

Dynamics Twin Cannibalism of Two Predator and Two Prey System with Prey Defense

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Abstract. The study discusses the condition of the existence of all non-negative equilibrium points. There are 9 realistic equilibrium points from the constructed model. A local stable condition is obtained, a point of equilibrium that is completely biologically feasible. The analytical method on the mathematically formed model is limited, so numerical simulation is also given to explore the model. Numerical simulation is intervened in a model that will show growth in trajectories. The tendency of trajectories in prey one and predator one species is relatively the same because the interactions that occur are intensive. Likewise, prey two and predator two occur, and the interactions that occur cause population growth grow to fluctuate. Differences occur in both types of species, namely predator-prey one and predator-prey two. In the one interaction group, growth tends to be more volatile and moves slowly towards the point of stability in population growth. Incidence is inversely proportional to the interaction of species two which tend to be faster towards the stability point. In general, the results of numerical simulations show that there is a pattern formation in the predator-prey system that grows sustainably.

Keywords. Twin Cannibalism, Predator-Prey, Defense

1 Introduction

Predation by predators follows the predation cycle according to the predation characteristics of the species. Species with predation characteristics waiting for their prey tend to be passive in the steps of the predation process [1][2]. In the types of predatory species that are actively preying on interactions occur gradually. The process of successful predation by predators is carried out repeatedly to complete the predation process, such as monitoring, catching, chasing, and consuming [3][4]. In real ecosystem conditions, there are also some special cases for predation, which do not go through these steps.

Some species are cruel in ecological terms, eating individuals of their own species. Predation like this often occurs in species that live in highly competitive ecosystems. Such predation is known as cannibalism [5]. In previous research, many have stated that cannibalistic traits are only owned by special predators [6]. Meanwhile, the latest research shows that most cannibalistic traits are owned by predators. The cannibalistic nature is much influenced by environmental changes that occur, especially in the marine ecosystem. Cannibalism provides an advantage for controlling population growth, but on the other hand, cannibalism increases the mortality of the predator itself. Fierce and gruesome battles are more common among predators than prey [7][8][9].

Several research reports indicate that cannibalism is a natural phenomenon that normally occurs in many populations, to reduce the population before food and survival resources are limited [10]. Many special cases of cannibalism occur also not because of the availability of food, it actually occurs naturally. In prey, most species avoid predation interactions with predators [11][12]. Predation interactions are considered a threat that can lead to fear and resistance. The resistance of prey species is commonly referred to as defense because prey species do not attack back to eat predators but to take refuge in groups [13]. Prey defense can provide a strong dynamic in predator-prey interactions. Predators themselves naturally avoid risky or dangerous prey. The number of predatory species is few and prey species are abundant, allowing the rate of predation to decrease. Another way to avoid unique predators is beneficial adaptations to prey, such as running and physical and chemical defenses [14]. Some prey species release chemicals from their bodies that can distance, injure and even paralyze predators.

The predation process formed in this research is a special interaction model in the predator-prey model with the effect of prey and predator defense cannibalism on passive species. Many previous studies have considered the interaction of cannibalism, but the specific interaction of two prey and two predators with twin cannibalism has not been discussed in previous

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empirical studies [15]. The mathematical assumptions that will be formed refer to the previous discussion related to cannibalism and species defense mechanisms [16]. The mathematical component of the model assumes that there are two species of prey and two species of predator. Each predator has predation characteristics according to the factors as the prey species. The special interaction that is interesting is that the process of predation on prey is softer than the interaction of cannibalism. Predation on prey, interactions follow the Holling type I concept, while cannibalism interactions follow the Holling Type II response function [17]. An event like this is very realistic to consider because in the ecological world, competition with other species will be much more sensitive for each cannibalism.

