Influence of the nonlinear behaviour of ballast on the dynamics of simply supported railway bridges

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Abstract. In this work, the vertical motion of a simply supported railway bridge which is subjected to the circulation of high speed trains was studied. A system consisting of two-layer beam was considered to model the dynamics of the bridge structure. The upper beam represents the rails with the sleepers and the lower beam the bridge deck. These two beams are coupled through distributed nonlinear springs that model the ballast action. The characteristics of these elements were identified from experimental measurements performed on real rail track. Considering the circulation of high speed train at given velocity, the influence of the nonlinear stiffness of the ballasted track on the response of the bridge system was analyzed. This was achieved by using the Galerkin method and the Runge-Kutta scheme to solve numerically the nonlinear partial differential equations governing the motion of the two beams. It was found that the nonlinear behaviour of the ballast affects notably the dynamics of the bridge, especially when the ballast stiffness is low. The proposed modelling enables to get more understanding regarding the vertical dynamics of ballasted track bridge in high speed line.

1 Introduction

The design of high-speed railway bridges requires studying their dynamic response. These structures may experience in fact severe transient accelerations under the passage of high speed trains. This happens at resonance which occurs for specific train speeds that depend on the general bridge characteristics: mass per unit length, bending stiffness, span length, damping as well as the configuration of circulating train. It was observed that simply supported ballasted track bridges can undergo high vertical acceleration levels during the passage of trains at velocities exceeding $200\, km/h$. Adverse effects may then result such as ballast destabilization, passenger discomfort, structural fatigue, loss of contact between wheel and rail, and even derailment [1-5].

In the actual design practice of railway bridges, the track is assumed to act as an additional mass and its stiffness is ignored. This hypothesis is made in order to get an evaluation on the safety side. However, in reality, track stiffness is not infinite and affects the bridge dynamics differently than by its mass. Improved modelling is then needed to predict realistically the vibration levels of ballasted track bridges under the circulation of high speed trains.

A large number of published papers have focused on different aspects of the complex coupled dynamics of vehicle-track-bridge-foundation system [6-7]. Authors have derived various models either under full 3D kinematics or by assuming 2D approximation of the problem. They have considered rigid, linear or nonlinear ballast deformation.

In the context of linear based investigations, Biondi et al. [8] proposed a 2D train-track-bridge model where the rail and the bridge have been represented as two beams coupled through a viscoelastic connection layer. By using the substructuring approach, they succeeded in extracting the dynamic response of the vehicles, rails and bridge. Regueiro et al. [3] investigated numerically the influence of ballasted track models on the dynamic response of medium span railway viaducts. They have considered 2D and 3D models for the track and two models for the loading. The authors noted that 2D models can be used for non-skewed viaducts. By comparing the predicted accelerations to field measurements, they proved that including the track superstructure has a favourable effect on the maximum acceleration. However, the authors have pointed out that the computations and measurements made do not reflect the resonance situation which needs further investigation. Zhai et al. [4] presented a review on the general problem of train-track-bridge interaction. This covers both the development of numerical models and experimental tests. The authors have argued that the consideration of track structure stiffness makes more practical analysis of the system vibration. Martinez-Rodrigo et al. [9] introduced a 2D numerical model that was fitted based on information from experimental tests. They concluded that the proposed approach enables to take into account the effects of the track components and the ballast coupling on the dynamic response of real

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bridges. They noted that high dispersion exists in the track parameters as given in literature even for similar track infrastructures. They mentioned that the rail-pad stiffness seems to have the most effect on the bridge maximum acceleration at resonance. Malveiro et al. [10] presented the calibration of a dynamic model for a railway viaduct by means of experimental data. Lou et al. [11] proposed a coupled numerical model to analyse the dynamic problem of train-track-bridge interaction system. The authors concluded that the double-layer track model, with sleepers considered, affects the natural frequencies of bridge in comparison with the single-layer track model where sleepers are ignored. The double-layer track model was found to be more accurate.

