Experimental approach to calculate the moments of inertia of a hexacopter unmanned aerial vehicle

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Abstract. Moments of inertia define the amount of torque needed to rotate an object at a defined angular acceleration around a given axis. With the limited information available for multi-rotor unmanned aerial vehicles (UAVs), this paper sets out to experimentally determine the moments of inertia of a hexarotor UAV with a mounted payload. The moments of inertia are calculated in three rotational axes using pendulum and torsion tests. The pendulum test is performed to verify the test setup by experimentally determining a parameter which is already known: gravity. In pitch and yaw orientations gravity could be experimentally calculated to within 1.33% and 1.22% respectively. Unfortunately, this was not the case in the roll orientation, indicating that the test setup in that orientation needs to be adjusted. The torsion test is then performed to determine the period which is used to calculate the moments of inertia. Ultimately, these calculated moments of inertia will be used in simulation from which a flight controller can be designed.

1 Introduction

Aerial robots, or unmanned aerial vehicles (UAVs), have been considered in several different application areas due to their small size, agility, and payload carrying ability [1, 2]. These application areas are often classified as the 3 D's: dull, dirty or dangerous. As such, UAVs, especially multi-rotors, are ideally suited to enter areas that are generally not safe for humans, such as caverns, mines or search and rescue areas. With their manoeuvrability UAVs can navigate and explore autonomously through complex environments [3]. Autonomous navigation and exploration of confined spaces or GPS-denied environments presents several technical challenges, especially regarding the flight control system [1].

To develop a flight control system for a UAV an accurate determination of the moments of inertia is essential as these are crucial in characterising the handling qualities of the UAV [2, 4-6]. The moments of inertia determine an object's resistance to a change in rotation about its centre of gravity. In the context of an aerial vehicle, this would be defined as the resistance to changes in the vehicle's roll, pitch, and yaw velocities. The moment of inertia also tends

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to vary during operation when a payload is carried [2, 6] that can move relative to the airframe. Consequently, inaccurate estimates of the moments of inertia will lead to inaccuracies in the multi-rotor equations of motion resulting in erroneous autonomous control of the flight dynamics [6]. Needless to say, this can ultimately lead to a potential disaster.

There are three methods to determine the moments of inertia of UAVs namely, mathematical, computer aided design (CAD) or experimentally [5, 6]. The mathematical approach breaks down the UAV components and assumes a simplified geometry to represent each component. Ashraf et al. [7] separated the quadrotor into the motors, electronic speed controller, arms and central hub representing each as a simplified geometric shape with a constant internal density. The moments of inertia for each component were then determined using the parallel-axis theorem and summed to provide an overall moment of inertia for the quadrotor. A disadvantage of this approach is that it assumes symmetry in the geometry of the components, which is not always the case in a UAV and could lead to poor UAV control performance [6]. Additionally, this method requires the weight of all UAV components and their distances to the centre of gravity, which is often impossible to obtain as disassembling an assembled UAV is not always possible [5].

In the CAD approach the moments of inertia are measured in a CAD software package, which increases the measured accuracy [6]. Hussein et al. [8] used Solid Egde's inspection tool to determine the moments of inertia for a quadrotor to design an autonomous control system for its flight dynamics.

Schedlinski and Link [9] presents a survey of the available experimental methods to determine the moments of inertia. These methods are classified as either static or dynamic. For UAVs, a dynamic approach referred to as the pendulum method is commonly considered. In the pendulum test the moments of inertia are determined by measuring the oscillation period of a pendulum [6]. Lehmkühler et al. [4] considered the pendulum test in both one and three degrees of freedom (DOF) for a fixed wing UAV. The advantage of the 3 DOF method is that it only requires a single pendulum test to determine all three moments of inertia, whereas the 1 DOF method requires three separate test setups. Both methods provided similar results. Teimourian and Firouzbakht [5] considered a torsional pendulum test on a fixed wing UAV where it is rotated about the three axes. They also showed that the accuracy of the calculated moment of inertia is dependent on the construction of pendulum experimental setup, dimensions, and the precision of the measurements.

There is limited information available to determine the moments of inertia of multi-rotors, and especially hexarotor UAVs. It is impractical to disassemble the DJI M600 hexarotor UAV. It is also time consuming to develop a CAD model. For these reasons, the most economical approach is the experimental pendulum method. This paper therefore considers the experimental procedures used for fixed-wing UAVs and applies these to a DJI M600 Pro hexarotor UAV to determine the moments of inertia.

