Extending Mononobe-Okabe theory to account for arbitrary backfill topography and variable density of surcharge

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Abstract. Coulomb proposed a formula for the static earth pressure acting on a rigid gravity retaining wall supporting a soil. Many researchers have generalized this theory and several analytical studies were achieved regarding the evaluation of seismic action. Following the work of Mononobe and Okabe, investigations have considered cohesive backfill, and with some restrictions the topography of the backfill and the pattern of surcharge acting on its surface. In this work an extension of the Mononobe-Okabe theory was proposed to deal with the most general case of backfill having an arbitrary shape and also vertical surcharge acting with variable density at the ground level. Equilibrium of Coulomb wedge was applied to express the active earth pressure coefficient related to a given orientation of the failure plane. Then, numerical optimization was performed to fix the critical angle providing its maximum value. Parametric studies were conducted after that. It was found that both the upward and downward vertical seismic acceleration should be considered. The obtained results indicate that the backfill surface shape and surcharge density have a significant effect on seismic action. The maximum relative variation of the pressure coefficient with regards to the reference case is however limited to 20%.

1 Introduction

Since the pioneering work performed by Coulomb for the determination of the static earth pressure acting on a rigid gravity retaining wall which is supporting the backfill soil through a plane face [1], many researchers have considered generalization of this simplified theory. Poncelet adapted the initial formulation of Coulomb theory to include the friction existing between the retaining wall and backfill and the inclination of the back face of the wall and that of the backfill surface [2]. Later on, various authors proposed to include earthquake loads in the Coulomb approach. Based on the pseudo-static approximation, Okabe [3] then Mononobe and Matsuo [4] introduced an extension known as Mononobe-Okabe theory which enables to take into account seismic loads. This was achieved by adding horizontal and vertical inertia forces to the equilibrium equations for a wall retaining cohesion-less backfill soil.

Assuming the superposition of active earth pressures resulting from three critical wedge sliding surfaces, which are associated respectively to the single action of weight, surcharge and cohesion, Prakash and Saran [5] and then Saran and Prakash [6] extended the Mononobe-Okabe theory to the case of backfill soil having uniform cohesion. They neglected the effect of the vertical acceleration and assumed that the backfill surface is horizontal along with the fact that unit adhesion at the wall-soil interface is equal to unit cohesion of the soil.

These assumptions were released by Das and Puri [7] who have allowed adhesion and cohesion to have different values and have taken into account both the inclination of the backfill and the vertical component of seismic acceleration. However, the previous solution which is based on the superposition of limit states at failure does not satisfy kinematics compatibility and gives an active earth pressure that may be different from that established under a unique limit state at failure. It was only in 2009 when Shukla et al. [8] have proposed an improved version of the Mononobe-Okabe model for the general case of a cohesive backfill soil that considers a unique critical wedge surface which holds simultaneously for all the acting loads.

Within the context of Coulomb theory, many analytical results were published in the literature regarding the particular pattern of a surcharge present on the backfill profile or the shape of the backfill surface. Among the numerous references one can find [9-11]. In this work an extension of the Mononobe-Okabe theory is proposed in order to deal with the case of a more general backfill surface that can be described as a curve having arbitrary form, and also in the situation where a variable density of surcharge acting vertically at the ground level is present. Equilibrium of a Coulomb sliding edge is expressed to get the expression of the coefficient of active earth pressure for a given orientation of the failure plane, and then numerical optimization is performed to fix the orientation giving the maximum value of the pressure coefficient. Parametric studies are then conducted to assess the effect of key parameters on the seismic active pressure in various conditions.

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2 Materials and method

Figure 1 shows a schematic representation of a rigid wedge enclosed in the backfill soil with all the forces acting on it including that due to the seismic active earth pressure. The retaining wall of height H is assumed to be rigid and its back face OM in contact with the soil is inclined by angle α with the vertical. The wall is supporting a cohesive backfill soil of uniform unit volumetric weight γ, uniform cohesion c and uniform friction angle φ. The adhesion c_a and wall friction δ on the wall - soil interface OM are assumed to be constant. The topography of the backfill is assumed to have an arbitrary profile which is described in the system of axes xOy by the function: $y = f(x)$. A variable vertical surcharge having density $q(x)$ per unit length is present on the top surface of the backfill.

