Vortex Identification in a Free-Surface Flow Problem Solved Using SPH

Petr Jančík[1,*], and *Tomáš* Hyhlík[1]

[1]Department of Fluid Mechanics and Thermodynamics, Faculty of Mechanical Engineering, Czech Technical University in Prague, Technická 4, 160 00 Prague 6, Czechia

Abstract. This article focuses on vortex structure identification methods and their implementation into smoothed particle hydrodynamics (SPH) framework. The most common criteria based on the local velocity gradient tensor analysis like Q-criterion, Δ-criterion, and λ_2-criterion are introduced together with their implementation for SPH data. A two-dimensional 'double dam break' problem was chosen as a test case because it results in a violent transient free-surface flow with emerging and vanishing eddies of various sizes and intensities. Q-criterion and Δ-criterion results were virtually identical, whereas λ_2-criterion appeared to be the most restrictive in vortex identification, so it was the best in suppressing incorrect findings due to the numerical solution imperfection. The analysis of the problem was therefore conducted using the λ_2-criterion.

Keywords: *CFD; vortex; SPH; free surface*

1 Introduction

Vortices are characteristic structures that are present in almost every flow. They significantly influence the transport of momentum and other quantities in fluids. Even though a vortex seems like a relatively simple structure, there is not even a clear definition. One of the most intuitive definitions was given by Lugt [1]: a vortex is the rotating motion of a multitude of material particles around a common center.

However, this definition is vague and not directly applicable to flow analysis. Therefore, some objective criteria have been proposed. The most obvious thing to do is the evaluation of vorticity in the flow field, but it did not turn out to be precise because it cannot distinguish between rotation and shear deformation [2]. More elaborate methods usually work with the velocity gradient tensor. The most widely used are Q-criterion [3], Δ-criterion [4], and λ_2-criterion [5]. Other methods have been proposed as well, for example, Γ_2-criterion suitable for PIV data analysis [6], residual vorticity originating from the triple decomposition [7], or relatively recently Ω-criterion [8].

This paper aims to present the implementation of the picked vortex identification criteria (Q-criterion, Δ-criterion, and λ_2-criterion) for 2D SPH data and find the one which gives the best results. A subdomain of the solution of 'two collapsing columns of liquid' served for the criteria comparison. Then, the problem of two collapsing liquid columns was analyzed using

* Corresponding author: petr.jancik@fs.cvut.cz

the best performing criterion (λ_2). The origin, development, and final dissipation of the most noticeable vortices are presented.

2 Methods

2.1 Smoothed particle hydrodynamics

Smoothed particle hydrodynamics (SPH) is a mesh-free particle numerical method suitable for transient free-surface problems. Many interacting particles of constant mass represent the fluid. A system of ordinary differential equations determines the evolution of particle properties and their motion. The continuity equation and the momentum equation used in this work are the following:

$$\frac{\mathrm{D}\rho_i}{\mathrm{D}t} = \rho_i \sum_j \frac{m_j}{\rho_j} \left(\mathbf{v}_i - \mathbf{v}_j\right) \cdot \nabla W_{ij} \tag{1}$$

$$\frac{\mathrm{D}\mathbf{v}_i}{\mathrm{D}t} = -\sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij}\right) \nabla W_{ij} + \mathbf{g} \tag{2}$$

where indices i and j denote different particles, ρ is density, p is pressure, m is mass, $\mathbf{v}$ is velocity vector, and $\mathbf{g}$ is gravity acceleration vector. The function W_{ij} is the weight function that mediates the interactions between particles. In this work, a truncated Gaussian smoothing function was used. The fluid is considered inviscid, but there is the artificial viscosity term Π_{ij} that serves for numerical stabilization [9].

In SPH, a common approach is to consider incompressible fluid as compressible one. Because of this, an equation of state is necessary to close the system of equations. In this work, we use

$$p = c^2(\rho - \rho_0) \tag{3}$$

where c is the numerical speed of sound and ρ_0 is the fluid density at zero gauge pressure. A too-small value of c would lead to very compressible fluid, and a too-large value would mean the necessity of prohibitively small integration steps. Usually, a good choice of c is when the variation in fluid density remains below 1 %.