2 Methodology

In this section the methodology behind the pendulum test is discussed providing the theoretical background for the gravitational and torsional pendulum tests, and the centre of gravity. Lastly, the experimental setup considered for this paper is presented along with a description of the DJI M600 Pro hexarotor UAV.

2.1 Theoretical background

In this section, we discuss the theory behind the gravitational and torsional pendulum test. We also present the associated mathematics required to calculate gravity and the moment of inertia.

2.1.1 Pendulum test

The bifilar pendulum test has been widely used since the 1930s to determine the moments of inertia for aircraft as it is simple, safe and has a relatively high accuracy [4,10]. This type of test is known as a free oscillation method since the pendulum oscillates only under the influence of gravity once displaced and released from rest [4, 6]. A bifilar pendulum is illustrated in Figure 1 and consists of a test object suspended by two wires of height, h, separated by a distance, D. For a gravitational test the pendulum will oscillate forwards and backwards about the horizontal axis. For a torsional test, the pendulum oscillates about the vertical axis.

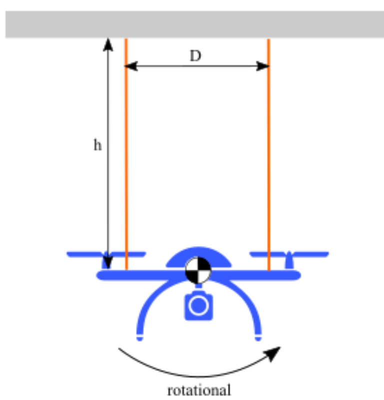


Fig. 1. Principle of the bifilar pendulum test (Redrawn from [9-11]).

2.1.1.1 Gravitational Pendulum Tests

A gravitational pendulum test is first considered to validate the experimental setup [11]. The aim of this test it to verify a known quantity such as the gravitational constant. This is achieved by simply displacing the hexarotor from rest and releasing it, and measuring the period based on the number of oscillations. The period (T) for a simple pendulum is determined from:

$$T = 2\pi\sqrt{\frac{h}{g}} \tag{1}$$

Rearranging equation (1) gravity (g) is then:

$$g = \frac{4\pi^2 h}{T^2} \tag{2}$$

If the value of gravity is approximately 9.81 m/s^2 the experimental test setup is verified, and the moments of inertia can be calculated using the torsional pendulum tests.

It is likely that these experiments will need to go through multiple iterations where certain parameters will be changed to see which values yield an acceptable result. Therefore, a sensitivity analysis needs to be conducted on the parameters of equation (2) [12]. These sensitivity analyses will assist in understanding the influence that dependant variables in the experiment will have on the independent variables. Using linear sensitivity analysis for equation (2), it was found that:

$$\frac{\Delta T}{T} = \pm \frac{1}{2}\frac{\Delta h}{h} \qquad (3)$$

This indicates that a small change in the length of the rope from which the hexarotor is hanging will influence the period with a factor of half that of the small change.

2.1.1.2 Torsional Pendulum Tests

The torsional pendulum test is conducted by first winding the hexarotor around the vertical axis before it is released allowing it to rotate while measuring the period. The moment of inertia for a known mass (m) is then determined from [9,10]:

$$I_{jj} = \frac{mgD^2}{4h\omega^2} \qquad (4)$$

The axis in which the moment of inertia is being calculated is represented by I_{jj} where the subscript j indicates the x, y or z axis. The angular frequency (ω) is defined as $2\pi f$ where f is the oscillation frequency and inversely proportional to the period (T). Substituting these into equation (4) results in the following description for the moments of inertia:

$$I_{jj} = \frac{mgD^2}{4h(2\pi f)^2} = \frac{mgD^2T^2}{16\pi^2 h} \qquad (5)$$

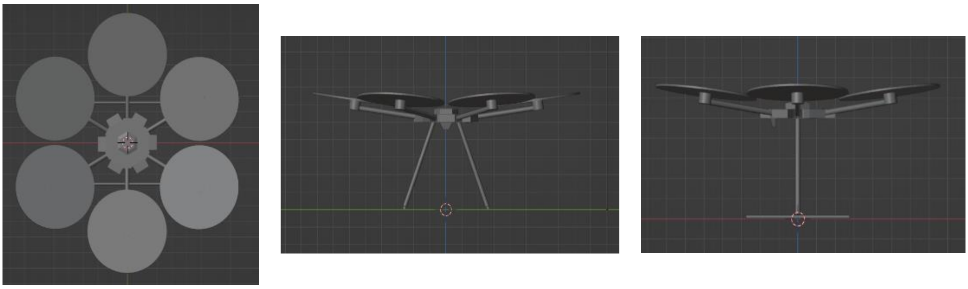
2.2 Centre of gravity

For a further understanding of the dynamics of the hexarotor, the centre of gravity will need to be determined. The centre of gravity, or centre of mass, is the location around which the combined mass of the system is concentrated [13]. Having knowledge of this point is important as it provides information about the dynamics of the system. From this, estimates can be made about how the system manoeuvres. For a discrete system of systems, the centre of mass can be determined by:

$$\overrightarrow{\mathrm{r_{cm}}} = \frac{(\Sigma \overrightarrow{\mathrm{r_i}}\mathrm{m_i})}{\Sigma\, \mathrm{m_{tot}}} \qquad (6)$$

Where m_i is the masses of an element in the system, $\vec{r_i}$ is the position of the centre of mass of the system relative to the reference point, and $\mathrm{m_{tot}}$ is the sum of all the masses.

To estimate the centre of gravity for the entire system (hexarotor with its payload), equation (6) is used. With assistance from [14], the point of origin from which the distance of each components' centre of gravity was calculated is shown in Figure 2 as a red circle. This point is in the centre of the hexarotor at the base of its legs.



(a) x-y plane (b) x-z plane (c) y-z plane

Fig2: Point of origin for the DJI M600 Pro centre of gravity determination

The weight of the hexarotor and its individual payload components are listed in Table 1 and using this information, along with equation (6), the centre of gravity of the hexarotor was calculated for the x-, y and z-axis. The centre of gravity of the UAV with its attached payload is detailed in Table 1. These results were acceptable because it was seen that during the moment of inertia tests, the hexarotor would hang horizontally, within a margin of 3°, when the ropes used to hang the hexarotor were the same length in the various orientations.

Table 1: Centre of gravity calculations

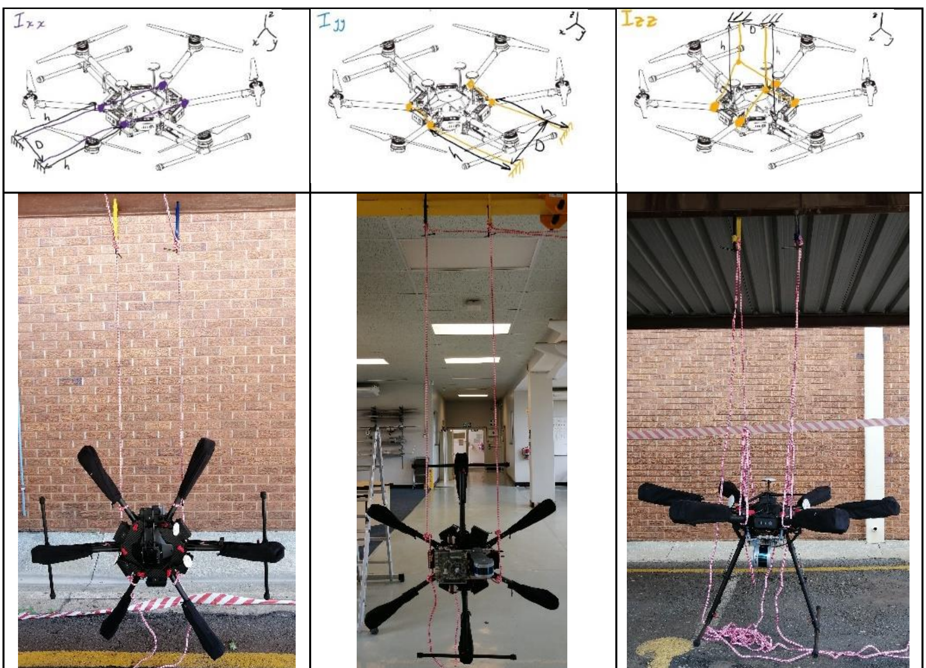
Item	**Description**	**Mass [kg]**	**Centre of mass distance from origin [m]**		
			x	**y**	**z**
1	Velodyne Puck LITE LiDAR (including cabling) *	0.89	0.0122	0.02	0.355
2	MicroStrain 3DM-GX5-25 IMU (including cabling) *	0.0665	0.041	0.02	0.43
3	Onboard PC (including CPU, RAM, fan, SSD) *	1	-0.075	0.02	0.41
4	Alfa Long Range WiFi Extender *	0.05	0	0	0.64
5	DC-DC converter *	0.3	-0.070	-0.017	0.455
6	Hardware mounts *	0.2	0.010	0.2	0.434
7	UAV (including propellers batteries, switches, fasteners, and connectors)	9.9535	0.003	-0.001	0.398
8	Velodyne controller*	0.1	0	0	0.615
	Total mass	12.56			
	Centre of mass		0.004	-0.002	0.395