Contrary to the above mentioned studies where the ballast stiffness was stated linear, it was recognized in literature that the nonlinear stiffness of the track is important to consider. It was found that it can play a positive role in the design of railway bridges [12-13]. Based on experimental testing performed on a single ballasted track railway bridge, Rebelo et al. [2] proved that there exists an important nonlinear behaviour related to the variation of the natural frequency with the amplitude of vibration. By using the continuous wavelet transform Ülker-Kaustell and Karoumi [14] demonstrated that, for an observed range of acceleration amplitudes, the natural frequency decreases and the damping ratio increases with increasing vibration amplitudes. The same conclusions were drawn by Fink and Mahr [15]. The influence of variations affecting resonance response characteristics in the case of simply supported railway bridge has been also studied by Ülker-Kaustell and Karoumi [6]. These authors have stated that these variations induce decreased critical speeds and resonant amplitudes. Based on experimental tests, Dahlberg [16] showed that the ballast in railway track has a highly nonlinear behaviour. He suggested that ballast foundation can be modelled by adding a cubic nonlinear stiffness term.

The nonlinear behaviour of ballasted track was also investigated by Ansari et al. [17] and Iwnicky [18]. But, this was performed for rails on foundation. The aim of this work is to analyze the influence of the nonlinear behaviour of the ballast on the vertical dynamic response of a railway bridge as function of the loading speed. The modelling is based on a 2D approach. It is assumed that the bridge and the rails behave as a system of two coupled beams which are connected continuously through vertical nonlinear springs and linear dampers which simulate together the ballast effect. The beams are assumed elastic and obeying to the Euler-Bernoulli theory. Torsion and the dynamics along the longitudinal direction are not studied.

The remainder of this paper is organized as follows. The partial differential equations of the two-layer beam system are derived at first. Then, the Galerkin method is employed for spatial discretization of these equations, before applying the Runge-Kutta method to calculate the time history of the dynamic response. A case of study is considered after that to analyse the effect of ballast nonlinearity on bridge dynamics as function of the train speed.

2 Materials and method

2.1 Modelling the ballasted track bridge

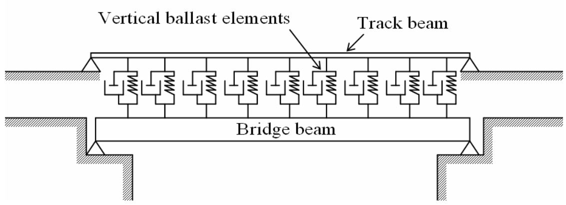

Fig. 1. 2D two-layer beam model considered for a simply supported ballasted track bridge.

Fig.1 illustrates the model considered for the description of the vertical motion of a simply supported bridge with ballasted track. The system consists of two uniform elastic beams which are connected between them by a series of vertical elastic nonlinear springs and dashpots. The upper beam represents the rails with the added mass of sleepers, whereas the lower one represents the bridge structure with added mass of the ballast. Both beams are assumed to be parallel and simply supported, with identical length L.

The assumption of the two beams having the same finite length constitutes a simplification with respect to the real rails conditions, as these can be considered to expand infinitely in both sides. In fact, there exists a transition zone at both ends of the bridge where the track is supported differently by soil, abutment and the bridge structure. The representation of boundary conditions according to the finite length assumption may not be always sufficient. However, to capture the real outcome of real boundary conditions in the zone of bridge ends, 3D modeling is needed. This is because complex phenomena take place there such stress concentration, and the Euler-Bernoulli beam theory is not sufficient.

The axial deformations are neglected and no explicit slip in the horizontal direction is considered. The influence of the track irregularities is neglected and the passing trains are introduced by means of moving loads model. These assumptions regarding track-bridge vertical kinematics enable to focus specifically on the interaction in the vertical direction resulting from the nonlinear stiffness of the ballast. Yang et al. [19] have used a synthetic beam model similar to that one given in Figure 1, but they have used linear elements between the track and bridge beams.