Cannibalism and predatory resistance behavior greatly influence the dynamics of the predator-prey model in the ecological world. Theoretically, cannibalism and prey resistance have the ability to reduce the number of populations or inhibit the growth of predators [18]. In the theory of population growth, the nature of cannibalism is very detrimental, because the population is decreasing. In extreme cases, predators can interact with cannibalism to destroy them. Species that have a predestined predatory nature and can do cannibalism are *Arachnoidea*. The *Arachnoidea*, the most extreme types of predators practicing cannibalism are the *Latrodectus genus* and *Thomisidae* [19][20]. The monotonic function used in the Holling Type I response function is simple and has many species that match the characteristics of this type. Mathematically the assumption used for this characteristic is that the process of predation, handling, and prey reconnaissance time is ignored simultaneously [6]. Characteristics that appear often occur in passive predators who prefer to wait for their prey. Predation that occurs is linear and highly dependent on prey density. Using the notation of Holling paper [21] we may express this relationship in the form $y = at_s x$, where y is the number of prey consumed by one predator, x is the prey density, t_s is the time available for searching and a constant of proportionality, termed the 'discovery rate', Holling. The mathematical form of Holling Type I above has been widely studied by previous research. Assuming that there is no handling time required by the predation process, all available time is maximized to pay attention to incoming prey $t_s = t_t$. In the case of a type I response function, it is assumed that the predator has p an independent density. In time t_t , the total number of preys will decrease in shape $y = at_s xp$. The Holling Type I response function formula is linear for small prey densities. In its long time, t_t the amount of prey consumed by one predator becomes, $y = \min\{at_s x, y_{\max}\}$.

Several research results also show that this form of cannibalism often occurs in predators and occurs naturally, not depending on food availability or density [22]. The predatory nature of cannibalism like this appears like an instinctive trait that arises from within

the species. A form of cannibalism $f(y) = \frac{\alpha y^2}{y + b}$, which

does not consider cannibalism that depends on factors such as food availability and density of prey. The

$-\frac{\alpha y^2}{y + b}$ is the mortality function for predators, where

α is the rate of cannibalism and b is the saturation constant.

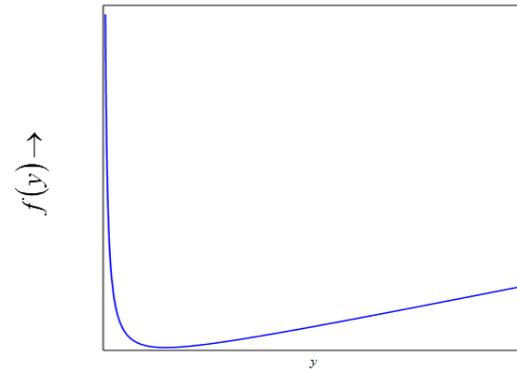


Fig. 1. Representation of cannibalism function

$$f(y) = \frac{\alpha y^2}{y + b} \cdot y \rightarrow$$

2 Formulation of System Model

Research on the predator-prey model with cannibalism mostly refers to research [14]. It discusses the specifics of cannibalism in predators and defense in prey. In this research, predators with cannibalism are considered to be developed in twin species, so we call it twin cannibalism. In the mathematical model compiled, four species are considered, each with two prey and two predators. The food chain model in the adopted model is a Gaussian type with four driving species. Species prey one $x_1(u, t)$, prey two $x_2(u, t)$, predator one, $y(u, t)$ and predator two $z(u, t)$. In prey, it is assumed that the characteristics of life have identical characteristics, while in predators each has identical characteristics in the form of cannibalism.

Predatory interaction that occurs in each predator is Holling Type I predation, while cannibalism predators use Holling Type III. Changes in the type of predation are strongly influenced by the type of prey eaten. While interactions with similar cannibalism characters are not meant to eat, the goal is to kill. This characteristic makes interaction cannibalism more extreme because there is a process of interaction duels. We make the following assumption to formulate the model, the population growth of prey one and prey two follows the intrinsic growth mechanism in r_1 and r_2 . All prey population growth is highly dependent on carrying capacity k .

Mathematically the assumed model is,

$$\frac{\partial x_1}{\partial t} = r_1 x_1 \left(1 - \frac{x_1}{k}\right) - \alpha x_1 y, \quad t > 0, \quad (1)$$

$$\frac{\partial x_2}{\partial t} = r_2 x_2 \left(1 - \frac{x_2}{k}\right) - wx_2 z, \quad t > 0,$$

$$\frac{\partial y}{\partial t} = wx_1 y - \delta_1 y - \frac{cy^2}{y + a_3}, \quad t > 0,$$

$$\frac{\partial z}{\partial t} = wx_2 z - \delta_2 z - \frac{cz^2}{z + a_3}. \quad t > 0,$$

with boundary and initial conditions,

$$x_1(u, t) > 0, x_2(u, t) > 0, y(u, t) > 0, \text{ and } z(u, t) > 0.$$

The study considers the dimensions of each variable, presented in the following table of parameters and units,

Table 1. Description and unit of Parameters in Model.

