Improving the load capacity of cylindrical gears and transmissions by parametric optimal design with CAD/CAM systems

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Abstract. Gears are an essential component of modern drives and systems. Cylindrical gears with involute profile have established themselves as one of the most widely used. Recent years have seen intensive research of both their profiling, as well as methodologies for improving their load carrying capacity. One way to increase productivity in the manufacture of cylindrical gears is the use of automated design systems. This paper presents an approach for optimal gear selection by applying the methods of parametric optimal design, modeling and study of gears with involute profile and study of their load carrying capacity.

1 Introduction

Gears are one of the most commonly used gears in industry and have established themselves as the most important gears in mechanical engineering [1]. Gears with involute profile are the most widely used worldwide and find applications in various fields. The study and further investigation of involute profile modeling respectively involute gear meshing, as well as new methods to improve the strength and load capacity is a topical issue that has been the focus of a number of works [2-4].

The selection of a gear transmission for a specific application is essential to achieve the desired indicators and parameters pf the drive, and improving the load capacity of the gear ensures an increase in its service life. One way to increase productivity in the manufacture of cylindrical gears is the use of automated design systems.

The problem is multivariate in terms of the total gear ratio, the structural-kinematic scheme, the distribution of the total gear ratio over the gear stages, the number of teeth on the two gears, the module, the correction factors and the helix angle. Depending on the specific application conditions and a number of other additional requirements, an optimization analysis is necessary. The task is time consuming but can be implemented programmatically using modern computer equipment and software products.

The paper presents an approach for the optimal selection of gears by applying the methods of parametric optimal design, modeling and study of gears with involute profile.

It is essential for gears to perform a comparative assessment of the quality of the transmitted motion, such as smoothness and noiselessness, to analyze the prerequisites for uniform wear of the working sections of the profile, etc., already at the design stage. As quality indicators, respectively as optimization criteria, the transverse contact ratio, the specific slip coefficient, the slip speed, the load capacity determined by the magnitude of the bending stresses and the contact strength are considered.

2 Formulating the problem

To synthesize cylindrical gear transmission by given torque and output angular velocity – $T_{out.}$, $\omega_{out.}$

The task for designing of cylindrical gears and transmissions is characterized by ambiguity of the solutions both in terms of the selection of input angular velocity, as well as of the transmissions scheme. The combinations that satisfy the input data are the possible variants or solutions. For the same input data, multiple variants can be designed depending on the gear ratio, the structural-kinematic scheme, the gear ratio distribution over the gear stages, the number of teeth $\boldsymbol{z}$, the module $\boldsymbol{m}$, the correction factors $\boldsymbol{x}$ and helix angle $\boldsymbol{\beta}$. The design is carried out according to known standard methodologies.

The main stages at the gear design are as follows:

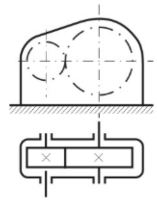
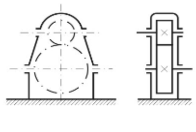
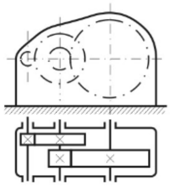
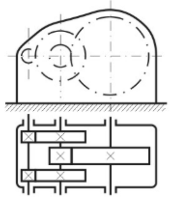
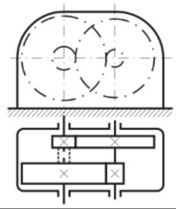
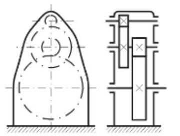
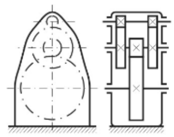
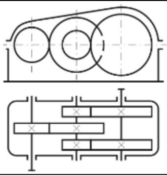
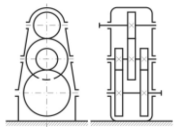
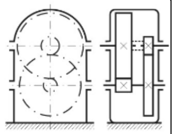
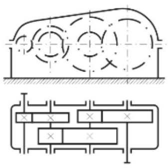
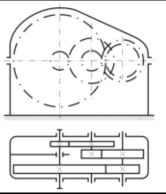
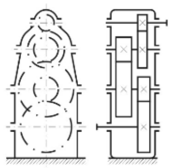
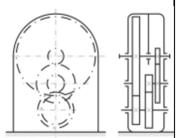
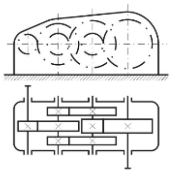
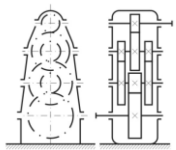
Stage 1- selection of electric motor

