An Energy Approach Applied to Define Elasto-Plastic Constitutive Models Describing Thermomechanical Metallic Materials Behavior During Forming Processes

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Abstract. A new formalism for the definition of metallic materials constitutive laws expressing the stress as a function of the plastic deformation energy it is proposed. This new approach, called energy approach, can integrate physical mechanisms governing the microstructure changes during a plastic deformation. It is also important to emphasize that the proposed energy formulation is more relevant since it can describe physical phenomena taking place in a material forming process characterizing at the different scales the material properties evolution. This formulation remains valid for a large field of deformation, the whole spectrum of loading conditions and remains able to predict rigorously the material response for all types of stresses states: static, transient or dynamic.

Keywords: Energy Approach, Differential Equation's Constitutive Models, Plastic Deformation Energy, Work Hardening and Dynamic Softening.

1 Introduction

Metallic materials are used intensively in the majority of engineering sectors through a very wide range of forming operations dedicated to the manufacture of various components and structures. It is therefore important to define their thermomechanical behavior to be used by predictive numerical simulations in order to limit the use of expensive experimental studies and to be able to achieve the forming processes optimization. The technical and scientific literature offers a very wide class of constitutive laws modeling the thermomechanical behavior of materials subjected to different loading conditions. The classic formulations of the rheological laws express the Cauchy stress tensor as a function of instantaneous internal variables such as the strain tensor, the strain rate tensor and the temperature, making it possible to give with sufficient precision the response of materials under quasi- static, but not very precise for fast or severe dynamic stresses. Indeed in these last cases resorted to parametric calibration techniques coming out of the physical framework which requires a rather fine description of the microstructural mechanisms Moreover in the majority of cases, in order to describe the behavior of work hardening under specific conditions, the traditional methods reduce the effect of strain history only through the cumulative plastic strain defined by generalized strain rate integration. However, during the hardening mechanisms, the stress undergone by the material on the scale of an infinitesimal volume element is rather linked to the locally dissipated plastic deformation energy responsible for any change in the structural state of the material.

This research work proposes the application of a new formalism for the definition of the hardening laws expressing the stress as a function of the energy dissipated by the plastic deformation. This approach was firstly mentioned by R. HILL [1], indicating that the work hardening depends on the plastic deformation energy. Then a new formalism is introduced by M. YOSHINO and T. SHIRAKASHI [2] by expressing a reference stress as a function of the plastic deformation energy. The energy approach can also be seen as a consequence of the thermodynamics principles and of the constructual law which postulates that any system finds its evolution over the time minimizing the losses energies and minimizing the entropy rate (Prof. A. BEJAN - [3]).

2 Computational Principles

After a short presentation of the problem formulation, it is described a more general mathematical formulation of the metals constitutive models with the consideration of dynamic softening phenomena (restoration and recrystallizations) through the above mentioned energy approach [4-5]. It is demonstrated that in certain special cases, linked only to work hardening for example, the expressions of usual laws van be obtained and, conversely, the constitutive equations known as energy formulations can be expressed from the classical formulations i.e.:

$$\sigma_h = H(\bar{\varepsilon}) \Leftrightarrow \sigma_h = f(W) \text{ with } W = \int \sigma_h d\bar{\varepsilon} \tag{1}$$

The new constitutive energetic model will be discussed with respect to a numerical implementation in the resolution of the Prandtl-Reuss equations specific to a 3D description of the elasto-plastic deformation starting from an energy description of the variation of the equivalent stress expressed by a set of differential equations defined in terms of plastic strain energy W.

$$g\left(W, \frac{d\bar{\sigma}}{d\bar{e}}\right) = 0, h\left(W, \frac{dW}{d\bar{e}}\right) = 0, W = \int \bar{\sigma} d\bar{\varepsilon} \Leftrightarrow \sigma_h = f(W) \tag{2}$$

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