Wall boundary conditions are modeled as free-slip using virtual particles according to [10]. The authors of this paper used a virtually identical solution method in their previous work [11]. For more details about the method, see, e.g., [12].

2.2 Vortex identification methods

In this section, the used vortex identification criteria are defined and their implementation for a 2D flow field. To do so, let us define the velocity gradient tensor

$$\nabla\mathbf{v} = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right]^T \cdot [u, v] = \begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix}. \tag{4}$$

To evaluate the elements of the velocity gradient tensor in the SPH framework, a rather standard SPH procedure was used. For example, the result for the first element is

$$(u_x)_i = \sum_j \frac{m_j}{\rho_j} \left(u_i - u_j\right) \frac{\partial W_{ij}}{\partial x}. \tag{5}$$

The remaining elements can be found in the same way.

2.2.1 Q-criterion

In Q-criterion [3], vortices are defined by a positive value of the second invariant of $\nabla\mathbf{v}$. It can be written

$$Q = \frac{1}{2}\left[\mathrm{tr}(\mathbf{\Omega}\mathbf{\Omega}^T) - \mathrm{tr}(\mathbf{S}\mathbf{S}^T)\right], \tag{6}$$

where $\mathbf{\Omega}$ is the anti-symmetric part of $\nabla\mathbf{v}$ and $\mathbf{S}$ is the symmetric part of $\nabla\mathbf{v}$.It can be interpreted as the excess of rotational motion over shear strain rate.

In two dimensions, it can be written as

$$Q = u_x v_y - u_y v_x - \frac{1}{2}\left(u_x + v_y\right)^2. \tag{7}$$

In this form, Q can be evaluated using formula 5. A vortex is identified where $Q > 0$.

2.2.2 Δ-criterion

According to Δ-criterion [4], vortices are present there, where the eigenvalues of $\nabla\mathbf{v}$ are complex numbers. In a 3D case, this condition is equivalent to

$$\Delta = \left(\frac{Q}{3}\right)^3, + \left(\frac{R}{2}\right)^2 > 0, \tag{8}$$

where Q is defined in equation 6 and $R = \det(\nabla\mathbf{v})$.

In a 2D case, the characteristic equation of $\nabla\mathbf{v}$ is

$$\lambda^2 - \lambda\left(u_x + v_y\right) - u_y v_x + u_x v_y = 0\,, \tag{9}$$

and the eigenvalues are complex when the discriminant of the characteristic equation is negative. Therefore, the criterium stands

$$\Delta = 4(u_y v_x - u_x v_y) + \left(u_x + v_y\right)^2 < 0. \tag{10}$$

2.2.3 λ₂-criterion

The λ_2-criterion [5] define vortex there, where the tensor $\mathbf{\Omega}\mathbf{\Omega}^T + \mathbf{S}\mathbf{S}^T$ has at least two negative eigenvalues. If the eigenvalues are ordered $\lambda_1 \geq \lambda_2 \geq \lambda_3$, then it is equivalent to $\lambda_2 < 0$.

In a 2D case, λ_2-criterion can be expressed [13]

$$\lambda_2 = u_y v_x - u_x v_y + \frac{1}{2}\left(u_x + v_y\right)^2 + \frac{1}{2}\left|u_x + v_y\right|\sqrt{\left(u_x - v_y\right)^2 + \left(u_y + v_x\right)^2}. \tag{11}$$

3 Results and discussion

3.1 Analyzed problem

The vortex identification methods were applied to the flow field resulting from the collapse of two liquid columns in the gravity field. The initial configuration is depicted in Fig. 1. The total number of particles was 60 000, and 3 seconds or the simulation are presented. The problem was solved as a free-surface problem, so the surrounding gaseous phase is omitted.

 https://doi.org/10.1051/matecconf/202236904003

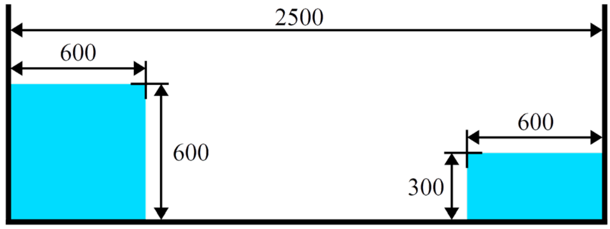

Fig. 1. The initial configutation of the analyzed problem. Dimensions in millimeters.