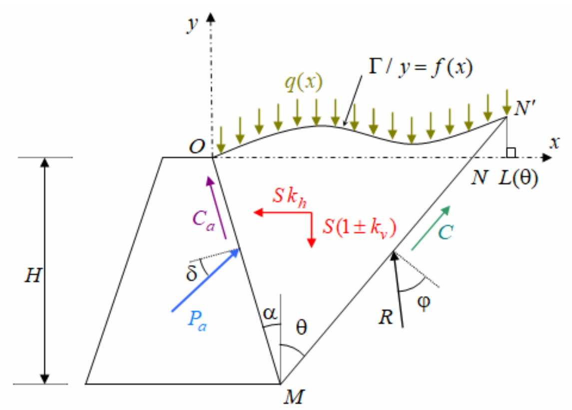

Fig. 1. Retaining wall and backfill soil wedge system with representation of all the acting forces.

The pseudo-static method is considered in order to model the seismic action during active state limit equilibrium. This is done by means of the seismic acceleration coefficients in the horizontal and vertical directions which are denoted k_h and k_v respectively. The inferior limit state of equilibrium of the backfill wedge is described by angle θ which forms the wedge sliding line MN with the vertical. The critical value of angle θ is defined as that yielding the maximum active force P_a acting on the retaining wall. This force is inclined by angle δ with the normal to the retaining back face of the wall OM. Its direction is located under this normal as indicated in Figure 1. The reaction acting on the soil at the sliding rupture line of the wedge MN is denoted R. According to the conventional Mononobe-Okabe theory which is based on Mohr-Coulomb criterion of rupture, the reaction R is inclined by φ with respect to the normal to the sliding line MN.

The total seismic weight is denoted S. This includes the total weight of the edge denoted $W + \Delta W$ with W the weight of the triangular wedge OMN and ΔW the additional weight associated to the earth sector located between the backfill surface, the horizontal line ON and the part of the rupture line NN'. The total seismic load includes also the total surcharge ΔQ resulting from the distributed load $q(x)$. The other forces are the total cohesion force and the total adhesion force which are denoted respectively by C and C_a.

Considering equilibrium along the horizontal and vertical directions of the backfill soil domain delimited by the lines OM, MN, NN' and the top surface Γ, one gets the following equations

$$\begin{aligned} P_a \cos(\alpha+\delta) - R\cos(\varphi+\theta) \\ + C\sin\theta - C_a \sin\alpha = Sk_h \end{aligned} \quad (1)$$

$$\begin{aligned} P_a \sin(\alpha+\delta) + R\sin(\varphi+\theta) \\ + C\cos\theta + C_a \cos\alpha = S(1\pm k_v) \end{aligned} \quad (2)$$

with $C = \dfrac{cH}{\cos\theta}$, $C_a = \dfrac{c_a H}{\cos\alpha}$, $S = W + \Delta W + \Delta Q$,

$W = \dfrac{\gamma H^2 \sin(\alpha+\theta)}{2\cos\alpha\cos\theta}$, $\Delta Q = \int_0^{L(\theta)} q(x)\, dx$,

$\Delta W = \int_0^{L(\theta)} \gamma\, f(x)\, dx - \dfrac{1}{2}\gamma L(\theta) f\left[L(\theta)\right]$.

The quantity $L(\theta)$ represents the distance separating O form the projection of point N' on the Ox axis, it satisfies the following implicit equation:

$$L(\theta) = H\left[\tan\alpha + \tan\theta\right] + f\left[L(\theta)\right]\tan\theta \quad (3)$$

Eliminating the reaction term R between equations (1) and (2), Substituting after that the expressions of C, C_a and S into the obtained equation, one arrives at the expression of the active force magnitude under the following form:

$$P_a = \frac{\gamma H^2 (1\pm k_v)}{2} k_a(\theta) \quad (4)$$

with $\psi = \tan^{-1}\left(\dfrac{k_h}{1\pm k_v}\right)$ and he coefficient of seismic active earth pressure given by

$$k_a(\theta) = \frac{\begin{pmatrix} A\sin(\alpha+\theta)\cos(\varphi-\psi+\theta) \\ -B\cos\theta\cos(\alpha+\varphi+\theta) - D \\ +E\left[\overline{\Delta W}(\theta) + \overline{\Delta Q}(\theta)\right]\cos\theta\cos(\varphi-\psi+\theta) \end{pmatrix}}{\cos\theta\sin(\alpha+\delta+\varphi+\theta)} \quad (5)$$

in which $A = \dfrac{1}{\cos\alpha}$, $B = \dfrac{2c_a}{\gamma H(1\pm k_v)\cos\alpha}$,

$D = \dfrac{2c\cos\varphi}{\gamma H(1\pm k_v)}$, $E = \dfrac{2}{H^2\cos\psi}$, $\overline{\Delta Q} = \dfrac{1}{\gamma}\int_0^{L(\theta)} q(x)dx$,

$\overline{\Delta W} = \int_0^{L(\theta)} f(x)\, dx - L(\theta) f\left[L(\theta)\right]$.

