

Study of the enwrapping of the front profiles of the active elements of a three-screw compressor

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Abstract. The paper proposes a method for identifying the front profile of a worm-type active element, consisting of a three-screw compressor. The purpose of this identification is to study the frontal enwrapping between the profiles of the driver and the driven element, in order to produce a possible replacement element. As is it well known, the two profiles are mutually enwrapping profiles, which means that the problem can be treated as a plane enwrapping problem. The identification of the profiles was performed by specific reverse engineering methods, the parts being scanned on an ATHOS 500 scanning system. Subsequently, the analytical shape of the driven screw was identified and, applying the "virtual pole" method, the corresponding shape of the driver screw profile was deduced. The obtained profile was compared with the real profile, obtained by 3D scanning. The obtained results demonstrated not only the good match between the theoretical and the real profile but also the simplicity and robustness of the method applied for the study of the enwrapping, namely the "virtual pole" method.

Keywords: reverse engineering, reciprocally enwrapping profiles, "virtual pole" method.

1 Introduction

Fluid transport requires specific equipment, such as pumps and compressors [1-4].

Currently, the emphasis is on the ease of transport of liquids and gases so that their handling can be done with the highest possible efficiency and safety.

One of the methods to increase efficiency is to change the front profile of the active elements of compressors and pumps. But, a side effect of this change is the difficulty of securing the necessary spare parts in case one of these items is damaged.

The usual solution, to completely change the pump or compressor, is not always possible or efficient.

The paper proposes a method for identifying the front profile of a worm-type active element, consisting of a compressor with three screws. The purpose of this identification is to study the frontal enwrapping between the profiles of the driver and driven element, in order to produce a possible replacement element.

The two profiles are reciprocally enwrapping profiles, which makes the problem can be treated as a plane enveloping problem.

The identification of the profiles was performed by specific reverse engineering methods, the parts being scanned on an ATHOS 500 scanning system.

Subsequently, the analytical shape of the driven screw was identified and, applying the “virtual pole” method, the corresponding shape of the driver screw profile was deduced.

The obtained profile was compared with the real profile obtained by 3D scanning.

2 The determination of the frontal profile

The determination of the frontal profile of the driven element was performed starting from the numerical model obtained by scanning.

The scan was performed using an ATHOS 500 system that allows a point cloud to be obtained and then processed and exported in “.stl” format.

The inspection of the driven rotor element is performed by means of the GOM Inspect program. The program allows the establishment of the dimensional characteristics of the part, serving to identify its front profile and to model it in a dedicated software program (in order to compare the scanned models with CAD models). Using a suitable chosen reference system and sectioning the point cloud with a plane perpendicular to the axis of the helical surface, the cross section of the driven rotor element is obtained.

Using the GOM Inspect program, the coordinates of some points belonging to the active profile of the section were identified. The active profile was considered to be the generating curve of the helical surface. The front section of that element and the coordinates of a few points on the active profile are shown in Figure 1.

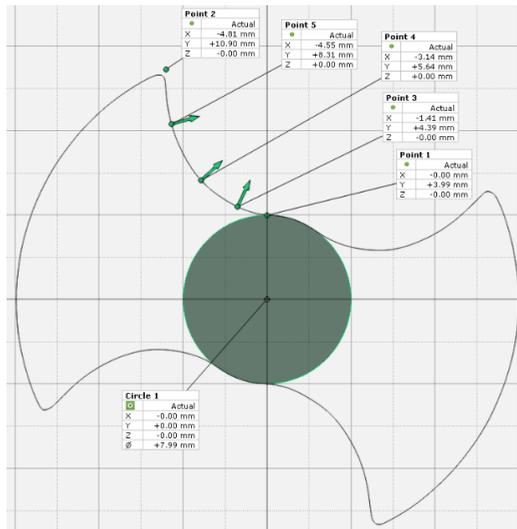


Fig. 1. The front section of the driven rotor element and points on the active profile

Identification of the frontal profile of the driven rotor

According to the literature, there are several important types of constant pitch cylindrical helical surfaces that are frequently used in the field of helical compressors [1-4].

To identify the type of the active surface of the driven rotor, it was scanned on a previously presented system.

The reference system was such that the Z axis coincided with the axis of the helical surface and the Y axis passed through the extreme point of the active profile, point at the minimum radius, figure 2.

