Analysis of the Influence of Boundary Condition on Simulation Accuracy of Solidification Thermokinetics Model

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Abstract. For a sample steel casting of a cylindrical shape that was cast in a metallic cylindrical mould, the analysis of influence and importance of the main boundary condition on the simulation accuracy of the temperature field in the system of casting-mould-environment was performed. For this system, the influence of the boundary condition on the frame of the mould, on the bottom of the mould, on the top of the casting, and on the casting-mould interface was analysed. As a comparing quantity for accuracy, the time-dependent temperature field (by means of isotherms) of the casting and of the mould and the total solidification time were selected. When the simulation considers the boundary conditions of one of these boundaries of the system, the heat transfer coefficient on the remaining three boundaries of the system is equal zero. The resultant heat flow along the axis is zero always. The conclusions can be used for fine-tuning of solidification models as well as for other applications; e.g. analysis and solution of inverse heat transfer problems.

1 Mathematical model

The solidification and cooling of a classically cast (i.e. gravitationally poured) casting (ingot) and simultaneous heating of the mould and chills is, from the viewpoint of thermokinetics, a very complicated case of 3D transient heat and mass transfer [1-5]. In systems comprising the casting (ingot), the mould, chills and environment, all three kinds of heat transfer take place. Since these problems cannot be solved analytically, even with the second-order partial differential Fourier equation (1) (where mass transfer is neglected and conduction is considered as the most important of the three kinds of heat transfer), it is necessary to engage numerical methods. The Fourier equation (1) for a casting (ingot) must be adapted to describe the temperature field of a casting in all its three phases: in the melt, in mushy zone and in the solid phase.

$$\frac{\partial(h\rho)}{\partial\tau} = \nabla\cdot\left(k_{eff}\nabla T\right) \tag{1}$$

Here it is necessary to introduce the enthalpy, which is

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$$h = \int_0^T \left(c(\zeta) - \Delta H \frac{\partial f_s}{\partial \zeta} \right) \mathrm{d}\zeta \tag{2}$$

where ΔH is the latent heat (J/kg) and f_s is the solid fraction (-). In equation (1), T [K] is temperature, τ [s] is time, k_{eff} [W/m.K] is the effective heat conductivity and ρ [kg.m^{-3}] is the density.

The original model of the transient temperature field of a system including the casting (ingot)-mould (chills)-ambient was developed [5], which is based on the first and second Fourier's laws on transient heat conduction, and the first and second law of thermodynamics. Further, the numerical method of finite differences with explicit formula for the unknown temperature of the mesh node (*i, j, k*) in the next time step is applied, which is a function of temperatures of the same node and six adjacent nodes in the Cartesian coordinate system from the previous time step. The model considers non-linearity of the task; it means the dependence of thermo-physical properties of all materials of the systems on the temperature [6-7] and the dependence of heat transfer coefficients (boundary conditions) on the temperature of all external surfaces. The model is equipped with and interactive graphical environment for automatic generation of a mesh and for evaluation of results, so-called pre-processing and post-processing.

2 Influence of boundary conditions

For a sample steel casting of a cylindrical shape that was cast in a cast iron cylindrical mould, the analysis of influence of boundary conditions on simulation accuracy of temperature field in the system casting-mould-environment was performed (Fig. 1). The case of solidification in a non-metal mould was not conducted because the boundary conditions here have a negligible effect [7-9]. The analysis was conducted with respect to the heat transfer coefficient on the surface of the casting and mould, the bottom of the mould, the outer surface of the frame of the mould, as well as the casting-mould interface (Fig. 2). When the calculation considers the boundary conditions of one of these boundaries of the system, the heat transfer coefficient on the remaining three boundaries of the system is equal to zero. The resultant heat flow along the heat axis is zero, $q = 0$.

Each of the four coefficients of total heat transfer h is given by the sum of conventional h_c and h_r component. The conventional component is determined using the similarity theory from the general criteria equation $Nu = C(Pr\ Gr)^n$, which applies to natural convection in limited and unlimited space. The Nu criterion (the dimensionless convective coefficient), according to this relationship, is a function of dimensionless properties of the flowing liquid (the Pr criterion) and the uplift (the Gr criterion). The numerical evaluation of h_c is performed in the general way. The same applies to the radiation component, which is transformed into the reduced heat transfer coefficient.