In the following, the vertical displacement of the rails beam and the bridge are denoted as $w_r(x,t)$ and $w_s(x,t)$ where the subscripts 'r' and 's' refer to the rails and bridge structure, respectively. t is the time and x is the spatial coordinate along the axis of the beams with the origin taken at the left extremity. The continuous layer connecting the two beams is characterized by the linear elastic stiffness k_l, nonlinear elastic stiffness k_{nl} and the damping coefficient c_w. The resulting vertical force per unit length is taken under the following form:

$$F_b = k_l\left(w_r - w_s\right) + k_{nl}\left(w_r - w_s\right)^3 + c_w\left(\dot{w}_r - \dot{w}_s\right) \qquad (1)$$

In Eq. (1), the quantity $w = w_r - w_s$ designates the relative displacement between rails and bridge structure.

The nonlinear constitutive part of the stiffness, namely the cubic term in Eq. (1), is conform to the dependency that was introduced by Dahlberg [16] based on experimental observations performed in the case of a railway ballasted track on rigid foundation. Dahlberg has identified the following values of the linear and nonlinear stiffnesses: $k_l = 22.75\ MN/m$ and $k_{nl} = 2.6\times10^8\ MN/m^3$.

By using standard techniques [20], the partial differential equations governing the transverse motion of the double-layer beam system can be expressed as follows:

$$\begin{aligned} EI_r \frac{\partial^4 w_r(x,t)}{\partial x^4} + m_r \frac{\partial^2 w_r(x,t)}{\partial t^2} + c_r \frac{\partial w_r}{\partial t} &= F_v - F_b \\ EI_s \frac{\partial^4 w_s(x,t)}{\partial x^4} + m_s \frac{\partial^2 w_s(x,t)}{\partial t^2} + c_s \frac{\partial w_s}{\partial t} &= F_b \end{aligned} \quad (2)$$

where EI_i and m_i are the bending stiffness and the mass per unit length of the ith beam, respectively, with $i=r$ or s for the rails beam or the bridge beam.

According to the recommendations of the EN 1991-2 [1], the damping ratio of the ballasted track and the bridge structure are attributed the same value. The damping depends on the span and the bridge type (filler beam, pre-stressed concrete, reinforced concrete…). The damping coefficients which are denoted c_r and c_s in Eq. (2) can then be calculated from the assigned damping ratio by using the mass and bending frequencies of the rails beam and the bridge beam. For steel and composite bridges having length exceeding $30m$, the Eurocode stipulations recommend: $\xi=0.005$.

It should be mentioned that additional damping resulting from the longitudinal slip of the ballast should be added to the previous damping. It was demonstrated that this damping is nonlinear and depends on the amplitude of vibration [21]. However, a linear approximation of this damping is made here. Considering the damping value given in [22], the global coefficient of damping is set equal to $\xi_r=\xi_b=0.07$.

The forcing term F_v in Eq. (2) is the railway excitation which is simulated by means of constant moving loads. From literature, consideration of the more complete train-bridge interaction leads in general to a reduction in the vertical acceleration of the bridge at resonance. The dynamic load is assumed to be applied vertically at the centerline of the rails beam. It is taken under the following general form [6,23]

$$F_v = -\sum_{k=1}^{N_v} F_k \left(H\left(t - \frac{d_k}{c}\right) - H\left(t - \frac{d_k + L}{c}\right) \right) \delta(x - ct + d_k) \quad (3)$$

where F_k is the force exerted by the kth load and acting at the rails beam point: $x_k(t)=ct-d_k$, N_v is the total number of axle loads, c is the constant train speed, d_k is the distance from the kth load to the entrance of the beam at time $t=0$, $H(.)$, $\delta(.)$ are Heaviside and Dirac delta functions.