Eq. (5) constitutes an extension of the Coulomb formula for active earth pressure in the presence of arbitrary backfill surface and variable surcharge load. In the absence of seismic action, $\psi = 0$ and $k_v = 0$, so Eq. (5) simplifies to yield the generalized case of static active earth pressure coefficient for arbitrary backfill topography and vertical surcharge.

It should be noticed that the previous modeling does not integrate the effect of tension crack. This can be added without major difficulties. However, according to Shukla [8] a reduced cohesion representing an average cohesion on the sliding line can be introduced. The modified cohesion is related to the initial cohesion and depth of tension crack z_c by the following relation:

$$\tilde{c} = c\left(1 - \frac{z_c}{2H}\right) \quad (6)$$

For a given topography of the backfill, fixing the value of the sliding line orientation θ enables to obtain the quantity $L(\theta)$ by solution of the nonlinear equation, Eq. (3). Use is made here of the Matlab function *fzero*. So, the quantities $\overline{\Delta W}$ and $\overline{\Delta Q}$ can be computed for given backfill profile and surcharge load density upon fixing the value of θ. Giving now the other geometrical and mechanical parameters of the retaining wall and supported backfill soil, the function $k_a(\theta)$ can be calculated for any value of θ. The seismic active earth pressure can then be maximized on the physical interval of sliding plane orientation angle $\theta \in [0, 90]$, resulting on the critical value of the orientation angle which is denoted θ_c. The active earth pressure coefficient is then given by $k_a(\theta_c)$. Note that because of the sign before k_v, two values of the pressure coefficient are to be determined. They are denoted $k_a(\theta_c^-)$ and $k_a(\theta_c^+)$. The Matlab function *max* is used to calculate the maximum value of $k_a(\theta)$.

3 Results and discussion

In the following, a parametric study is conducted in order to analyze the effects on the coefficient of seismic active pressure which are due to the topography of the backfill and the surcharge profile. The parameters are taken to be as follows: $H = 5m$, $\gamma = 18kN/m^3$, $\alpha = 20°$, $\varphi = 20°$, $\delta = 15°$, $c = 15kPa$, $c_a = 10kPa$, $k_h = 0.3$ and $k_v = 0.2$.

The topography of backfill soil is assumed to be described by the following sinusoidal function:

$$f(x) = \frac{\pi ab}{H\left[\pi b + 1 - \cos(\pi b)\right]}\left[1 + \sin\left(\frac{b\pi}{H}x\right)\right] \quad (7)$$

with $a \in \{0,1,2\}$ and $b \in \{H, 2H, 3H\}$.

The surcharge distribution is taken of the same form:

$$q(x) = \frac{9\times10^4 \pi d}{H\left[\pi d + 1 - \cos(\pi d)\right]}\left[1 + \sin\left(\frac{d\pi}{H}x\right)\right] \quad (8)$$

with $d \in \{H, 2H, 3H\}$.

The expressions of $f(x)$ and $q(x)$ as defined respectively by Eq. (7) and Eq. (8) were chosen to be always positive and such that they ensure the same total weight over the domain $x \in [0, H]$ of backfill soil which is located above the horizontal line passing at the head of the retaining wall, as well as the same total surcharge over this domain. So, comparison will hold only on the effect of the wavy form of the backfill surface and surcharge distribution on the coefficient of seismic active earth pressure.

Figure 2 represents the three backfill surface for the case $a = 1$ and the three values of $b = H, 2H, 3H$. Changing the value of parameter a affects only the magnitude and the curves for $a = 2$ are not represented. The case $a = 0$ gives a flat backfill which is taken as a reference in the comparison study. The curves giving the surcharge density have the same form than those given in Figure 2, while their amplitude is different.

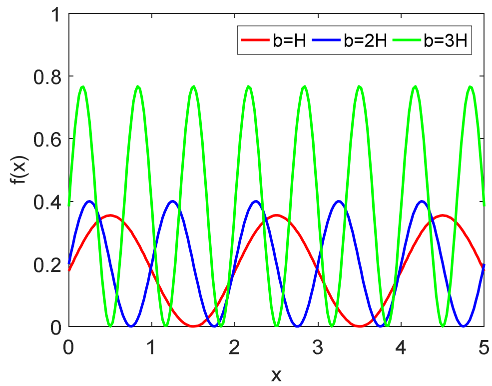

Fig. 2. Curves giving the topography of the backfill soil surface for $a = 1$ and $b = H, 2H, 3H$.