At given $T_{out.}$, $\omega_{out.}$ the power of the output shaft P_{out} of the gear transmission is determined, on bases of which at given tentative efficiency coefficient the power of the drive electric motor can be determined. The resulting power can be realized with electric motors of the same rated power, but with different synchronous speed, hence the multivariate nature of the motor selection problem.

Stage 2 – selection of structural-kinematic scheme of the gear transmissions

Several possible gear ratio values result from the multivariate nature of the motor selection. The same gear ratio can be realized with several possible structural-kinematics schemes of gears. The task is characterized by multivariance. It allows a large number of possible solutions (Table 1).

Table 1. Schemes of commonly used cylindrical gear reducers.

Type		Horizontal			Vertical		
No. of stages	gear ratio	With consecutively situated gears	With division of the power flow in 1st, 2nd or 3rd stage	Coaxial gear reducers	With consecutively situated gears	With division of the power flow in 1st, 2nd or 3rd stage	Coaxial gear reducers
single-stage	**i = up to 8**						
two-stage	**i = 8 - 40**						
							
three-stage	**i = up to 200**						
							

Stage 3 - distributing the overall gear ratio between the individual stages

At the multi-stage gears, the overall gear ratio is determined by the following dependence:

$$i = i_1 . i_2 . i_3 \dots i_k \quad (1)$$

where i_1, i_2, $i_3, \dots, i_k$ are the ratios of the separate stages.

The distribution of the overall gear ratio between the individual stages significantly affects a number of their important properties.

In many cases, an incorrect distribution leads to so-called design incompatibility between gears and shafts.

By appropriately distributing the overall gear ratio in multi-stage gear units, the overall design and processability of the gear unit components can be improved, the overall dimensions and mass reduced and the service life increased.

The distribution of the overall gear ratio between the individual stages, based on data from the reference literature, is carried out from diagrams. The final values of the gear ratios of the stages are obtained after specifying the number of gear teeth.

In the case of multistage gears, the distribution of the total gear ratio over the stages, performed according to different criteria, also determines multivariability and increases the number of solutions.

Different variants have better or worse technical indicators. Depending on the specific conditions and a number of additional requirements, the optimal variant must be selected. For this purpose, the resulting large number of possible solutions is analyzed in terms of the selected evaluation criteria. In some cases, the exact solution cannot be found.

The problem is time-consuming, but it is amenable to software implementation with modern computer technology. A block diagram illustrating the multivariate nature of the problem is presented on Fig. 1.

Stage 4 – geometrical synthesis of the gear pair

For every value of the gear ratio, an approximate geometrical synthesis of the gear pair is carried out. It is expedient to use existing methodology for accelerated selection of cylindrical gears and gear pair using nomograms.