3.2 Method comparison

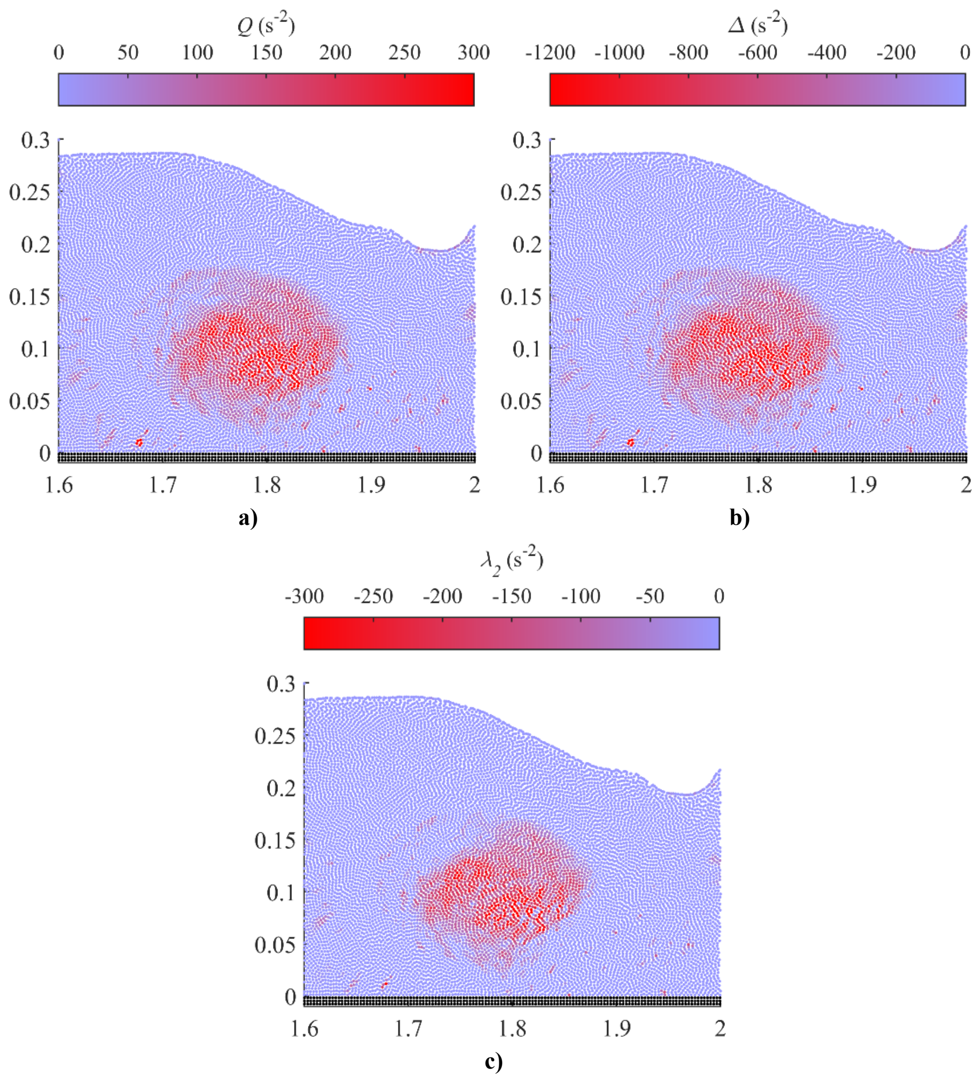

Fig. 2. The comparison of the vortex identification criteria for a chosen part of the flow domain at time 2 s. a) Q-criterion, b) Δ-criterion, c) λ_2-criterion. Dimensions in meters.