To perform a parametric study, the reference case is taken to be that with flat backfill topography and without surcharge. The calculated seismic active earth pressure coefficients are given by: $k_{a0+} = 0.4829$ and $k_{a0-} = 0.4174$. These values are used to define dimensionless pressure coefficients as follows: k_{a+} / k_{a0+} and k_{a-} / k_{a0-}. This enables to make more easily the analysis of the effects yielded by the topography and the surcharge on the seismic action applied on the retaining wall. Also, a combination number is introduced according to Table 1 for the combined effect of backfill topography and surcharge density. Table 2 gives the combination number used for the case of a flat backfill and when the effect of surcharge is considered alone.

The optimization procedure used in the determination of the seismic active earth pressure coefficient is based on the Matlab function which enables to get the global maximum, contrary to other methods which can get only a local minimum. This is an essential part of the problem. To emphasize this aspect, Figure 3 depicts the variations of the function $k_{a+}(\theta)$ versus the orientation angle θ. It is related to the parameters: $a = 1$, $b = 2$ and

$d = 3$. One can see that multiple local maxima are present. If one fails to attain the global maximum, then the seismic earth pressure will be underestimated.

Table 1. Combination number as function of the backfill surface parameters a and b.

Combination number	a	b
1	1	H
2	1	$2H$
3	1	$3H$
4	2	H
5	2	$2H$
6	2	$3H$

Table 2. Combination number as function of the surcharge parameter d.

Combination number	d
1	H
2	$2H$
3	$3H$

Figure 4 gives the obtained results for the case without surcharge. Figures 5 to 8 give the obtained results in terms of dimensionless seismic earth pressure coefficients for the three cases of surcharge and the six cases of backfill topography.

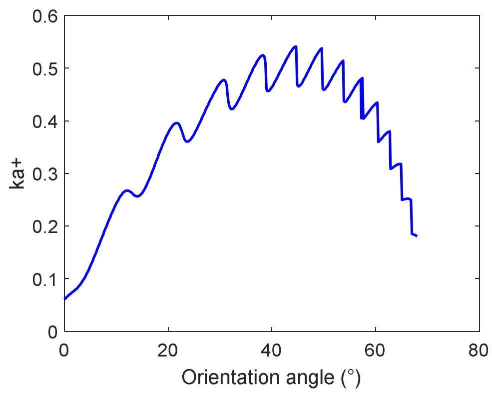

Fig. 3. Variation of the calculated function $k_{a+}(\theta)$ versus the sliding line orientation angle θ; $a = 1$, $b = 2$ and $d = 3$

From figures 4 to 8, it can be seen that in most cases the combinations with $-k_v$ gives the critical seismic earth pressure coefficient. This holds for all the studied cases except for combination 3 in the absence of surcharge. So, in general the two cases $\pm k_v$ should always be considered.

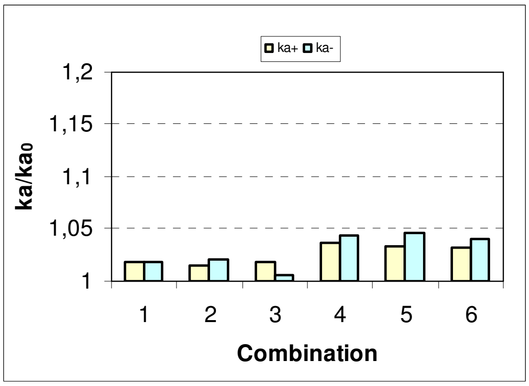

Fig. 4. The coefficient of seismic active earth pressure related to the case without surcharge and for parameters $a = 1,2$ and $b = H, 2H, 3H$; the combination number is given in Table 1.

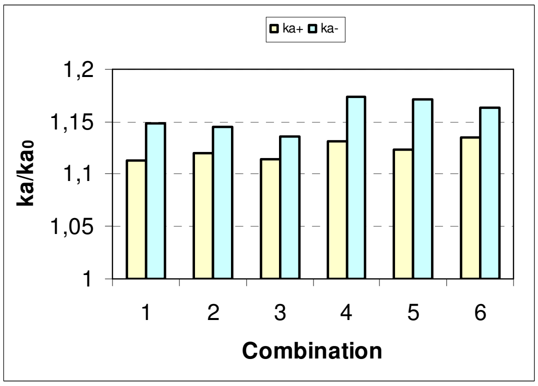

Fig. 5. The coefficient of seismic active earth pressure related to the case with a surcharge for which $d = H$ and as function of backfill topography $a = 1,2$ and $b = H, 2H, 3H$; the combination number is given according to Table 1.

