Seismic analysis of reinforced concrete building accounting for local uplift, large rotation of the base and soil plasticity

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Abstract. This study provides an analysis of multistory reinforced concrete residential buildings to earthquakes considering local uplift of foundation, soil behaviour, site category and seismicity of area as suggested by the Moroccan seismic regulation. The superstructure was modeled as a flexible column with lumped mass placed at the top of the column under the hypothesis of large rotations or small rotations. The governing equations of the coupled system were obtained by expressing momentum conservation principle and were solved by means of adequate time integration numerical scheme. Then, the maximum seismic response of the coupled system was determined under El Centro ground motion for various configurations of the coupled mat-foundation system: elastic or elastic-perfectly plastic, as well as soil class and seismic magnitude. It was found that the nonlinear seismic demand of a reinforced concrete building, in terms of maximum values of horizontal and vertical displacements and rotation, depends considerably on the assumption regarding rotation magnitude. It is governed also by the height of the structure and the soil stiffness.

1 Introduction

In earthquake engineering, there are two aspects that are of primary importance when considering the building foundation interaction. Firstly, it is known that the response of a structure supported by a flexible foundation, under the action of an earthquake motion, can be significantly different to that obtained in case of rigid foundation. Secondly, the response of the structure in terms of displacement and shear effort is sensitive to the contact interface conditions at the foundation level and soil stiffness.

To take into account foundation uplift on the dynamic response of flexible multi-storey structures during earthquakes, a model of rigid foundation mounted on two-springs to represent soil deformation was introduced [1-3]. The authors have found that uplift of foundation is not always conservative. They have found also that uplift effect depends on the considered structure and on the characteristics of the ground input motion. Karapetrou et al. [4] investigated the influence of interaction between soil and structure and site effects on fragility curves of a 9-story reinforced concrete building. Both linear elastic and nonlinear soil behavior have been considered. The obtained results have indicated that soil structure interaction plays a significant role, for both linear and nonlinear soil, and modifies the structural response and resistance to earthquake.

This work is a continuation of a previously published work [5] where the foundation soil was assumed to be elastic of Winkler type. It aims at investigating further the influence of local uplift, soil plasticity, site category and seismic intensity zone as suggested by the Moroccan seismic regulation [6] on the response of buildings. Scaled ground-motion recording obtained from El Centro 1940 earthquake is selected as seismic excitation. Two cases of study consisting of three and five storey reinforced concrete residential building are considered. These structures were modelled by equivalent flexible mat which is assumed to undergo small deformation, while the rigid base may undergo large rocking rotations. Both assumptions of small and large rotation motion of the system about its base are taken account.

2 Materials and method

2.1 Structure and soil models

The structures analyzed in this work correspond to common reinforced concrete residential buildings in Morocco having regular design. The structural system consists of a set of frames made of columns and beams that are linked at joints. According to the Moroccan seismic regulation, this kind of regular buildings can be represented by a single linear degree of freedom (SDOF) system with assuming the foundation to be rigid [7]. The associated SDOF system has mass, m , height, h and is characterized by its natural frequency $\omega=\sqrt{k/m}$ and damping ratio $\xi=c/(2m\omega)$ where k stands for lateral stiffness and c for lateral damping. The SDOF system can have the merit to be enough in representing, in some situation, even complex multi-story buildings provided their behavior is close to that of a single oscillator.

In the following the previous assumption regarding

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infinite rigidity of foundation is relaxed. The mat is assumed to be raised on a foundation which is rigid, but that may undergo some rotation because of soil deformation. The foundation is idealized here as a rigid segment having very small thickness. Its mass, denoted m_0, is taken to be uniformly distributed; the width of the base is denoted $2b$ and the total moment of inertia about the center of the foundation is referred to as I_0.

Fig. 1 illustrates the mat-foundation system proposed to model the effect of soil flexibility on the coupled dynamics. In this figure, $\vec{d}_r$ is the mat tip rigid displacement in the horizontal direction due to rotation of the base, $\vec{d}_e$ is the mat tip elastic displacement in the horizontal direction relatively to the rigid footing, $\ddot{u}_g$ is the acceleration resulting from input ground motion, v is the mat base centre of gravity displacement in the vertical direction, θ is the mat base angle of rotation and ψ the mat flexible rotation angle due to the deformation of the structure. The mat elastic deformation is assumed to be small so as $sin\,\psi$ and $cos\,\psi$ can be respectively approximated by ψ and 1.