The boundary conditions for $w_r(x,t)$ and $w_s(x,t)$ are identical for the two beams. They are expressed as:

$$w_i(x,t) = \frac{\partial^2 w_i(x,t)}{\partial x^2} = 0 \quad at \quad \begin{cases} x = 0 \\ x = L \end{cases} \quad ; i = r, s \quad (4)$$

2.2 Numerical solution of the nonlinear equations

The system of Eq. (2) is nonlinear. The Galerkin method is employed in order to perform spatial discretization into a system of ordinary differential equations. This consists at first in selecting a set of pertinent trial functions. These can be for example the modal functions of a linear elastic beam satisfying the boundary conditions in Eq. (4). Then, the vertical displacements are expanded as explicit series in terms of these basic functions [24]:

$$\begin{aligned} w_r(x,t) &= \sum_{i=1}^{\infty} q_i(t) \sin\left(\frac{i\pi x}{L}\right) \\ w_s(x,t) &= \sum_{j=1}^{\infty} u_j(t) \sin\left(\frac{j\pi x}{L}\right) \end{aligned} \quad (5)$$

where $q_i(t)$ and $u_j(t)$ are, respectively, sets of generalized displacements which describe time variations of the bending displacement of the rails beam and bridge beam.

Truncation to the first Nm Galerkin terms of Eq. (5) is next operated before substituting Eq. (5) into Eq. (2). Then, multiplying both sides of the obtained q and u equations by $\sin\left(\frac{m\pi x}{L}\right)$ and $\sin\left(\frac{n\pi x}{L}\right)$, respectively, and finally integrating the resulting terms over the interval $[0, L]$, a double set of second-order differential ordinary equations is derived as:

$$\begin{aligned} & m_r \omega_{rm}^2 q_m(t) + m_r \ddot{q}_m(t) + c_r \dot{q}_m(t) \\ & + c_w \left(\dot{q}_m(t) - \sum_{j=1}^{Nmu} \dot{u}_j(t) \delta_{mj} \right) + k_l \left(q_m(t) - \sum_{j=1}^{Nmu} u_j(t) \delta_{mj} \right) \\ & + k_{nl} \frac{2}{L} \int_0^L \sin\left(\frac{m\pi x}{L}\right) \left(\sum_{i=1}^{Nmq} q_i(t) \sin\left(\frac{i\pi x}{L}\right) - \sum_{j=1}^{Nmu} u_j(t) \sin\left(\frac{j\pi x}{L}\right) \right)^3 dx = \\ & -\sum_{k=1}^{N_v} \frac{2F_k}{L} \left(H\left(t - \frac{d_k}{c}\right) - H\left(t - \frac{d_k + L}{c}\right) \right) \sin\left(\frac{m\pi(ct - d_k)}{L}\right) \end{aligned} \quad (6.1)$$

$$\begin{aligned} & m_s \omega_{sn}^2 u_n(t) + m_s \ddot{u}_n(t) + c_s \dot{u}_n(t) + c_w \left(\dot{u}_n(t) - \sum_{i=1}^{Nmq} \dot{q}_i(t) \delta_{ni} \right) + \\ & k_l \left(u_n(t) - \sum_{i=1}^{Nmq} q_i(t) \delta_{ni} \right) \\ & + k_{nl} \frac{2}{L} \int_0^L \sin\left(\frac{n\pi x}{L}\right) \left(\sum_{j=1}^{Nmu} u_j(t) \sin\left(\frac{j\pi x}{L}\right) - \sum_{i=1}^{Nmq} q_i(t) \sin\left(\frac{i\pi x}{L}\right) \right)^3 dx = 0 \end{aligned} \quad (6.2)$$

In Eqs. (6.1) and (6.2), the quantities $\omega_{rm}^2 = \left(\frac{m\pi}{L}\right)^4 \frac{EI_r}{m_r}$

and $\omega_{sn}^2 = \left(\frac{n\pi}{L}\right)^4 \frac{EI_s}{m_s}$, for m=1,2,...,Nmq and n=1,2,....,Nmu are respectively the natural bending frequencies of the rails beam and the bridge beam. By applying $\xi_{rm} = \xi_{sn} = \xi$ for the damping ratios, one gets $c_r = 2\xi\omega_{rm} m_r$ and $c_s = 2\xi\omega_{sn} m_s$.

Considering the situation of forced vibrations, the initial conditions are all set to zero such that:

$$\begin{aligned} q_i(t)\big|_{t=0} &= 0 \quad , \quad q_{i,t}(t)\big|_{t=0} = 0 \\ u_j(t)\big|_{t=0} &= 0 \quad , \quad u_{j,t}(t)\big|_{t=0} = 0 \end{aligned} \tag{7}$$

The time interval used for the calculation of the system forced response corresponds to the duration of train passage which depends on the actual train speed.