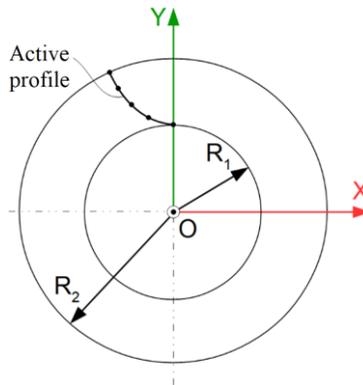


Fig. 2. Frontal active profile of the driven rotor

By measuring the scanned surfaces, the values for the driven rotor were obtained: $R_1=12$ mm; $R_2=4$ mm. Similar measurements were made for the driver rotor, obtaining: $R'_1=20$ mm; $R'_2=12$ mm.

According to the data from the literature [3], the active flank of the helical surface of the driven rotor is an epicycloid obtained by rolling on the base circle of radius $R_1 = 12$ mm of the roulette circle of radius $R'_2 = 12$ mm.

The point that generates the epicycloid is at $a = (R_1 + R'_2 - R_2)$ distance from the center of the roulette. In this case, $R_1 = R'_2$.

According to the data presented, the parametric equations of the frontal profile of the driven rotor are:

$$\Sigma: \begin{cases} X = 2 \cdot R \cdot \sin u - a \cdot \sin(2 \cdot u); \\ Y = 2 \cdot R \cdot \cos u - a \cdot \cos(2 \cdot u). \end{cases} \quad (1)$$

3 Study of the enwrapping, in frontal plane, between the profiles of the driven and driver rotors

As presented in the previous section, the active profiles of the two rotors are reciprocally enwrapping curves. Therefore, their frontal enwrapping can be studied by known methods, in particular by the "virtual pole" method [5, 6].

According to the "virtual pole" method, the identification of the profile of the driver rotor, reciprocally enwrapping of the profile known by equations (1), involves the consideration of four reference systems, figure 3: xy - fixed reference system, having the origin in the center of the rolling circle of the driven rotor, the C_1 circle; x_0y_0 - fixed reference system, having the origin in the center of the rolling circle of the driver rotor, the C_2 circle; X_1Y_1 - mobile reference system, joined with the driven rotor; X_2Y_2 - mobile reference system, joined with the driver rotor.

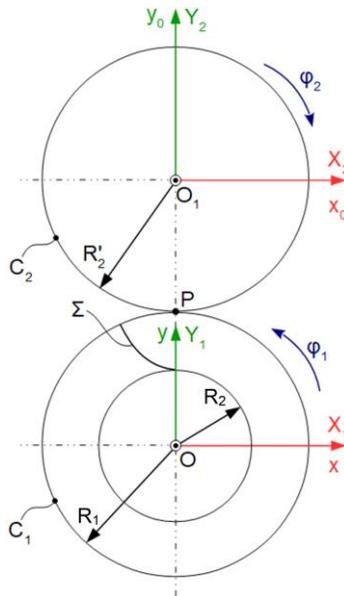


Fig. 3. Establishment of reference systems to identify the profile of the driver rotor

Between the reference systems xy and x_0y_0 , the coordinate transformation relationship can be established:

$$x_0 = x + A; \quad A = \begin{pmatrix} 0 \\ -A_{12} \end{pmatrix}; \quad A_{12} = R_1 + R_2' = 2 \cdot R. \quad (2)$$

According to known algorithms for the study of enwrapping curves [5], the two mobile reference systems have the absolute movements:

$$x = \omega_3^T(\varphi_1) \cdot X \quad \text{and} \quad x_0 = \omega_3^T(-\varphi_2) \cdot X_1. \quad (3)$$

The same algorithms [5] involve the determination of relative movements, the movements that occur between the mobile reference systems:

$$\begin{aligned} x_0 &= \omega_3^T(-\varphi_2) \cdot X_1 = x + A = \omega_3^T(\varphi_1) \cdot X + A \Leftrightarrow \\ \Leftrightarrow X_1 &= \omega_3(-\varphi_2) \cdot [\omega_3^T(\varphi_1) \cdot X + A] \Leftrightarrow X_1 = \omega_3^T(\varphi_2) \cdot [\omega_3^T(\varphi_1) \cdot X + A] \end{aligned} \quad (4)$$

and reverse movement:

$$X = \omega_3(\varphi_1) \cdot [\omega_3(\varphi_2) \cdot X_1 - A]. \quad (5)$$

If the rolling condition is also taken into account, according to which the relative speed in the gearing pole must be zero, meaning:

$$\varphi_1 \cdot R_1 = \varphi_2 \cdot R_2 \Rightarrow \frac{\varphi_1}{\varphi_2} = \frac{R_2}{R_1} = 1, \quad (6)$$

the following expressions are obtained for relative speeds:

$$X_1 = \omega_3^T(\varphi) \cdot [\omega_3^T(\varphi) \cdot X + A]; \quad (7)$$

$$X = \omega_3(\varphi) \cdot [\omega_3(\varphi) \cdot X - A], \quad (8)$$

with $\varphi_1 = \varphi_2 = \varphi$.