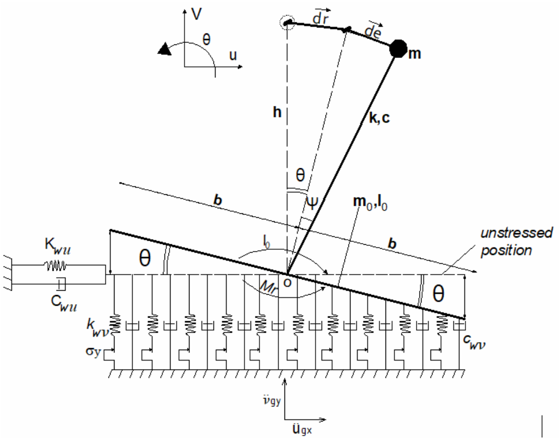


Fig. 1. Flexible structure on elastic-perfectly plastic foundation

The soil-structure interaction occurs at the interface between the foundation soil and the rigid footing. The behavior of this coupling interface is modeled by distributed vertical Saint-Venant's units. Each unit consists of an elastic spring connected in series with a plastic element which is activated if the strain from elastic spring reaches the yield stress σ_y, according to the constitutive behavior shown on Fig. 2. k_{wv} denotes the initial soil modulus of elasticity per unit length of the base, while c_{wv} soil is soil damping per unit length.

In the horizontal direction, the presence of laterally deformable soil causes the foundation input motion to deviate from that associated to a free-field ground motion. This lateral interaction is modeled here as restricted by the surrounding soil lateral stiffness K_{wu} and damping C_{wu}.

Fig. 2 illustrates the hysteresis model describing both foundation yielding and uplift. In the elastic range of strain, the soil stress-strain relationship is given by: $\sigma = -k_{wv}\left(v_0 + x\,sin\,\theta\right)$ where x is the abscissa along the foundation base with the origin taken at its centre. In loading conditions, when the stress reaches the threshold $-\sigma_y$ for some plastic frontier point x_p, then $\sigma = -\sigma_y$. Also, local uplifting occurs if $\sigma = 0$ for some critical position point which position is denoted x_c.

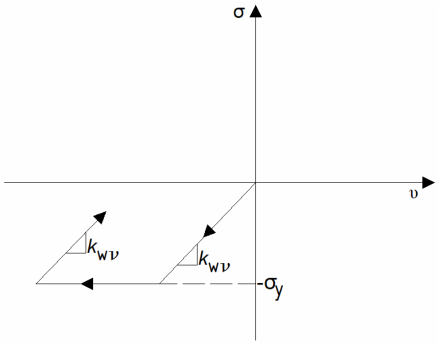


Fig. 2. Elastic perfectly plastic behavior of a Saint-Venant's unit, uplift occurs to the right of the origin and elastic-plastic displacement in compression to the left of the origin.

Let us introduce the following notations:
- slenderness ratio $\alpha = h/b$, where b is the half width of the foundation;
- natural frequency of the rigidly supported structure $\omega = \sqrt{k/m}$;
- horizontal vibration frequency of the mat-foundation base system $\omega_u = \sqrt{K_{wu}/(m_0 + m)}$;
- vertical vibration frequency of the mat-foundation base system $\omega_v = \sqrt{2k_{wv}b/(m_0 + m)}$;
- horizontal frequency ratio $\beta_u = \omega_u/\omega$;
- vertical frequency ratio $\beta_v = \omega_v/\omega$;
- foundation mass to superstructure mass ratio $\gamma = m_0/m$;
- ratio of yield stress to total vertical rigidity of the foundation soil $\delta = \sigma_y h^2/(m_0 g)$, where g is the gravitational acceleration;
- damping ratio of the rigidly supported structure $\xi = c/\left(2m\omega\right)$;
- horizontal damping ratio the mat-foundation system $\xi_u = c_{wu}/\left[2\sqrt{K_{wu}(m_0 + m)}\right]$;
- vertical damping ratio of the mat-foundation system; $\xi_v = c_{wv}/\left[2\sqrt{2k_{wv}b(m_0 + m)}\right]$.

The horizontal and vertical stiffnesses per unit length are taken to be those given in [8]. They write respectively:

$$K_{wu} = \frac{2a\rho V_s^2}{2-\nu}\left[2 + 2.5\left(\frac{b}{a}\right)^{0.85}\right], \quad k_{wv} = \frac{\rho V_s^2}{1-\nu}\left[0.73 + 1.54\left(\frac{b}{a}\right)^{0.75}\right]$$

where a is the footing length, ρ soil density, V_s soil shear wave velocity, ν soil Poisson coefficient . An empirical correlation for V_s as given in [8] is used here: $V_s = 38.55N^{0.176}z^{0.481}$ where N is the value of blow count in standard penetration test and z the foundation depth.