Considering similar orders of truncation of the two series $Nmq = Nmu = N$, Eqs. (6.1) and (6.2) yield the following system of equations:

$$\begin{aligned} & m_r \ddot{q}_m(t) + c_r \dot{q}_m(t) - c_w \dot{\alpha}_m(t) + m_r \omega_{rm}^2 q_m(t) \\ & -k_l \alpha_m(t) + k_{nl} \frac{2}{L} I_N^m(t) \\ & = -\sum_{k=1}^{N_v} \frac{2F_k}{L} \left(H\left(t - \frac{d_k}{c}\right) - H\left(t - \frac{d_k + L}{c}\right) \right) \\ & \qquad \sin\left(\frac{m\pi(ct - d_k)}{L} \right) \\ & m_s \ddot{u}_m(t) + c_s \dot{u}_m(t) + c_w \dot{\alpha}_m(t) + m_s \omega_{sm}^2 u_m(t) \\ & +k_l \alpha_m(t) - k_{nl} \frac{2}{L} I_N^m(t) = 0 \\ & m = 1, 2, ..., N \end{aligned} \tag{8}$$

with $\alpha_m(t) = u_m(t) - q_m(t)$ and the nonlinear functions $I_N^m(t)$ being calculated from the following iteration:

$$\begin{aligned} I_n^m(t) &= I_{n-1}^m(t) + \frac{3\alpha_n^3(t) L}{8} \delta_{n,m} + \frac{3\alpha_n^2 L}{4} \sum_{j=1}^{n-1} \alpha_j(t) \delta_{j,m} \\ &+ \frac{3\alpha_n L}{8} \left[-\sum_{j=1}^{n-1} \alpha_j^2(t) \left(\delta_{2j,n-m} + \delta_{2j,n+m} \right) \right] \\ &+ \frac{3\alpha_n L}{4} \left[\sum_{j=1}^{n-2} \sum_{i=j+1}^{n-1} \alpha_i(t) \alpha_j(t) \left(\delta_{i,n-m+j} + \delta_{i,n+m-j} - \delta_{i,n-m-j} \right) \right] \end{aligned} \tag{9}$$

where $I_1^m(t) = \frac{\alpha_1^3(t) L}{8} \left(3\delta_{1,m} - \delta_{3,m} \right)$ and δ is the Kronecker symbol.

Eq. (8) is a nonlinear system of $2N$ coupled second order ordinary differential equations. Each equation contains nonlinear terms which are due to the cubic ballast stiffness nonlinearity.

The restriction of motion to the first bending mode is not generally sufficient. This was recognized by Ding et al. [12] who have analyzed the convergence of the Galerkin method for the vibration of a single finite length Euler–Bernoulli beam resting on a nonlinear foundation and subjected to a moving concentrated force. The authors have found that the convergence of the Galerkin truncation depends on the intervening parameters and large truncation terms may be needed in some cases. Here, the retained number of modes is fixed by reference to the EN 1991-2 [1] stipulating the need to cover the frequency band ranging from 0 to $30Hz$, or for the superior limit 1.5 times the frequency of the first mode if this exceeds the bound of $30Hz$.

By taking into consideration the initial conditions given in Eq. (7) and for given geometrical characteristics of the double-layer beam system, the generalized coordinates $q_m(t)$ and $u_m(t)$ can be numerically calculated from Eq. (8). Consequently, the vertical displacements and accelerations can be solved for any point location on the bridge. This requires only substituting the numerical solutions of $q_m(t)$ and $u_m(t)$ into Eq. (5).

In the following, the nonlinear model will be compared to the linear model that can be retrieved from Eq. (8) by dropping the nonlinear terms.

3 Results and discussion

In this section, the Skidträsk bridge [6,14] is modeled as a simply supported two-layer beam. This bridge has a medium length of $L = 36m$. The real bridge is a steel-concrete composite structure which presents a small horizontal skew, but skewness was not included in the present modeling. The objective is to assess in this particular case the effect of ballast nonlinear behavior as it can be predicted by Eq. (8). Comparison is performed between the predicted results based on the present proposed model and those of the linear model.