When analyzing equations 7, 9, and 11, it is noticeable that they are equivalent if the fluid is incompressible because the terms containing $(u_x + v_y)$ are zero (the continuity equation). All the discussed criteria thus reduce to

$$u_x v_y - u_y v_x > 0. \tag{12}$$

However, the used computational model considers a compressible fluid (very weakly), so the results may slightly differ. Fig. 2 illustrates the comparison of the criteria. The Q-criterion and Δ-criterion yield virtually the same results. Since they differ in the relative weight of the term $(u_x + v_y)^2$, the conclusion is that this term makes a negligible difference. Apart from the vortex identified in the middle (Fig. 2), there are small areas identified as vortices as well. However, these are numerical artifacts rather than actual vortices. The results by the λ_2-criterion are slightly different. This criterium tends to be more restrictive and identifies smaller areas than the previous, as it suppresses the numerical artifacts better. Even though the term $\frac{1}{2}|u_x + v_y|\sqrt{(u_x - v_y)^2 + (u_y + v_x)^2}$ is close to zero, it affects the result. From the compared criteria, the λ_2-criterion seems to yield the best results, so it was used in the following analysis.

3.3 Problem analysis

After the collapse of the two liquid columns, the surges met approximately in the middle of the tank. That resulted in a violent flow with rolling waves. The impacts of these waves were the general source of vortex structures that were identified in the flow field. Since the walls were a free-slip boundary condition, there were no vortices originating from the turbulent boundary layer. Chosen instances of the flow at the various time are presented in Fig. 3, where the most distinct vortices are marked for easier orientation.

At $t = 1$ s, there are no distinct vortical structures. There are some small areas of high λ_2, but they are only a result of the violent impact of the rolling wave on the surface, and they disappear rather quickly.

At $t = 1.5$ s, there appeared two vortices in the right part of the tank (Ia and Ib). In the middle of the tank, there were two cavities as a result of rolling wave impact, and the fluid around them was going to form another two vortices (II and III).

At $t = 2$ s, vortices Ia and Ib merged into one larger vortex, but the intensity decreased due to dissipation. Vortices II and III evolved from the cavities, and they were relatively strong and well defined. There were also two more cavities on the left (IV) and on the right (V) that became vortices as well.

At $t = 2.5$ s, vortex I was still there and well defined, while vortex III became considerably weaker, and vortex II vanished completely. Vortices are noticeable now but not very strong or well defined. Once again, another vortex was emerging as a result of a rolling wave impact (VI).

At $t = 3.0$ s, all the followed vortices became relatively weak due to dissipation, and only the most recently created vortex (VI) remained relatively strong. Overall, the occurrence and intensity of vortices decrease from this point as the flow becomes calmer.