In this correlation z is expressed in m and V_s in m/s. The dashpot coefficients for horizontal and vertical base motion are given according to that reference respectively by $C_{wu}=2a\rho V_s\,\chi_u\left(\frac{a}{b},a_u\right)$ and $c_{wv}=\frac{1.0823\rho V_s}{1-\nu}\chi_v\left(\frac{a}{b},a_v\right)$, with $a_u=\frac{\beta_u\omega a}{V_s}$, $a_v=\frac{\beta_v\omega a}{V_s}$ and χ_u, χ_v are given in [8].

2.2 Dynamic equilibrium equation

In the following, the equations of motion are derived by expressing equilibrium conditions for the SDOF oscillator that schematizes the coupled foundation-mat system with the total mass of the structure concentrated at the mat tip. Fig. 3 gives the free body diagram of the system including the intervening inertial forces under large rotation of the footing. The resulting equilibrium equations for horizontal and vertical forces, and moments acting around the center of the foundation were obtained in [9].

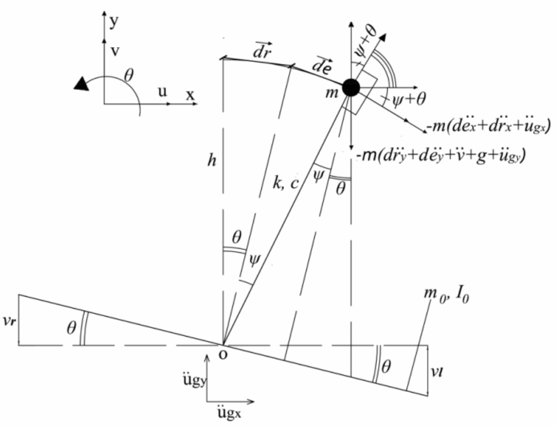


Fig. 3. Free body diagram of the mat-foundation system considering the different types of displacements, uplift and the intervening inertial forces.

The coupled equations of motion are given by [9]:

$$\begin{aligned}&\ddot{u}+2\xi\omega\dot{u}+\omega^2u=\\&-\frac{h}{cos\theta-\frac{u}{h}sin\theta}\left(\ddot{\theta}cos\theta-\dot{\theta}^2sin\theta\right)-\frac{1}{cos\theta-\frac{u}{h}sin\theta}\left(\ddot{u}_0+\ddot{u}_{gx}\right)\end{aligned}\tag{1}$$

$$\begin{aligned}&\ddot{u}_0+2\xi_u\beta_u\omega\dot{u}_0+\beta_u^2\omega^2u_0=\\&-\ddot{u}_{gx}-\frac{h}{\gamma+1}\left(\ddot{\theta}cos\theta-\dot{\theta}^2sin\theta\right)-\frac{1}{\gamma+1}\ddot{u}\left(cos\theta-\frac{u}{h}sin\theta\right)\end{aligned}\tag{2}$$

$$\begin{aligned}&\ddot{v}_0+\frac{F_v}{m_0+m}=-\ddot{v}_{gy}-g+\\&\frac{h}{\gamma+1}\left(2\frac{\dot{u}}{h}\dot{\theta}cos\theta-\frac{u}{h}\dot{\theta}^2sin\theta+\ddot{\theta}sin\theta+\dot{\theta}^2cos\theta-\frac{u\ddot{u}}{h^2}cos\theta\right)\end{aligned}\tag{3}$$

$$\begin{aligned}&\ddot{\theta}+\frac{3(1+\gamma)\alpha^2}{\gamma h^2}\frac{M_r}{m_0+m}=-\frac{3\alpha^2}{\gamma}\left(\ddot{\theta}cos\theta-\dot{\theta}^2sin\theta\right)\\&-\frac{3\alpha^2}{\gamma h}\left(\ddot{u}_0+\ddot{u}_{gx}\right)-\frac{3\alpha^2}{\gamma}\left(\frac{\ddot{u}}{h}cos\theta-\frac{u\ddot{u}}{h^2}sin\theta\right)\end{aligned}\tag{4}$$

where M_r designates the resultant moment applied at the mat base, u_0 and v_0 are the horizontal and vertical displacements of the foundation centre O, and $u=\vec{d}_e.\vec{e}_t$ where $\vec{e}_t=cos(\theta+\psi)\vec{e}_x-sin(\theta+\psi)\vec{e}_y$.