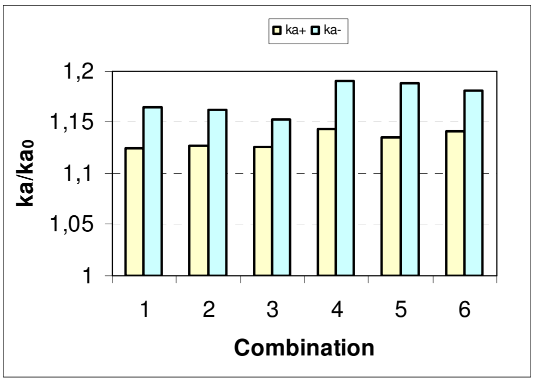

Fig. 6. The coefficient of seismic active earth pressure related to the case with a surcharge for which $d = 2H$ and as function of backfill topography $a = 1,2$ and $b = H, 2H, 3H$; the combination number is given according to Table 1.

Figures 4 to 8 show obviously that increasing the magnitude a results in increased seismic earth pressure. They show also that the wavy form of the surcharge has an effect. The most unfavorable case corresponds to

$b = H$ for $d = H$ and $d = 2H$, while it is given by $b = 2H$ for $d = 3H$ and the cases with flat backfill or without surcharge.

The results in figures 4 to 8 indicate also that the relative variation, with respect to the reference case with flat backfill surface and without surcharge, does not exceed 20%.

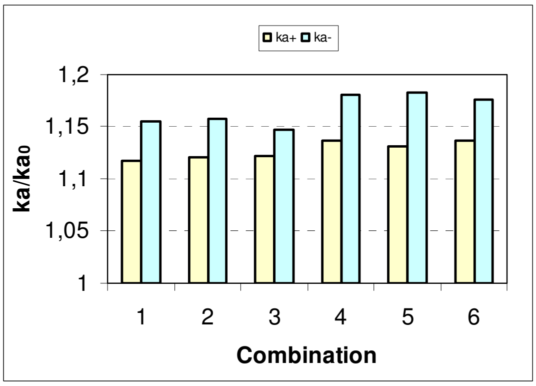

Fig. 7. The coefficient of seismic active earth pressure related to the case with a surcharge for which $d = 3H$ and as function of backfill topography $a = 1,2$ and $b = H, 2H, 3H$; the combination number is given in Table 1.

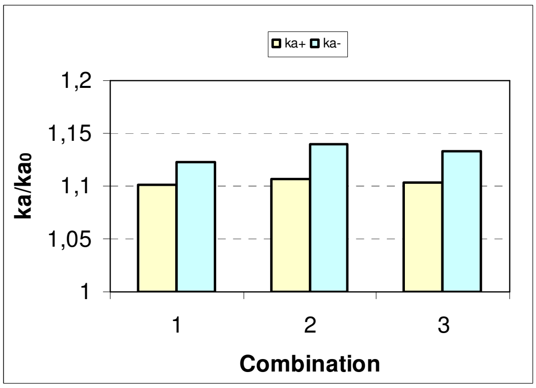

Fig. 8. The coefficient of seismic active earth pressure related to the case with a flat backfill surface and as function surcharge density $d = H, 2H, 3H$; the combination number is given according to Table 2.

4 Conclusions

In this work, an extension of the conventional Mononobe-Okabe method was performed. This integrated the general case of cohesive soil when the backfill topography is arbitrary and in the presence of a surcharge having variable density. Applying the concept of Coulomb wedge equilibrium, the sliding line was determined by means of an optimization procedure. This relies on finding the additional resultant forces representing the backfill domain above the horizontal line passing through the top of the wall and the distributed surcharge, and then maximizing the earth pressure versus the sliding line orientation angle. The problem requires solution of a nonlinear equation. A parametric study was conducted regarding the conjugate effect of backfill wavy form and surcharge density. The obtained results revealed that both the upward and downward vertical seismic acceleration should be considered. They indicate also that for the investigated topography and surcharge profile, the relative variation of the seismic active earth pressure coefficient with regards to the reference case, with flat backfill and without surcharge, is limited to a maximum of 20%. The shapes of the backfill surface and surcharge density were found to affect significantly the seismic active earth pressure coefficient. It should be noticed that the modeling does not integrate explicitly the effect of tension crack, but this can be taken through modifying the value of cohesion.

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