The "virtual pole" method, developed in the paper [6], involves identifying a point called the virtual pole, defined as the intersection between the normal to the profile Σ , taken through the current point on the profile and the centrode conjugated to the profile, the C_l circle.

Subsequently, the value by which the XY system must be moved so that the virtual pole reaches the gearing pole is determined. In this position of the XY mobile system, both the point on the profile to be generated and the one on the generating profile are on the contact curve, as defined: "the contact curve represents the geometric locus of the contact points between two mutually enveloping profiles, in the fixed reference system" [5].

To determine the virtual pole, it starts from the equations (1). The position vector of the current point M is, see figure 4:

$$\vec{r} = [2 \cdot R \cdot \sin u - a \cdot \sin(2 \cdot u)] \cdot \vec{i} + [2 \cdot R \cdot \cos u - a \cdot \cos(2 \cdot u)] \cdot \vec{j}. \quad (9)$$

The normal to this profile, taken from M , to the intersection with the C_l centrode, has the equation:

$$\begin{aligned} \vec{N} &= \lambda \cdot (\dot{Y}_u \cdot \vec{i} - \dot{X}_u \cdot \vec{j}) = \lambda \cdot [-2 \cdot R \cdot \sin u + 2 \cdot a \cdot \sin(2 \cdot u)] \cdot \vec{i} - \\ &- \lambda \cdot [2 \cdot R \cdot \cos u - 2 \cdot a \cdot \cos(2 \cdot u)] \cdot \vec{j}, \end{aligned} \quad (10)$$

where λ represents the modulus of the vector \vec{N} .

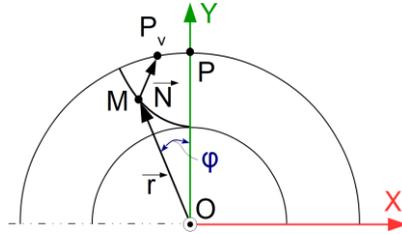


Fig. 4. Position vector of the current point M

The position vector of the virtual pole P_v , as a point belonging to the circle C_l of radius R , is given by:

$$\vec{r}_p = (R \cdot \sin \varphi) \cdot \vec{i} + (R \cdot \cos \varphi) \cdot \vec{j}. \quad (11)$$

There is a relationship between the vectors (9), (10) and (11):

$$\vec{N} + \vec{r} = \vec{r}_p, \quad (12)$$

relationship that allows the elimination of the parameter λ :

$$\begin{cases} 2 \cdot R \cdot \sin u - a \cdot \sin(2 \cdot u) + \lambda \cdot (-2 \cdot R \cdot \sin u + 2 \cdot a \cdot \sin(2 \cdot u)) = R \cdot \sin \varphi; \\ 2 \cdot R \cdot \cos u - a \cdot \cos(2 \cdot u) - \lambda \cdot (2 \cdot R \cdot \cos u - 2 \cdot a \cdot \cos(2 \cdot u)) = R \cdot \cos \varphi. \end{cases} \quad (13)$$

or

$$\begin{aligned} & \left[2 \cdot R \cdot \sin u - a \cdot \sin(2 \cdot u) + \lambda \cdot (-2 \cdot R \cdot \sin u + 2 \cdot a \cdot \sin(2 \cdot u)) \right] \cdot \vec{i} + \\ & + \left[2 \cdot R \cdot \cos u - a \cdot \cos(2 \cdot u) - \lambda \cdot (2 \cdot R \cdot \cos u - 2 \cdot a \cdot \cos(2 \cdot u)) \right] \cdot \vec{j} = \\ & = (R \cdot \sin \varphi) \cdot \vec{i} + (R \cdot \cos \varphi) \cdot \vec{j} \end{aligned} \quad (14)$$

Taking the relation (14), the enwrapping condition is obtained, meaning the value φ with which the XY system must be rotated so that the P_v point, corresponding to a certain value of the u parameter, overlaps the gearing pole:

$$\varphi = u + 2 \cdot \arctg \left(\frac{R - a \cdot \cos u}{a \cdot \sin u} \right). \quad (15)$$

The identification of the point on the profile of the driver rotor, which is in contact with the current point on the profile of the driven rotor, is made by applying the movements (3).