NOTE: Items marked with * have estimates for mass

The results in Table 1 illustrate that the estimated centre of mass of the system lies a few millimetres off centre from point of origin in the x- and y-directions. The z-coordinate of the centre of mass lies approximately at the position of the payload. The centre of mass x- and y-coordinates provide an indication that the mass of the payload is evenly distributed across the weight of the hexarotor. For example, the heavier mass and the lowered centre of mass would require more propellor thrust for lift than what is specified in the user manual.

2.3 Experimental Setup

As explained in the previous section two pendulum tests will be performed. First a gravitational pendulum test to validate the experimental setup, followed by a torsional pendulum test to determine the moments of inertia. Before conducting the experiments, the hexarotor is weighed to determine its mass. For each experimental setup, the hexarotor is hung from a stable structure using two ropes in the desired orientation as shown in Figure 3.



(a) *Ixx* Orientation	(b) *Iyy* Orientation	(c) *Izz* Orientation

Fig 3: Three orientations to experimentally determine the moments of inertia.

The rope is attached to the hexarotor at two points of contact, for stability and security, and at one location on the structure to allow it to swing or rotate for the various tests. After the hexarotor is suspended, its orientation is verified with a digital spirit level ensuring that it is within a 5° error. This will verify if the centre of gravity of the hexarotor is located in between the two ropes from which it is hanging. Once the hexarotor is hanging securely, the length of the rope from the knot on the structure to the knot on the hexarotor is measured as well as the distance between the two connecting points on the structure.

With the experiment properly set up, the gravitational pendulum test can be conducted. A stopwatch will be used to record the time it takes for the hexarotor to complete a single swing, that is, from the starting position forwards and then back to the starting position. Using the lap function of the stopwatch, at least 10 times should be recorded, and the average of those times is then used as the period. Once the gravitational pendulum tests have been completed and the value of gravity has been verified using equation (2), the torsional pendulum tests can be conducted.

For the torsional pendulum test, the hexarotor is wound around the vertical axis until both ropes are completely spiralled around each other. The hexarotor is then released and the time it takes to complete one revolution is recorded. Similar to the gravitational pendulum test, at least 10 revolutions should be recorded, and the average period is then determined. The moment of inertia for that specific orientation is determined from equation (5). Both experiments are then repeated for the next orientation.

2.3.1 UAV Description

The UAV used in the pendulum experiments is the DJI M600 Pro, depicted in Figure 4. The hexarotor has an attached payload which consists of an onboard computer, inertial measurement unit (IMU), Velodyne LiDAR sensor and a wi-fi adaptor. The hexarotor, with its propellers and landing gear extended, has dimensions of 1668 mm x 1518 mm x 759 mm and weighs 12.54 kg. The covers on the UAV's propellers were kept on during the experiments and were considered to have negligible influence on the experiment. They were kept on as a safety precaution to mitigate against unexpected take-off if the UAV was turned on accidentally during the experiment.