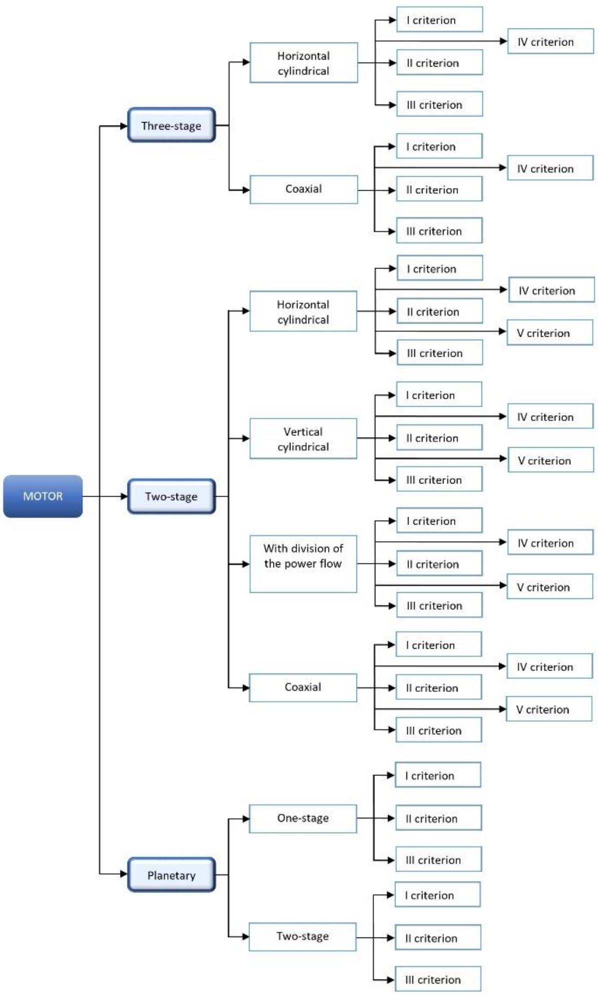


Fig. 1. Illustration of the multivariate problem to be solved.

Using the nomograms and at given specific values of the input data we can determine the number of teeth of the drive gear z_1, the normal module of the gears m_n, the coefficient of relative tooth width ψ_m, the helix angle β^o, the displacement coefficient of the output contour x and the volume of the drive gear V_1, corresponding to a certain number of teeth z_1.

This makes it possible to determine approximately the basic dimensions of the gears, respectively of the gear transmission. The nomograms are drawn with the minimum safety factors and limit stresses, therefore the selected gear pairs are guaranteed to be durable and no further control calculation is necessary.

The values of the determined and calculated parameters and indicator of the gear transmission are tabulated, the general form being shown in Table 2.

For each gear ratio value, twenty options are obtained. The alternative that meets the requirement that the deviation from the given gear ratio ΔU is equal or approximately equal to 0 shall be selected.

Table 2. Parameters and indicators of the gear transmission

Alternative	***1***	***2***	***3***	***... n***
Normal module m_n				
Coefficient of relative tooth width ψ_m				
Correction coefficient x_1				
Helix angle β^o				
Number of teeth of the driving gear z_1				
Number of teeth of the driven gear $z_2 = z1*U$				
Deviation of the given gear ratio $\Delta U = (u - z_2/z_1)/u\ 100, \%$				

Stage 5 – parametrization of the involute profiles of the gear pairs

For the optimal variants thus obtained with minimum error in the gear ratio, the involute profiles of the gear pairs are modelled and a parametric model is formulated.

The polar parametric equations of the involute are as follows:

$$v_y = tg\alpha_y - \alpha_y = inv\alpha_y \;\; and \;\; r_y = \frac{r_b}{cos\alpha_y} \qquad (2)$$

On Fig. 2 left and right involute are graphically illustrated. Fig. 3 shows a pair of conjugated involutes, obtained as the trajectories of points on the common tangent of two principal circles in rolling and on one and the other without sliding. When the involutes are rotated, their point of contact lies on the common tangent B_1B_2, denoted as the line of intercept. The angle α_w is the angle of engagement and the angle of force transmission between the two contours.

The correct and cooperative operation of the conjugated tooth profiles and the quality of the transmitted motion depend on the accuracy of the involute contours, i.e., the accuracy of the forming [5]. The more accurate the contour shape, the quieter, more reliable and more suitable for high-speed gear transmissions for the respective gears. The accuracy of the gear contour shape is primarily down to the accuracy of the involute contouring.

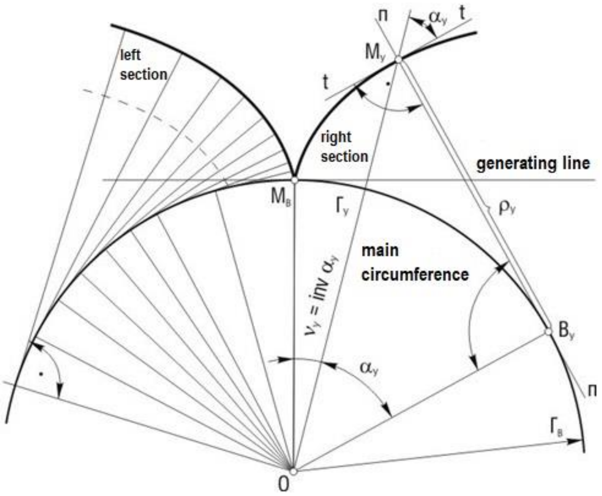


Fig. 2. Left and right involute.

