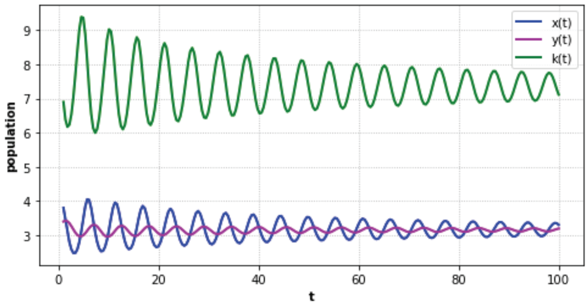

Fig. 2. Times series the solution system (1) represent time trajectories for prey, predator, and environmental carrying capacity at at $\rho = 1.6$

The time series graph with Python simulation shows that the system's positive equilibria (1) are locally asymptotically stable for $\rho = 1.6 < \rho^* = 1.614$. (see Fig. 2). Hopf bifurcation occures for $\rho = 1.61475 > \rho^* = 1.614$. (see Fig. 3).

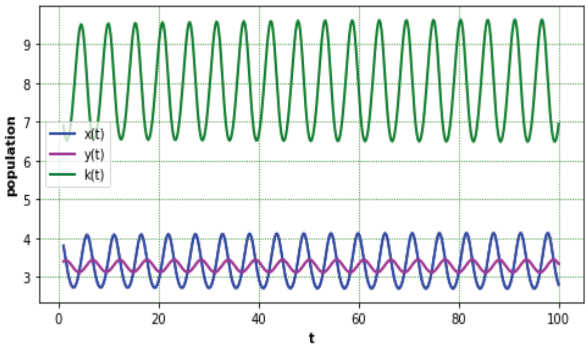

Fig. 3. The dynamics of the system (1) represent time trajectories for prey, predator, and environmental carrying capacity at at $\rho = 1.61475$

The probability of occurrence of limit cycle dynamics (Hopf bifurcation) near the interior point $T_2{}^*(x^*, y^*, k^*)$ should be analyzed by varying ρ and setting all other parameters as constan using the parameter values in Table 1. The equilibria point T_2 generates an unstable limit cycle as bifurcation as parameter ρ (see Fig. 4).

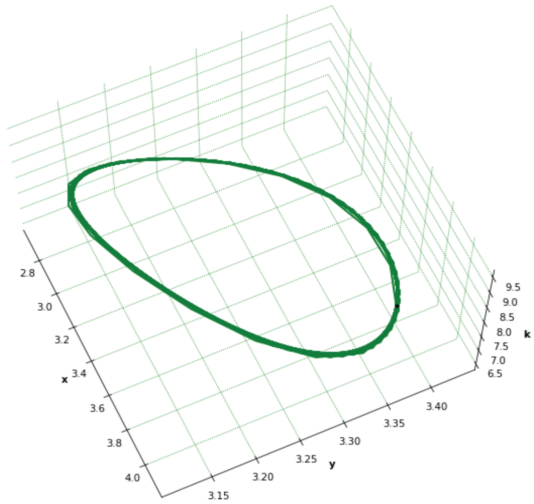

Fig. 4. The stability of equilibria point changes from stable to unstable and indicated the appearance of Hopf bifurcation

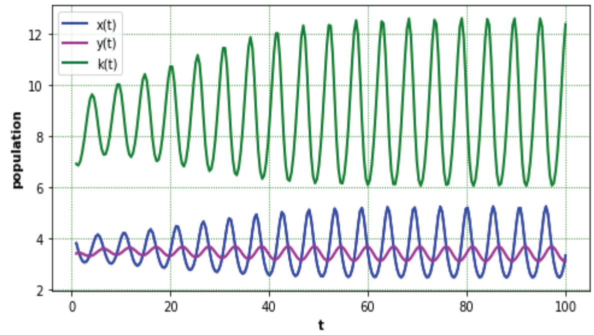

Fig. 5. The time series graph shows the system's solution (1) $\rho = 1.69$

Finally, we vary the parameter of the growth rate of environmental carrying capacity, which is $\rho = 1.69$, the other parameter values remain constant in Table 1. Therefore, phase portrait solution system (1) in equilibria points for T_2 (see Fig. 4).

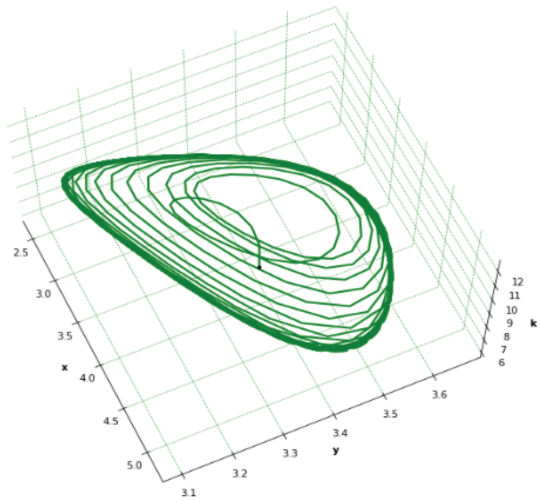

Fig. 6. A phase portrait with a solid green circle shows unstable focus if $\rho = 1.69$.

4.2 Hopf Bifurcation

However, we perform the numerical simulation showing that the limit cycle undergoes periodic. We find the Hopf bifurcation when $\rho = 1.61475$ and stable. After this point, the stability of equilibria points to T_2 unstable as shown in Figure 7.

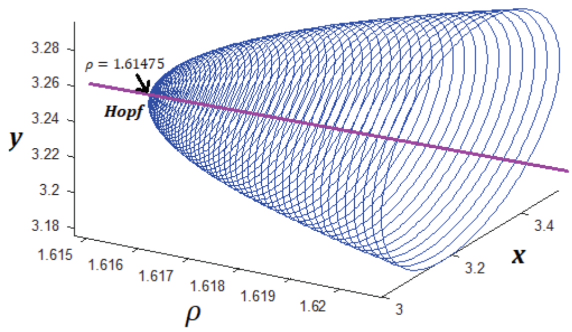

Fig. 7. Hopf Bifurcation in point at $\rho = 1.61475$

5 Conclusions

Our model uses the Holling-Tanner functional response of predation. The dynamics of a predator-prey system can be quite complex if one considers other Holling-type functional responses. Our model also considers the carrying capacity of Mangrove ecosystems for prey growth rate and prey-dependent growth rate for predators. The results of the numerical continuation of the variation of biological growth parameters carrying capacity obtained by the Hopf bifurcation.

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