Considering the coordinates of the current point M as:

$$M: \begin{cases} X = X_M; \\ Y = Y_M, \end{cases} \quad (16)$$

and these being known for a value of the parameter u , when rotating the XY system with the φ angle, given by the relation (15), in the fixed reference system, the point M will have the coordinates, according to equation (3):

$$\begin{aligned} x &= \omega_3^T(\varphi) \cdot X \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix} \cdot \begin{pmatrix} X_M \\ Y_M \end{pmatrix} = \begin{pmatrix} X_M \cdot \cos \varphi - Y_M \cdot \sin \varphi \\ X_M \cdot \sin \varphi + Y_M \cdot \cos \varphi \end{pmatrix} \Rightarrow \\ \Rightarrow M_x: & \begin{cases} x_M = X_M \cdot \cos \varphi - Y_M \cdot \sin \varphi; \\ y_M = X_M \cdot \sin \varphi + Y_M \cdot \cos \varphi. \end{cases} \end{aligned} \quad (17)$$

At this point, point M is on the contact curve. In the fixed reference system, x_0y_0 , the same point has the coordinates given by equation (2):

$$M_{x_0}: \begin{cases} x_{0M} = x_M; \\ y_{0M} = y_M - 2 \cdot R. \end{cases} \quad (18)$$

Taking into account (3), the coordinates of M_I point, the contact point with M , were determined:

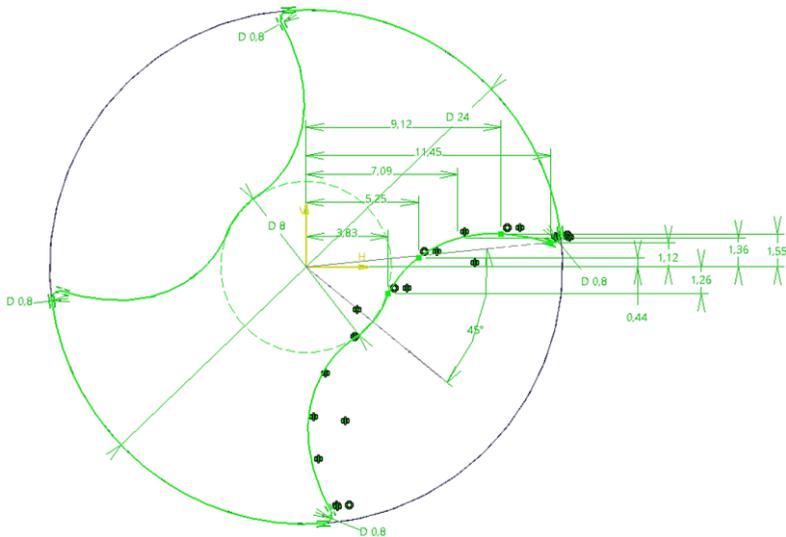
$$X_1 = \omega_3^T(\varphi) \cdot x_0; M_1: \begin{cases} X_{1M_1} = x_{0M} \cdot \cos \varphi - y_{0M} \cdot \sin \varphi; \\ Y_{1M_1} = x_{0M} \cdot \sin \varphi + y_{0M} \cdot \cos \varphi. \end{cases} \quad (19)$$

In this way, when u parameter varies so that the current point runs entirely along the side of the driven rotor, the coordinates of the corresponding points on the profile of the driver rotor can be identified.

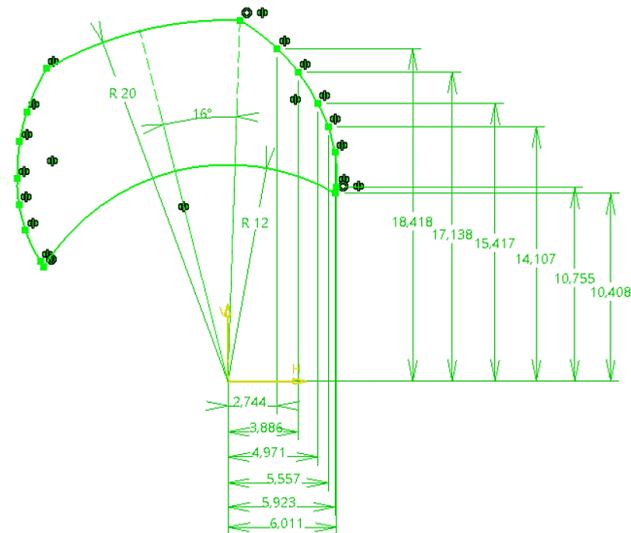
Applying the described algorithm, the driver and the driven rotor were modeled and the obtained numerical models were compared with the scanned models.