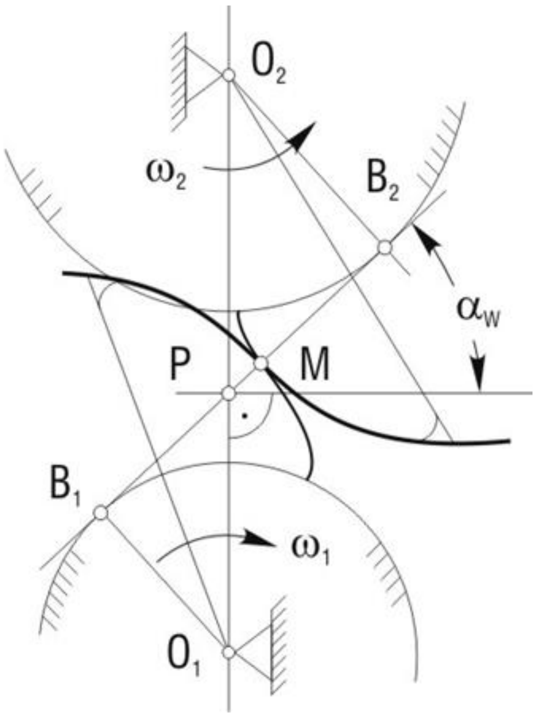


Fig. 3. A pair of conjugated involutes.

The most commonly used method for obtaining tooth profiles is the crawling method with the tool rack-shape cutter. It is known, that at the shaping there are three characteristic positions of the dividing line of the output contour in relation to the pitch circle of the cut gear (fig. 4) – tangent to the pitch circle, shifted outward or inward relative to the pitch circle at a distance of *x.m*.

On fig. 5 the profiles of three gears with the same module *m* and number of teeth *z*, cut with the same instrument but with different displacement are shown.

The displacement of the dividing line of the output tooth contour is necessary to avoid the phenomena of undercutting and tooth tip sharpening. Tooth undercutting, when significant, has a negative effect on the kinematics and load capacity of the gear and weakens the bending strength of the teeth. It significantly reduces the transverse contact ratio, which in individual cases can even become $\varepsilon_\alpha < 1$. The tooth tip sharpening occurs when for the tooth thickness along the tip circumference the inequality $s_a < s_{amin}$ is valid, as for minimum allowable tooth thickness is considered $s_{amin} = 0{,}2\ m$.

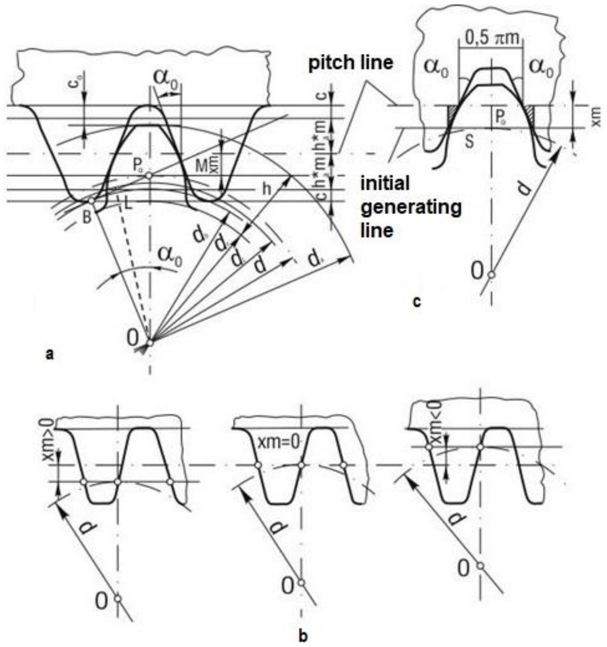


Fig. 4. Forming and position of the dividing line of the output contour relative to the pitch circle of the cut gear wheel.