One of the most important issues to deal with in the context of nonlinear model as given by Eq. (8), is how to identify the mechanical characteristics of the ballast elements in terms of the spring stiffness and damping defined in Eq. (1), namely the constants: k_l, k_{nl}, c_w. In this work, reference is made to the experimental work performed by Dahlberg [16] to fix the parameters of the nonlinear ballast model and to [22] to estimate damping value. Dahlberg has identified the nonlinear force-displacement characteristics from measurements performed on a newly built track in Sweden which is supported by a rigid foundation. It was done under the circulation of the Swedish high speed train X2000 with the speed $c = 198\ km/h$. The author has noted that the ballasted track presents highly nonlinear behavior with hardening characteristics. Besides, he pointed out that a linearized track model can only be used for one single axle load.

The track parameters used in the following are recalled in Table 1. The flexural rigidity of the upper beam is due to that of two UIC-60 rails. The mass per unit length is taken as that of the track superstructure which includes the rails and the sleepers [25]. They are assumed to remain constant along the railway track bridge.

Table 1. Geometrical properties of the Skidträsk bridge, including track beam and connection layer [6,16,25].

Parameter	**Value**
Length $L(m)$	36
Mass per unit length of the bridge including that of the ballast $m_s(kg/m)$	16454
Bending stiffness of the bridge beam $EI_s(GN.m^2)$	172.2
Mass per unit length of the rails beam including that of the sleepers $m_r(kg/m)$	545.6
Bending stiffness of the rails $EI_r(MN.m^2)$	12.831
Damping ratio $\xi(\%)$	7
Linear initial stiffness of the ballast $k_l(MN/m)$	22.75
Non-linear stiffness of the ballast $k_{nl}(MN/m^3)$	2.6×10^8
Viscous damping $c_w(kN.s/m)$	1125

The Swedish Steel Arrow train [6] will be used as a railway excitation to the coupled track-bridge structure. The train consists of 24 wagons with each of them having a length of $13.9m$. Axle loads of the locomotives and wagons are given respectively by $19.5\,tons$ and $22.5\,tons$. Train speeds investigated in the following are ranging from $50\,km/h$ to $300\,km/h$. The other geometrical characteristics of this train are given in [6].

Considering the linear model of the bridge, the resonance occurs when the train crosses the bridge at critical speeds which are given according to [23,26] as:

$$v_{res}^{j,n} = \frac{f_n D_w}{j} \quad , j = 1,2,3... \tag{10}$$

where D_w is the wagon length, which is equal to $13.9m$ for the Steel Arrow train and f_n is the nth natural frequency of the bridge.

As for the bridge considered here, the first mode frequency is $f_1 = 3.855Hz$, one can predict the following three critical speeds belonging to the considered interval of speeds:

$$v_{res}^1 = 192\,km/h;\ v_{res}^2 = 97\,km/h;\ v_{res}^3 = 64\,km/h \tag{11}$$

In the following, the maximum values of acceleration and displacement at mid-span section are calculated for various Steel Arrow speeds in the interval $c \in [50,300]\,km/h$. The predictions of the nonlinear model are also compared to those of the linear model. Convergence of the Galerkin series in terms of bridge and rail displacements was verified for the truncation order $N = 27$ for both the nonlinear and linear models. Convergence was not yet reached in terms of rail acceleration in case of the nonlinear model. The obtained results corresponding to this truncation order are plotted as curves in Fig. 2 and Fig. 3.