Parameters	Description	Unit
r_1	Intrinsic growth rate of prey one	$[T]^{-1}$
r_2	Intrinsic growth rate of prey two	$[T]^{-1}$
k	Carrying capacity of prey in the environment.	$[N]$
w	Reduction rate of prey due to predation	$[T]^{-1}$ $[N]^{-1}$
δ_1	Mortality rate of predator one	$[T]^{-1}$
δ_2	Mortality rate of predator two	$[T]^{-1}$
c	Cannibalism rate among top predator	$[T]^{-1}$ $[N]$
a_3	Half saturation constant of predator	-

3 Result and discussion

3.1 Equilibrium model

This section describes the equilibrium point analysis in model (1). Mathematically, model (1) is solved by a differential equation model. Model analysis (1) shows a total of 9 equilibrium non-negatif. The overall equilibrium points are shown in the following table,

Table 2. Equilibrium in Model.

Equilibrium	Description
w_0	$(0,0,0,0)$
w_1	$(k,0,0,0)$
w_2	$(0,k,0,0)$
w_3	$(k,k,0,0)$
w_4	$(x_1,0,y,0)$
w_5	$(0,x_2,0,z)$
w_6	$(x_1,x_2,y,0)$
w_7	$(k,x_2,0,z)$
w_8	(x_1,x_2,y,z)

where for equilibrium involving the values of A and B is,

$$A = -\frac{\delta_2 a_3}{c + \delta_2} \text{ and } B = -\frac{\delta_1 a_3}{c + \delta_1},$$

for both values, we ignore them, because they do not meet the rationale for taking equilibrium or by using the

Routh-Hurwitz criteria, both values are not valid. Will never be positive. In other words, the equilibrium is not desired to lead to model equilibrium (1). Meanwhile, for equilibrium containing a value of 0, no meaningful analysis is given to model (1).

Each equilibrium point has characteristics in meeting the equilibrium model (1), with $\frac{\partial x_1}{\partial t} = 0$,

$$\frac{\partial x_2}{\partial t} = 0, \frac{\partial y}{\partial t} = 0 \text{ dan } \frac{\partial z}{\partial t} = 0. \text{ The existence of an}$$

equilibrium point in model (1) is analyzed on the dynamic system in that model. Of the 9 equilibrium, there are several that exist in the dynamic system in a model (1), which are non-negative, including the following,

- (i) Equilibrium w_0 always exists, trivial extinction state solution.
- (ii) Equilibrium w_1 and w_2 always existence, where growth exists in the prey one population and ignores growth in the other population. Meanwhile, the same thing also applies to the existence of prey two population growth, ignoring the growth of other populations. It can be said that, in the absence of predatory species, prey species' existence grows in existence.
- (iii) Equilibrium w_3 , w_4 and w_5 always existence, population prey one grows with predator one continuously, without population for prey two and predator two. The opposite is also true for growth for the population prey two and predator two, meaning without species prey one and predator one.
- (iv) Equilibrium w_6 and w_7 always existence, with the condition that the prey population grows continuously but not with predatory species, the growth of the existing model (1) is in a dynamic system without predator one or predator two.
- (v) Let us suppose w_8 , is a positive solution model (1). At this equilibrium the model can maintain the existence of the four populations on an ongoing basis.

The following will show the local stability analysis of the model form (1). All equilibrium points will be tested for stability because in ecosystem ecology all have the opportunity to live sustainably or have no hope of population growth, in other words, extinct populations. The following are all possible equilibrium values in a model (1) show by the Jacobian matrix.

$$J(w_i) = \begin{bmatrix} j_{11} & j_{12} & j_{13} & j_{14} \\ j_{21} & j_{22} & j_{23} & j_{24} \\ j_{31} & j_{32} & j_{33} & j_{34} \\ j_{41} & j_{42} & j_{43} & j_{44} \end{bmatrix} \quad (2)$$

where,

$$j_{11} = r_1 \left(1 - \frac{x_1}{k}\right) - \frac{r_1 x_1}{k} - wy,$$

$$j_{13} = -wx_1,$$

$$\begin{aligned}
 j_{22} &= r_2 \left(1 - \frac{x_2}{k} \right) - \frac{r_2 x_2}{k} - h z, \\
 j_{23} &= -h x_2, \\
 j_{13} &= w y, \\
 j_{33} &= w x_1 - \delta_1 - \frac{2cy}{y+a_3} + \frac{cy^2}{(y+a_3)^2}, \\
 j_{42} &= h z, \\
 j_{44} &= h x_2 - \delta_2 - \frac{2cz}{z+a_3} + \frac{cz^2}{(z+a_3)^2}.
 \end{aligned}$$

The characteristic equation associated with the jacobian matrix $J(w_8)$ is,

$$f(\lambda) = \lambda^4 + A_1 \lambda^3 + A_2 \lambda^2 + A_3 \lambda + A_4, \quad (5)$$

The characteristic equation will show the eigenvalues that meet the Routh-Hurwitz criteria and at the same time answer the local asymptotic stability for equilibrium w_8 .