In order to avoid the above-mentioned occurrences, the displacement coefficient *x* for a gear wheel with given number of teeth *z* can vary within certain limits:
$x_{min} \le x \le x_{max}$, as $x_{min} = h_a^*\left(1 - \frac{z}{z_{min}}\right)$ and x_{max} are determined from the maximum allowable tooth thickness condition.

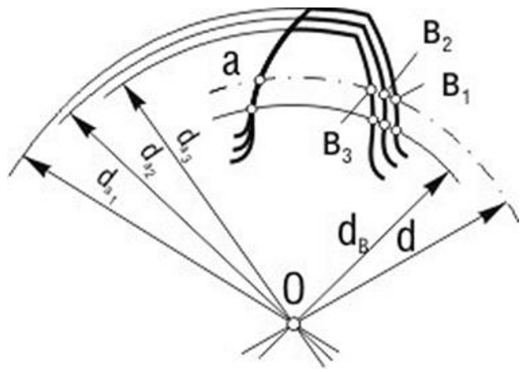


Fig. 5. Profile of the teeth of three gears with different displacement.

At determining the corrections of a gear with given number of teeth *z* the displacement coefficient *x* can vary in certain limits, that depend on the conditions that there are no undercutting and sharpening of the teeth. When meshing two gears, there are other requirements that determine the limits of variation of the gear wheel displacement coefficients.

Taking all conditions together mathematically, which is a voluminous job, the permissible limits of variation of the displacement coefficients for a particular combination of the number of teeth of the gear wheels can be found. The permissible area of variation represented graphically is the so-called blocking contour (Fig. 6). For different combinations of z_1 and z_2, the permissible areas differ from each other. The coefficients x_1 и x_2, which are selected, has to pertain in the area. A large number of blocking contours are given in the literature, allowing the selection of an optimal tooth geometry [6, 7].

Blocking contours are a very useful way of representing complex dependencies in a simple and convenient way. With their help, the choice of the displacement coefficients is safe, fast and easy and a great freedom in the design is achieved. This makes it possible to design gears to meet different requirements - for uniform tooth bending strength, for maximum tooth strength or for equalized specific slip.

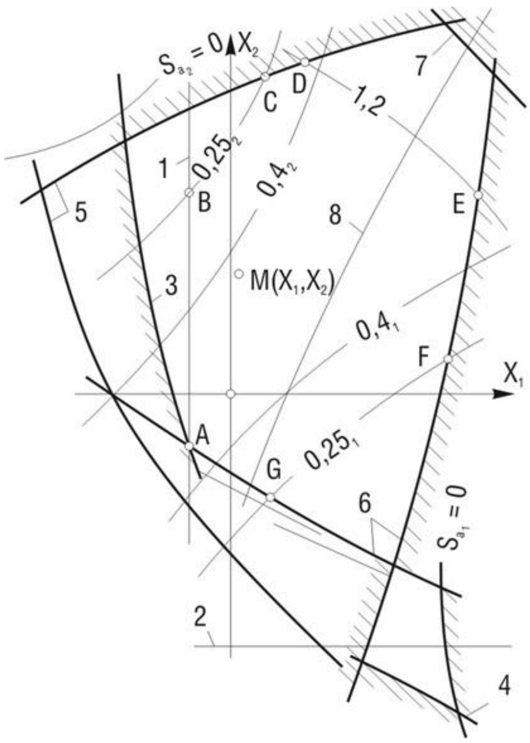


Fig. 6. Blocking contour.

It is known, that depending on the values of the correction coefficients of the gear pair that are working jointly, several types of external meshing are realized with cylindrical gears with involute teeth.

Stage 6 – realizing the types of external meshing at cylindrical gears with involute teeth, respectively involute pairs

Zero meshing – it is formed by two zero gears ($x_1=x_2=0$).

The zero meshing has the advantage, that geometrically it is the simplest one. Its main disadvantage is that the bending stresses of the teeth of both gear wheels are not uniform, and the difference is bigger when the difference in the number of teeth is bigger. At this type of meshing, the material is not used very rationally. Moreover, smaller number of teeth z_1 results in unfavorable values for the specific slip θ_1 for the root of the tooth of the smaller gear wheel 1. In some cases, the fact that an arbitrary center distance cannot be achieved is also felt to be a disadvantage. Zero meshing is applied with good reason when the number of teeth on the smaller gear wheel is large, or when the sum of the number of teeth is large.