The vertical displacement measured from the initial unstressed position is given, see Fig. 4, by:

$$v=v_0+xsin\theta \tag{5}$$

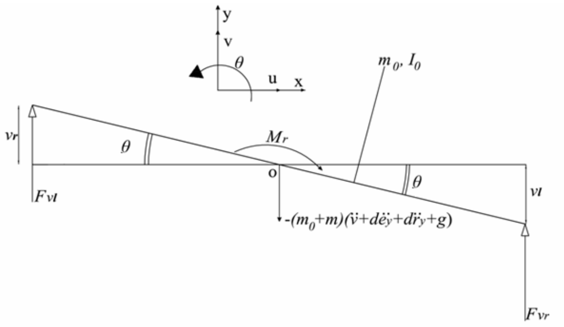


Fig. 4. Free body diagram for the base.

The foundation can lift up at an edge at a time instant for which the following condition is fulfilled

$$v>0 \tag{6}$$

Four cases can then be distinguished according to the signs of v_0 and θ.

The force F_v in Eq. (3) and the resistant moment M_r in Eq. (4) can be calculated analytically by integration over the domain of contact between the base and the foundation soil. Considering the elastic, respectively, the elastic-plastic range of deformation, the following compact forms are obtained [9]:

$$\frac{F_v}{m_0+m}=\begin{cases}\left(\beta_v^2\omega^2v_0+2\xi_v\beta_v\omega\dot{v}_0\right)\eta_1+\\\left(\beta_v^2\omega^2sin\theta+2\xi_v\beta_v\omega\dot{\theta}cos\theta\right)\eta_2 & if\ \dot{v}_0-\frac{\dot{\theta}\left(k_{wv}v_0+\sigma_y\right)}{k_{wv}tan\theta}>0\\-\sigma_y\eta_4+\left(\beta_v^2\omega^2v_0+2\xi_v\beta_v\omega\dot{v}_0\right)\eta_5+\\\left(\beta_v^2\omega^2sin\theta+2\xi_v\beta_v\omega\dot{\theta}cos\theta\right)\eta_6 & otherwise\end{cases}\tag{7}$$

$$\frac{M_r}{m_0+m}=\begin{cases}\left(\beta_v^2\omega^2v_0+2\xi_v\beta_v\omega\dot{v}_0\right)\eta_2+\\\left(\beta_v^2\omega^2sin\theta+2\xi_v\beta_v\omega\dot{\theta}cos\theta\right)\eta_3 & if\ \dot{v}_0-\frac{\dot{\theta}\left(k_{wv}v_0+\sigma_y\right)}{k_{wv}tan\theta}>0\\-\sigma_y\eta_7+\left(\beta_v^2\omega^2v_0+2\xi_v\beta_v\omega\dot{v}_0\right)\eta_6+\\\left(\beta_v^2\omega^2sin\theta+2\xi_v\beta_v\omega\dot{\theta}cos\theta\right)\eta_8 & otherwise\end{cases}\tag{8}$$

where η_i, $i=1,2,3$ are functions of b and x_c, and η_i, $i=4,...,8$ are functions of b,x_c,x_p and x_d, with this last computed from the condition $\dot{v}=0$.

The equations of motion of the system correspond to system of equations (1) to (4), (7) and (8). An appropriate numerical procedure was devised in this work in order to integrate these equations for any potential state: occurrence of elastic perfectly plastic range of deformation or uplifting nonlinearity.

3 Results and discussion

Two reinforced concrete (RC) residential buildings having three and five stories were studied. Their total seismic weight is respectively $G+0.2Q=392.6\times10^3\,kg$ and $G+0.2Q=654.3\times10^3\,kg$.