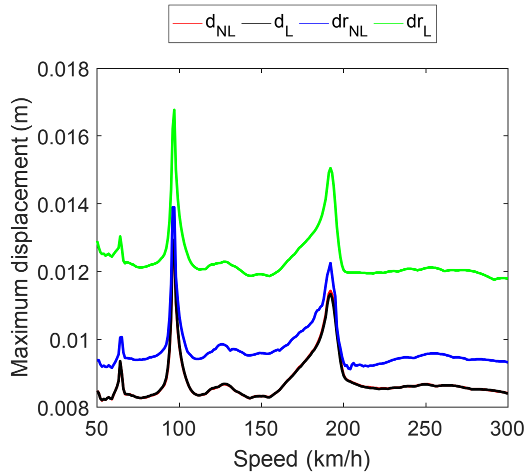

Fig. 2. Displacements at mid-span of the bridge (d) and rail (dr) beams versus the Steel Arrow speed; the subscript NL stands for nonlinear model while L is for linear model.

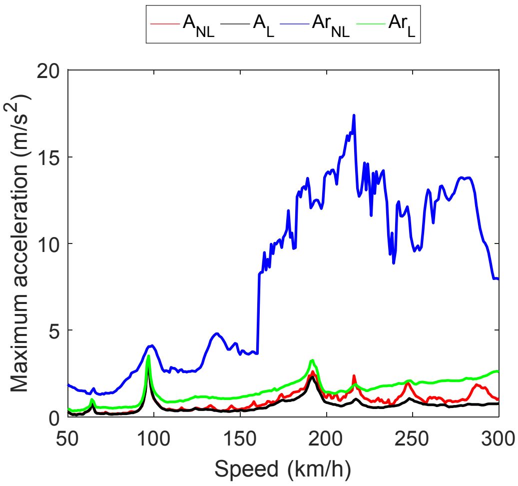

Fig. 3. Acceleration at mid-span of the bridge (A) and rails (Ar) beams versus the Steel Arrow speed; the subscript NL stands for nonlinear model while L is for linear model.

From Fig. 2, one can verify that the three linear resonant peaks predicted according to the theoretical values given in Eq. (11) appear in the displacement curves. However, other local peaks are present in the bridge acceleration curves of Fig. 3. This occurs for both the linear and nonlinear models with the amplitudes of the new peaks of acceleration not exceeding those associated to the theoretical peaks.

Fig. 2 shows that for the bridge displacement, the two models predict the same response. However, for the rails, the nonlinear model predicts reduced level of relative displacement. This can be understood from the fact that the nonlinear model presents a strengthening effect in comparison to the linear model.

As shown in Fig. 2 and Fig. 3, a significant difference exists in the results produced by the linear and nonlinear models. For the bridge, the acceleration peak predicted by the nonlinear model is slightly higher than that of the linear model. This variation can be of importance in practice if one considers the justification of ballast stability limit state. The nonlinear model predicts more adverse acceleration in this case. However, for the purpose of justification of the relative displacement between the rails and the bridge, the nonlinear model predicts more favorable results.

One should notice that the previous results are relative to a bridge which was not designed for the circulation of high speed trains. The maximum acceleration under the Steel Arrow loading, for train speed exceeding $200\,km/h$, is the highest according to the nonlinear model, while the linear models do not predict that. One can conclude that the ballasted track nonlinearities which are accounted for here by means of Eq. (8) induce quite different dynamics than that due to the linear model.

It is important to moderate the previous statements. The simplified modeling proposed in this work is according to the double-layer beam approach. It does not represent accurately the inertial interaction occurring between the rails and the sleepers, neither that between the sleepers and the ballast. Moreover, linear damping was assumed in this work and a more complete description of damping as function of the relative amplitude of bridge and rails beams should be performed.

4 Conclusions

In this work, the dynamic response of simply supported ballasted track bridges was investigated. Focus was on the effect of the nonlinear behavior of the ballast under resonance and non-resonance conditions occurring during the passage of high speed trains. The bridge was modeled as a composite beam with two layers which represent the rails with the sleepers and the bridge deck structure. They are interacting through a continuous interface consisting of vertical nonlinear springs and dashpots. By using the Galerkin method and the Runge-Kutta scheme the solution of the equations of motion was calculated. Simulations have shown that the nonlinear behavior of the ballast material affects the track-bridge dynamics when the ballast stiffness is low. Further investigation is needed in order to tackle other effects which may result from the inertia of sleepers and ballast induced damping by slip in the horizontal direction.

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