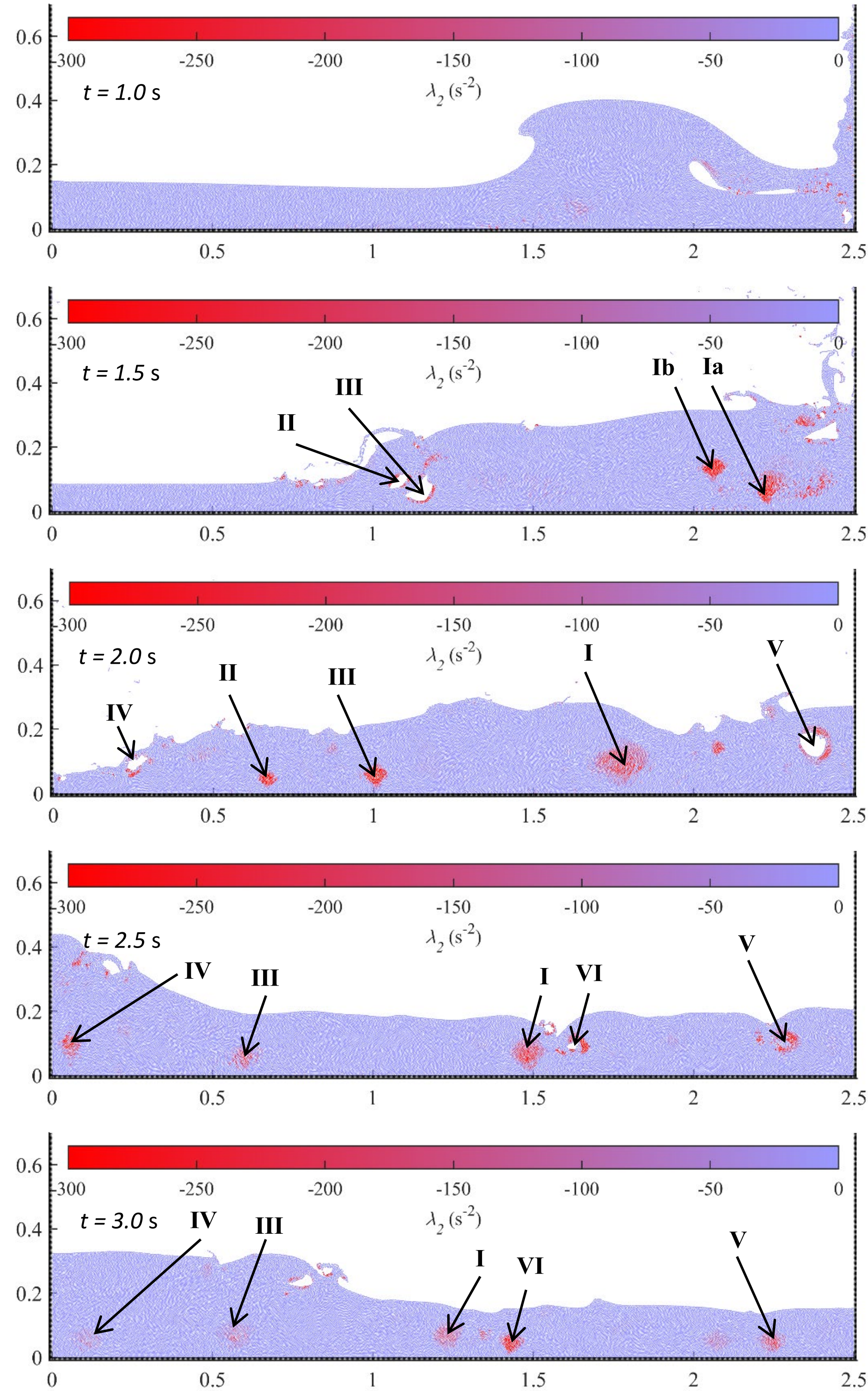

Fig. 3. The results of λ_2-criterion at various time instances with marked vortices. Dimensions in meters.

4 Conclusion

The most widely used vortex identification criteria (Q-criterion, Δ-criterion, and λ_2-criterion) were introduced together with their possible implementation for 2D SPH data. Quite expectably, they yielded almost identical results. However, the λ_2-criterion proved to be the most suitable because it managed to suppress errors most likely originating from the data imperfection.

The problem of two liquid column collapse was analyzed by the λ_2-criterion. It became apparent that the most significant vortices originate from the impact of rolling waves on the liquid surface. The boundary layer was not modeled, so no significant vortices were identified near the walls.

In the future, the criteria could be further refined to give better results, or some other criteria could be implemented as well. Not only methods based on velocity gradient tensor are available. A combination of these criteria with, for example, previously tested the Finite-time Lyapunov exponent (FTLE) for identification of Lagrangian coherent structures [14] may give remarkable results.

This work was supported by the Grant Agency of the Czech Technical University in Prague, grant No. SGS20/112/OHK2/2T/12.

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