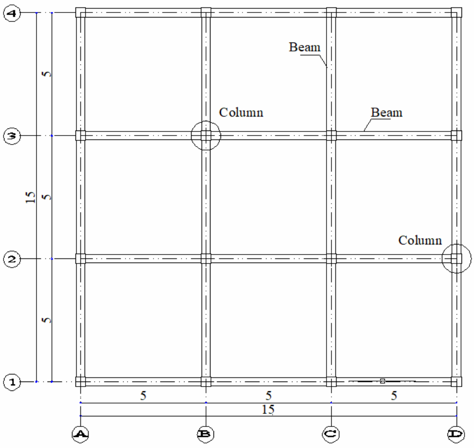
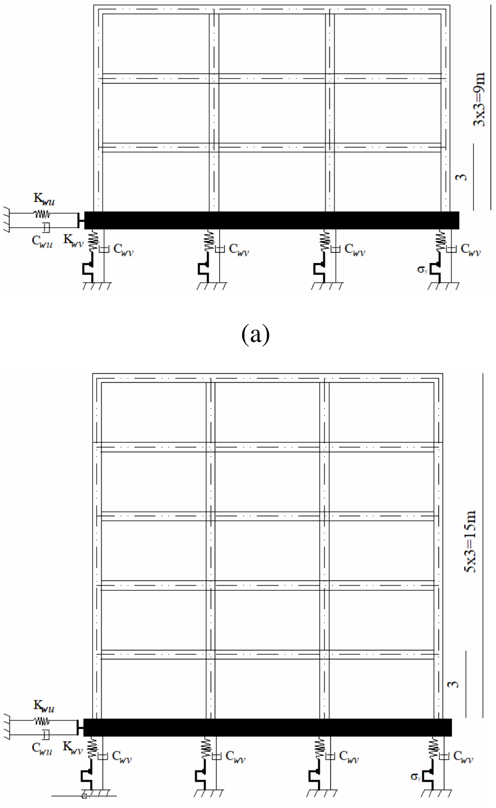


Fig. 5. Base of the RC buildings considered in this study.



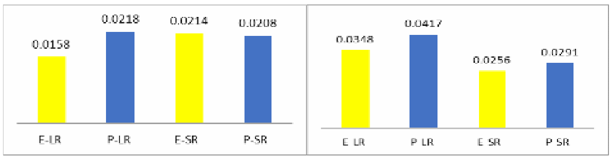
(a)

(b)

Fig. 6. Frames of the three (a) and five (b) stories reinforced concrete buildings considered in this work

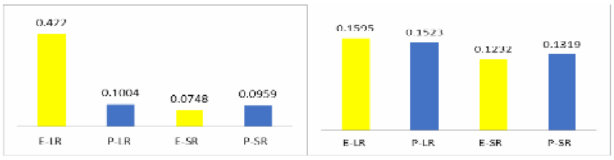
Fig. 5 gives the implantation symmetric base which is the same for both of them. Fig.6 gives their frame structures in any seismic direction parallel to axes shown in Fig. 5.

For each building, four models have been considered for each building: elastic soil with large rotations (E-LR), elastic-plastic soil with large rotations (P-LR), elastic soil with small rotations (E-SR) and elastic-plastic soil with small rotations (P-SR). These were considered for all the soil site categories and seismic zones as defined in the Moroccan seismic regulation RPS2011 [6], while the depth of foundation was varied in the set: $z=\{1,2,3\}\,m$.



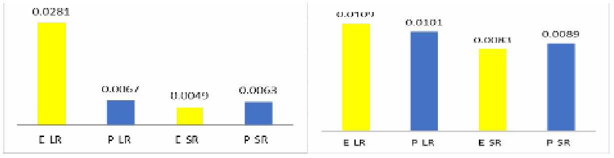
(a) (b)

Fig. 7. Maximum horizontal displacement of mat-tip as function of the considered model: (a) three storey RC building; (b) five storey RC building.



(a) (b)

Fig. 8. Maximum vertical displacement of base as function of the considered model: (a) three storey RC building; (b) five storey RC building.



(a) (b)

Fig. 9. Maximum rotation of foundation base as function of the considered model: (a) three storey RC building; (b) five storey RC building.

Figs. 7 to 9 give the obtained maximum value for mat-tip displacement, base vertical displacement and base rotation, respectively. As seen in these figures, the maximum values of horizontal, vertical displacements and foundation base rotation depend largely on the adopted assumption for rotation magnitude, height of structure and also on the hypothesis regarding soil deformation and depth of foundation base.

In comparison to small base rotation hypothesis, considering large rotation yields always the highest value of horizontal, vertical displacements and foundation base rotation for elastic-plastic models and for both heights of buildings. Except from vertical displacement and rotation foundation in three storey building, this holds also for elastic models.

Comparing now the results of elastic to those of elastic-plastic models, no general conclusion can be drawn.

The obtained results indicate that large rotations and soil plasticity should be considered in modeling mat-foundation system. Neglecting either one of these may lead to under-estimation or over-estimation of the building displacement.

4 Conclusions

In this study, the dynamic analysis of multistory reinforced concrete residential building to scaled El Centro earthquake motion was performed. This included local uplift of foundation, elastic-plastic behavior of the soil, large rotation about the system base, site categories and seismic zones, different number of stories and depths of foundation.

The obtained results have shown that the maximum values of rotation, horizontal and vertical displacements are considerably depending on the assumption used regarding the magnitude of rotation, height of structure and soil behaviour. So, modelling should integrate the real characteristics of building and its foundation.

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