The heat transfer outside the frame of the mould h_f does not affect the total time of solidification of the analysed cylinder. If h_f changes from 0 to 10^6 W/m^{2}K (the real values range close to 10^1 W/m^{2}K), then not only the total time of solidification is constant but the temperature field along the axial section of cylinder, illustrated using isotherms, is the same. Even the temperature field through the section of the mould does not differ significantly. The course of the isotherms (Fig. 3), for example 180 s after casting, for $h_f = 0$ W/m^{2}K and $h_f = 10^6$ W/m^{2}K is unexpectedly very close.

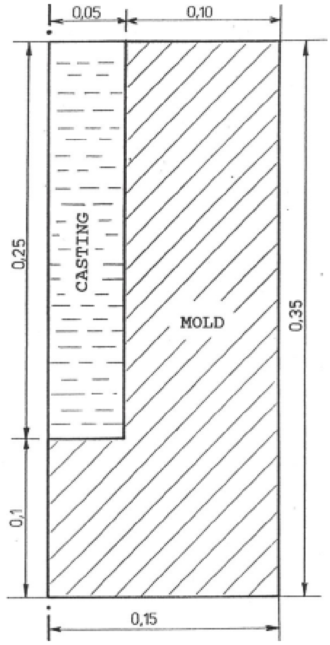

Fig. 1. Longitudinal axis section of the system casting-mould-environment.

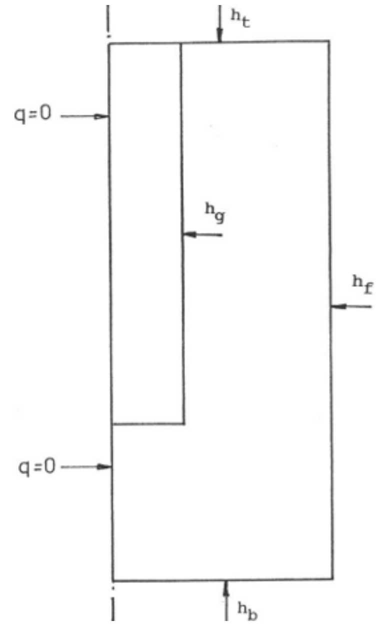

Fig. 2. The boundary conditions.

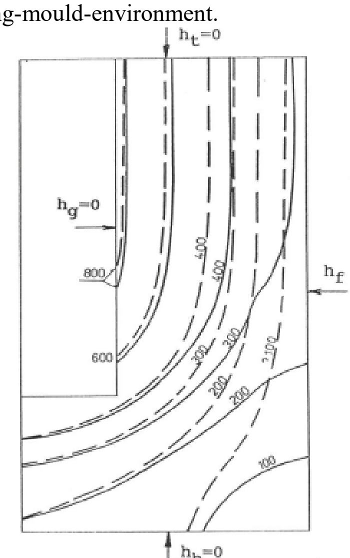

Fig. 3. The analysis of the boundary condition condition on the frame of the mould. Time: 180 s, $h_f = 0$, $h_f = 10^6$.

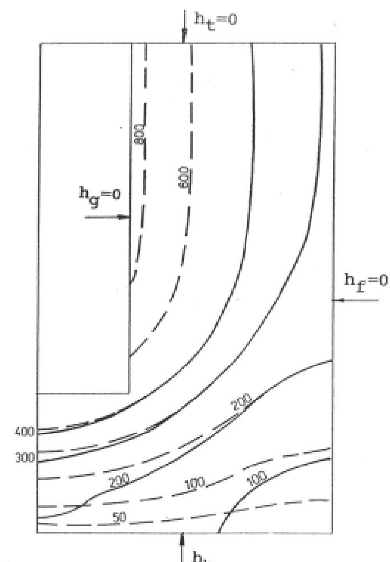

Fig. 4. The analysis of the boundary on the bottom of the mold. Time: 180 s, $h_b = 0$, $h_b = 10^6$.

The heat transfer from the bottom of the mould h_b also has no effect on the temperature field of the casting during solidification. The effect on the mutual position of the isotherms inside the mould, for both extreme values of h_b, (Fig. 4) is even smaller than in Fig. 3. For example, the 600 - 800 °C isotherms coincide. Even the range of values of the heat transfer coefficient for the top of the casting and mould h_t in a wide range from 0 to 10^6 W/m^{2}K simply does not affect the total time of solidification, even though it does affect the dynamics of the temperature field (Fig. 5), however not very significantly.