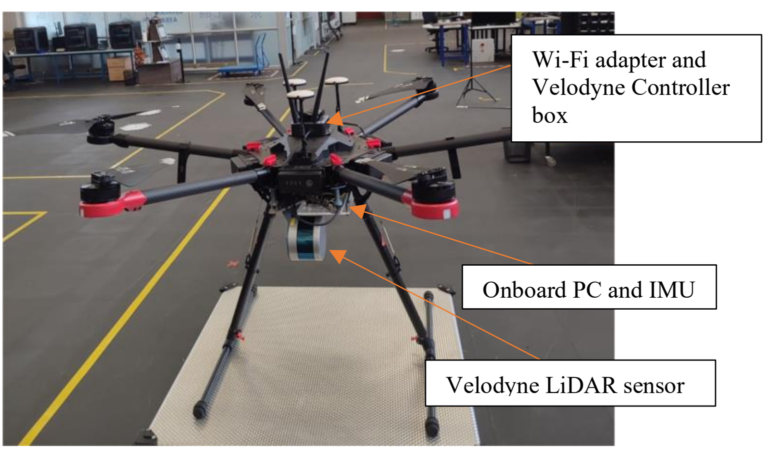


Fig. 4. DJI M600 Pro with attached payload.

3 Results and Discussion

The results for both the gravitational and torsional pendulum tests for all orientations are presented in Table 2 and Table 3 respectively. The average period of three separate tests were taken for each experiment and orientation.

The effect of the landing gear extended or folded is also considered especially as the flying configuration of the hexarotor during its operations will require the landing gear to be folded. The moments of inertia for the flying configuration are therefore of interest to develop an autonomous flight control system for navigation.

Table 2. Measurements and results from the gravitational pendulum experiments.

Orientation	h (m)	D (m)	T_1 (s)	T_2 (s)	T_3 (s)	Gravitational T_{avg} (s)	g (m/s^2)	Error (%)
I_{xx} (Roll)	1.95	0.31	2.88	2.88	2.88	2.88	9.28	5.4
I_{yy} (Pitch)	2.00	0.39	2.858	2.857	2.848	2.85	9.69	1.22
I_{zz} (Yaw)	1.45	0.30	2.43	2.41	2.417	2.42	9.93	1.33

Table 3. Measurements and results from the torsional pendulum experiments.

Orientation	h (m)	D (m)	g (m/s^2)	T_1 (s)	T_2 (s)	T_3 (s)	Torsional T_{avg} (s)	I (kg/m^2)
I_{xx} (Roll)	1.95	0.31	9.28	2.209	1.97	1.96	2.05	0.15
I_{yy} (Pitch)	2.00	0.39	9.69	2.049	1.93	1.9	1.96	0.22
I_{zz} (Yaw)	1.45	0.30	9.93	3.868	3.7025	3.81	3.79	0.70

3.1 Effect of landing gear and Yaw moment of inertia (I_{zz})

For the I_{zz} orientation the hexarotor is oriented as shown in Figure 3(c) which is its normal take-off and landing orientation. The conventional flying orientation for the hexarotor is with its landing gear folded. For this reason, the effect of landing gear on the calculated moments of inertia were investigated for the normal take-off and landing orientation only. The results from this experiment are listed in Table 4, where gravity is determined from equation (2) using the average period (T_{avg}).

From Table 4 the margin of error for gravity from 9.81 m/s^2 is only 0.2 % when the landing gear is extended. With the landing gear folded the gravitational pendulum test estimated gravity with an error of 1.33 %, which is within an acceptable error margin to continue the experiment. The landing gear therefore did not have a considerable influence on the accuracy of the experimental setup. It will therefore be folded for the remainder of the experiments, thereby representing its normal state of operation.

Table 4. Effect of the landing gear on the moment of inertia.

I_{zz}	T_1 (s)	T_2 (s)	T_3 (s)	T_{avg} (s)	g (m/s^2)	Margin of error from 9.81 m/s^2 (%)
Extended	2.43	2.41	2.417	2.419	9.78	0.2
Folded	2.411	2.398	2.393	2.401	9.93	1.33

The yaw moment of inertia was then determined from equation (5) to be 0.70 kg/m^2 as shown in Table 3. It is also noted that the yaw moment of inertia is larger than that of the roll and pitch moments of inertia. This is expected and correlates well with observations from literature where the yaw moment of inertia is always larger for an object whose height is less than its length and width [7,8].

3.2 Pitch moment of inertia (I_{yy})

For the I_{yy} orientation the hexarotor is oriented as shown in Figure 3(b). Similar to the I_{zz} orientation, the margin of error for gravity in the gravitational pendulum tests is 1.22 % as seen in Table 2. This is within an acceptable error margin for the continuation of the experiment. It is interesting to note that the LiDAR of the payload is more towards one side potentially offsetting the centre of gravity as the hexarotor is not symmetrical about the vertical axis. This, however, does not seem to influence the experimentally determined value of gravity to a large extent.