3.2 Numerical simulation

Simulation in model (1) is carried out to provide an initial description of the growth simulation of each species population. Parameters taken are based on assumptions and mostly based on relevant references in this study. The effect of interactions that occur in each species will be shown in the trajectories population. Numerical simulation will also be shown in the equilibrium point and equilibrium point stability section of model (1).

The parameters taken are $r_1 = r_2 = 1.5$, $k = 100$, $h = 0.22$, $a = 0.9$, $b = 0.8$, $m = 0.3$, $\delta_1 = 0.052$, $\delta_2 = 0.5$, $a_1 = 8$, $a_2 = 2$, $a_3 = 3$, $\sigma = 0.9$, $\beta = 0.05$, $w = 0.55$ and $c = 0.01$. The analysis of numerical simulations, the focus is on discussing non-negative equilibrium points that are relevant and affect population growth. As for the non-negative equilibrium points, there are 8 of them, $w_0 = (0,0,0,0)$, $w_1 = (100,0,0,0)$, $w_2 = (0,100,0,0)$, $w_3 = (100,100,0,0)$, $w_4 = (0.1031987807, 0.2.724458215, 0)$, $w_5 = (0, 2.304067087, 0, 6.661086335)$, $w_6 = (0.1031987807, 100, 2.724458215, 0)$, $w_7 = (100, 2.304067087, 0, 6.661086335)$, and $w_8 = (0.1031987807, 2.304067087, 2.724458215, 6.661086335)$.

Consideration to preserve all population species used in this study, mathematically the w_8 realistic equilibrium to be selected is carried out using the Routh-Hurwitz criteria. The characteristic equations associated with the Jacobian matrix at equilibrium w_8 are, $\lambda^4 + 0.04074418458 \lambda^3 + 0.8281006896 \lambda^2 + 0.006124619266 \lambda + 0.06318712782 = 0$ so that the eigenvalues of the characteristic equations are $\lambda_1 = -0.0020210943815$, $\lambda_2 = -0.0020210943815$, $\lambda_3 = -0.0183509979125$ and $\lambda_4 = -0.0183509979125$.

All eigenvalues show the results $\lambda_1 < 0$, $\lambda_2 < 0$, $\lambda_3 < 0$ and $\lambda_4 < 0$, this confirms that the equilibrium point towards asymptotic stability. All species can grow and live in predator-prey interactions for long periods of time

The following will show the overall population growth trajectories. The initial values taken for the simulation process are $x_1 = 0.1$, $x_2 = 2.3$, $y = 2.7$ and $z = 2.7$. All initial values are taken based on the time period for the t growth of the species population. Population growth trajectories can be seen as follows,

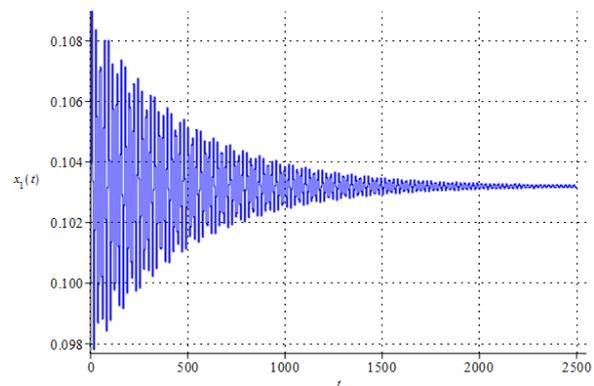


Fig. 2. Trajectories Population Prey One.