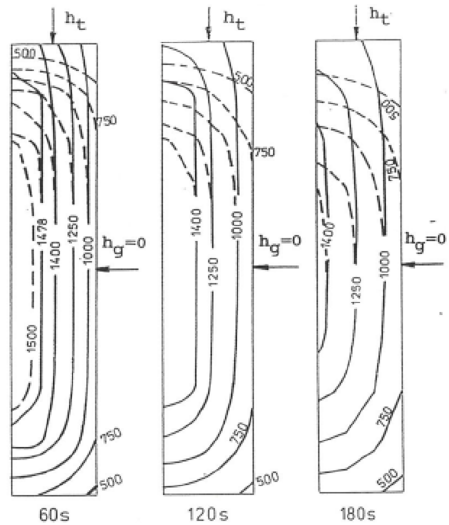

Fig. 5. The analysis of the boundary condition on the top of the casting and mould Time: 60, 120, 180 s, $h_t = 0$ W/m^{2}K, $h_t = 10^6$ W/m^{2}K.

The most difficult task is to define the boundary condition for the casting-mould interface (contact surface) and in the gap. A discussion covering the derivation of this is not the subject of this paper. Furthermore, it has significant effect on the total time of solidification, as well as on the formation of the temperature field within the casting and the mould. It could be stated that the majority of recognized authors dealing with numerical solidification models in sand (non-metal) moulds assume an ideal physical contact between these two surfaces. This simplification is fully justifiable. During the solidification in metal moulds, this assumption is ruled out. In most cases, it is necessary to consider a change in the temperature resistance of the interface, i.e. gaps as reciprocal values h_g for the contracting casting and expanding mould. In order to simplify the analysis, it is presumed that the selected size of the gap between the casting and the mould is constant during solidification. It should be noted that here too, h_g is the sum of the conventional and radiation components. Fig. 6 illustrates the influence of the size of the gap on the total time of solidification and the relative extension of the total time of solidification in percentage. If the gap widens from 10^{-4} m to 10^{-2} m, then the total solidification time moves from 342 s to 1035 s, i.e. to 202.6 %.

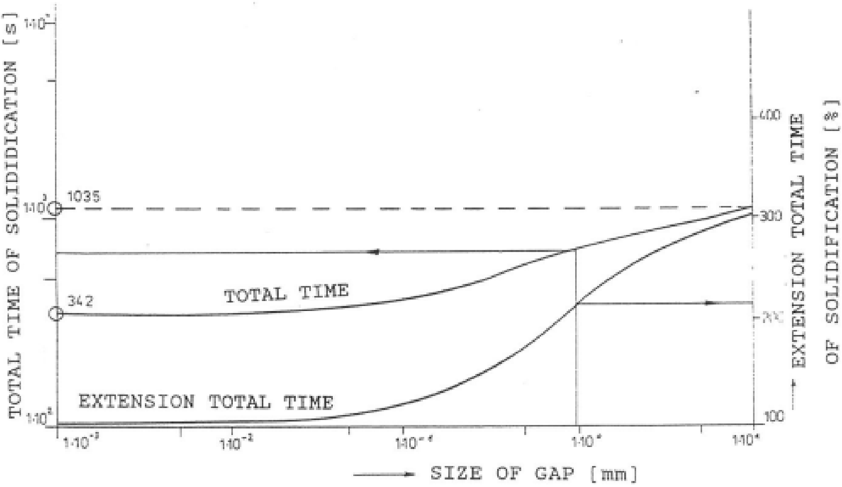

Fig. 6. The analysis of the boundary condition on the casting-mould interface (in the gap).

3 Conclusion

The main boundary conditions in the solution of heat transfer in a casting-nonmetalic mould-environment system have significant influence on the actual casting process of gravitationally cast steels, and on the accuracy of numerical simulation and optimization. It is anyway necessary to reduce energy and material consumption in foundry and metallurgical production using the numerical thermokinetics models. An original numerical model of the 3D transient temperature field was used for the influence analysis of boundary conditions on the accuracy of simulated temperature field and its numerical optimization. The influence of the boundary condition on the frame of the mould, on the bottom of the mould, on the top of the casting and on the casting-mould interface was analysed.

This work was supported by the project Adaptive Soft Computing Framework for Inverse Heat Transfer Problems with Phase Change, reg. no. 22-31173S funded by the Czech Science Foundation and by the project Computer Simulations for Effective Low-Emission Energy Engineering, reg. no. CZ.02.1.01/0.0/0.0/16_026/0008392 funded by Operational Programme Research, Development and Education, Czech Republic.

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