4 Verification of the obtained results

Applying the methodology presented above, for the values obtained by measuring the worms, the theoretical frontal profiles of the driven and driver screws were obtained. These profiles are shown in Figure 5.



a.



b.

Fig. 5. Frontal profile of driven (a) and driver (b) screws

Numerical models of the parts were built based on these frontal profiles. In the GOM Inspect program, the comparison between the scanned models (real) and those obtained by design (theoretical) was made. The results of these comparisons are shown in Figures 6 and 7.

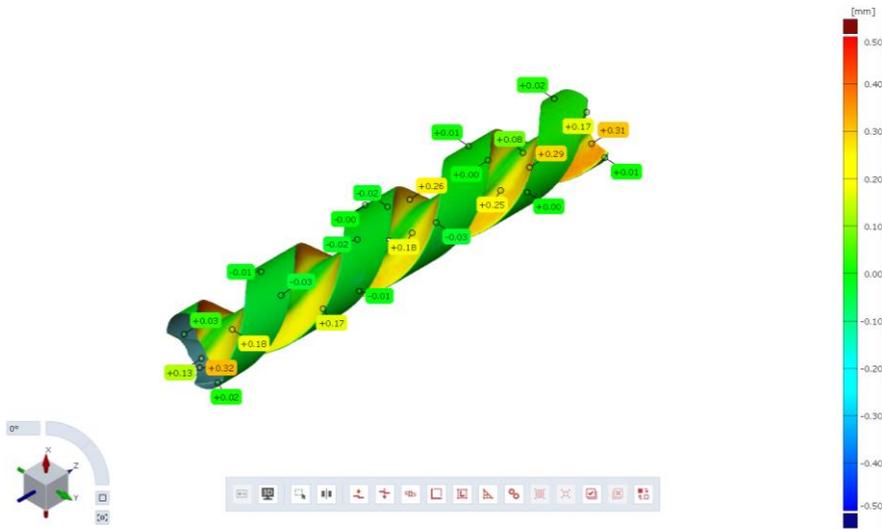


Fig. 6. The comparison between the scanned model and the numerical model in case of the driven screw

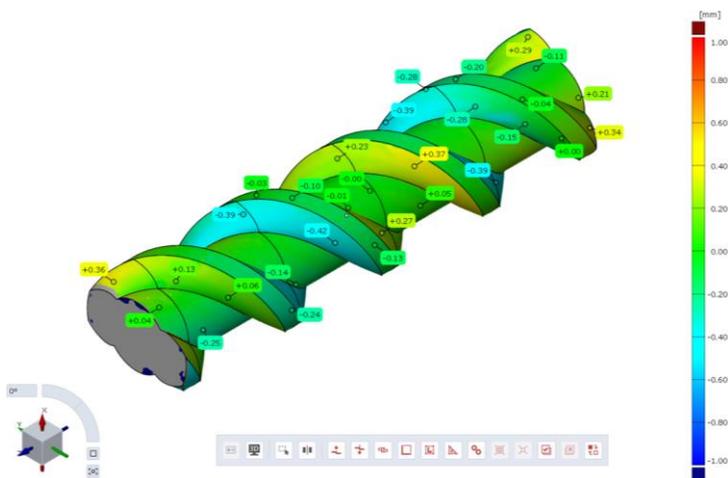


Fig. 7. The comparison between the scanned model and the numerical model in case of the driver screw

5 Conclusions

Starting from the frontal profile of the driven rotor, the reciprocally enwrapping profile was determined, corresponding to the driver rotor and, based on this profile, the element was modeled, the numerical model obtained being subsequently compared to the model obtained by scanning, using GOM Inspect software.

The comparison between the scanned model and the CAD model, based on the measured dimensions, can demonstrate the quality of the profiling method used, the “virtual pole” method and that it can be used as a basis for making the execution documentation of the respective parts.

The areas in which exist important deviations compared with the physical model are limited in extent and are considered exceptions.

Following the comparison between the scanned models and the CAD models of the driver and driven rotors, it was observed that the deviations are more pronounced on the top of the coil, which indicates a more accentuated wear.

This may be due to the measurement difficulties (because of the complex shape of the parts and there may be no errors in the numerical parts that were obtained by design) or due to the application of the anti-reflective spray, being one of the stages of preparing the parts for the scanning process (if the coverage is not uniform, errors can occur in the form of gaps in the point cloud; if the layer thickness is too large, dimensional shape deviations may occur, which may be outside the admissible tolerance limits).

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