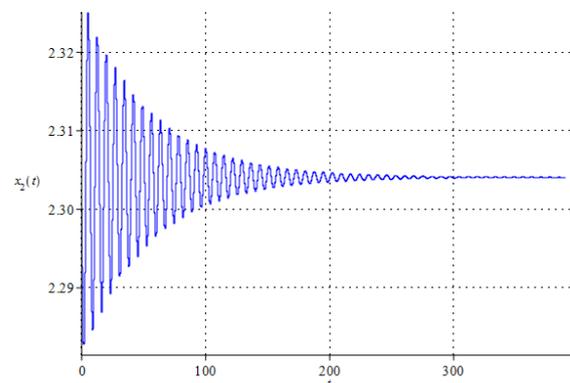


Fig. 3. Trajectories Population Prey Two.

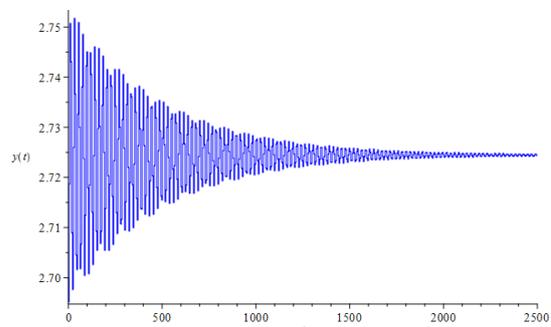


Fig. 4. Trajectories Population Predator One.

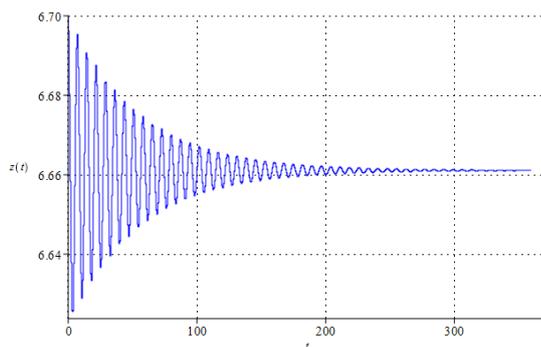


Fig. 5. Trajectories Population Predator Two.

Fig. 1, fig. 2, fig. 3, and fig. 4, it can be seen that population growth fluctuated significantly. All populations experience large fluctuating movements, depending on the coefficient of each population growth. In the prey one species, it is clear that the fluctuating density is very large so that population growth tends to rise and fall sharply. A similar incident occurred in the growth of the predator one species which became the opposite of the prey one species interaction. Events like this have been studied a lot, this is in line with previous research studies, species that interact with each other have a strong influence on each population growth. The same thing also happened to the prey two and predator two species, the effect of fluctuating interactions was very visible. The movement of prey one population growth is faster than the growth of predator two, it is possible that attacks from predator two are seen more, so the growth is very volatile. When compared to the two interactions, predator-prey one and predator-prey two, it tends to be more stable in a short time in the predator-prey two interaction model. Meanwhile, in the predator-prey one interaction model, it takes a long time to reach a stable growth point. All species growing in a model (1) reach a point of stability over a long period of time. It can be said that all species populations do not experience extinction, but experience significant fluctuations in the early interactions between these species.

4 Introduction

In the mathematically formulated model, provides an explanation that the dynamic model of population growth in species that have cannibalism traits forms special characteristics. Ecologically, species that have these characteristics tend to have different characteristics from most species. The tendency for traits and characteristics to be formed can be the result of environmental, genetic, or even internal conditions of the species such as being threatened, the transition of food sources, and existence. Model (1) provides a simple description of the species with special characteristics. The assumption of the model used is close to the condition of living species. The results of model (1) with mathematical differential equations provide 9 non-negative equilibrium points. The analysis of the stability of the equilibrium point is continued at a realistic equilibrium point both mathematically and

ecologically, $w_8 = (x_1, x_2, y, z)$. The equilibrium point w_8 met the local asymptotic stability criteria in the Routh-Hurwitz criteria. The research results are reflected in trajectories in the numerical simulation section. Trajectories species prey one, prey two, predator one and predator two each have their own characteristics. The tendency of trajectories in prey one and predator one species is relatively the same because the interactions that occur are intensive. Likewise, prey two and predator two occur, and the interactions that occur cause population growth grow to fluctuate. Differences occur in both types of species, namely predator-prey one and predator-prey two. In the one interaction group, growth tends to be more volatile and moves slowly towards the point of stability in population growth. Incidence is inversely proportional to the interaction of species two which tend to be faster towards the stability point. Overall population growth can occur and with the interaction of cannibalism, population growth is maintained for the survival of the species.

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