The pitch moment of inertia from Table 3 is 0.22 kg/m^2. This is smaller than the yaw moment of inertia, which is to be expected based on observations from literature [7,8].

3.3 Roll moment of inertia (I_{xx})

For the I_{xx} orientation the hexarotor is oriented as shown in Figure 3(c). For the gravitational pendulum test the value for the calculated gravity had an error margin of 5.4 %. This is not within an acceptable margin of error. To reduce this margin of error, several experiments were conducted varying the length of the rope to change the pendulum period. The results from these attempts are shown in Table 5. An error margin of 5.4 % is the best. This is unexpected as the I_{xx} and I_{yy} orientations are similar – apart from the direction of the landing gear and payload – and should therefore have similar results. In fact, the hexarotor and payload are symmetrical about the vertical axis and theoretically it should be possible to accurately estimate gravity. The digital spirit level did indicate that the hexarotor was hanging horizontally at an angle smaller than 3° compared to the I_{yy} orientation which was at a perfect 0°. This could be the potential source of error, but further investigation is needed to confirm. Another potential source of error could be air resistance because in the I_{xx} and I_{yy} orientations, the UAV has a greater surface area in the swinging direction, and therefore would experience greater air resistance as it swings. This would affect the period of the swing measured in both orientations.

Despite not verifying the experimental test setup, the torsional pendulum tests were still completed. The roll moment of inertia from Table 3 is 0.15 kg/m^2. This is smaller than the pitch moment of inertia, which is unexpected. Based on [7,8] the roll and pitch moment of inertia should be equal or within a 2 % margin of error as these orientations are supposed to

Table 5. Effect of varying the rope length on the gravitational pendulum test for the roll moment of inertia orientation.

h (m)	T_1 (s)	T_2 (s)	T_3 (s)	T_{avg} (s)	g (m/s^2)	Margin of error from 9.81 m/s^2 (%)
1.99	3.034	3.046	2.644	3.03	8.53	12.96
1.8	2.890	2.888	2.891	2.89	8.5	13.27
1.47	2.602	2.631	2.618	2.62	8.47	13.57
1.43	2.621	2.642	2.648	2.64	8.07	17.65
1.4	2.492	2.511	2.480	2.49	8.9	9.18
1.3	2.398	2.397	2.420	2.4	8.92	8.98
1.2	2.309	2.297	2.327	2.31	8.87	9.49

be similar apart from the directional changes of the landing gear and payload components (e.g., onboard PC and LiDAR) for the I_{xx} and I_{yy} orientations.

4 Conclusion

This paper shows how we experimentally determined the moments of inertia of a DJI M600 Pro hexarotor. The approach that we used is similar to one that has been used for fixed-wing UAVs, as there is limited information and experimentation available for multi-rotors.

First a gravitational pendulum test is performed to determine gravity within a reasonable accuracy, thereby ensuring the experimental setup is correct. For both the pitch (I_{yy}) and yaw (I_{zz}) orientations the experimental setup was validated where gravity was determined with a margin of error of 1.22 and 1.33 % respectively. Unfortunately, the experimental setup could not be validated for the roll (I_{xx}) orientation with a margin of error of 5.4 % for the gravity estimate. The unexpected gravity values were due to fluctuating periods. The reasons behind these unexpected results still need to be investigated further.

After the validation of the experimental setup, a torsional pendulum test was then conducted to calculate the moments of inertia. The moments of inertia were found to be 0.15 kg/m^2 for the roll (I_{xx}), 0.20 kg/m^2 for the pitch (I_{yy}) and 0.70 kg/m^2 for the yaw (I_{zz}) orientations. The moment of inertia for the roll orientation is not accepted as accurate due to the failure to validate gravity during the gravitational pendulum test.

These moments of inertia will be used to design a flight controller for the DJI M600 Pro hexarotor. These parameters ensurethat the developed model of the hexarotor, including its flight dynamics, closely resembles that of the physical hexarotor. With these, an accurate flight controller can be designed in a simulated environment before implementing it on the hexarotor. The flight controller will be used for stable hovering and path following during navigation. This is especially important for autonomous flight in confined or GPS-denied environments.

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