Equally displaced – it is obtained by two gear wheels, respectively by two involutes with equal in absolute value and opposite in sign direction coefficients of displacement x_1 и x_2, at which the smaller gear wheel is positively corrected $x_1 > 0$ and the bigger gear wheel is negatively corrected $x_2 < 0$ and $|x_1| = |x_2|$, as a result of which the coefficient of the total displacement is $x_\Sigma = x_1 + x_2 = 0$.

The equally displaced meshing has the advantage that it can increase the load capacity for bending of the teeth of the smaller gear wheel in expense of the bigger gear wheel, and thus of the gear transmission as a whole. The geometrical calculations at this type of meshing are relatively simple. Its application does not involve any technological difficulties or cost overruns. However, it is not universally applicable, for example for gear ratio $u=1$. In some cases, it is not possible arbitrary center distance to be achieved, which is a disadvantage. Compared to zero meshing, at the equally displaced meshing the meshing loses increase. The equally displaced meshing can be applied successfully at gear transmissions with smaller number of teeth z_1 for the smaller gear wheel 1 and bigger gear ratio u, and both at designing new gear transmissions and repairing working gears. In the design of new gear transmissions in can avoid undercutting of the teeth of the smaller gear wheel, produce a more compact gear transmission by a more rational use of material, and create more favorable conditions for friction and tooth wear. When repairing larger gears, which have been operational and damaged gear transmissions it is possible in some cases to have a significant economic impact by only rebuilding the smaller gear wheel and the larger gear wheel, which is more expensive, could be re-cut.

Displaced meshing – it is obtained by gear wheels, the smaller of which is positively corrected $x_1>0$ and the bigger gear wheel can be uncorrected $x_2=0$ or positively $x_2>0$ or negatively $x_2<0$ corrected. This type of meshing has the biggest geometrical possibilities at designing of gear transmissions.

When the number of teeth z_1 and z_2 of both gear wheels, the displacement coefficients x_1 and x_2 and the module m are given, here the problem of finding the pressure angle α_w, and through them the other geometrical parameters of the gear wheels is posed. This is the so-called forward problem.

The inverse problem seeks at which displacement coefficients x_1 and x_2 the prescribed center distance a_w can be realized.

The displaced meshing has the biggest geometrical and design possibilities – the largest of the three types of meshing. Its application does not involve any technological difficulties or cost overruns. By the displaced meshing is possible to avoid undercutting of the teeth, to increase the load capacity for bending and the contact strength of both gears, to improve the friction conditions between the teeth and to achieve arbitrary center distance.

As a disadvantage of the displaced meshing, it is considered that the geometrical calculations are relatively the most complex and this places increased demands on the designer. In addition, a disadvantage is at increasing the pressure angle the transverse contact ratio decreases, which is detrimental to the noise performance of the gear transmission.

Displaced meshing is applied in the design of new gears and in repairs. In repair, it is applied to large and expensive gears for modernizing and increasing their load capacity.

In general, the possibilities and flexibility offered by the corrections are viewed as one of the main advantages of involute meshing compared to other meshes.

Stage 7 – performing comparative assessment of the quality of the transmitted motion by determining a group of quality indicators, respectively optimization criteria

The aim of this study is to determine the influence of correction coefficient on the shape of the involute and at what way it influences the operation of two co-operating involutes.

It is essential for gears at the design stage to perform a comparative assessment of the quality of the transmitted motion, such as smoothness and noiselessness, to analyze the prerequisites for uniform wear of the working sections of the profile, etc. The following are considered as qualitative indicators or optimization criteria: transverse contact ratio, specific slip coefficient, slip speed, load capacity determined by the magnitude of bending stresses and contact strength.

Transverse contact ratio ε_α

This coefficient takes into account the smoothness of the meshing of the conjugated tooth profiles. For smooth and continuous transmission of the motion, it is necessary before a pair of co-cooperating profiles has come out of the meshing the next pair of profiles to has entered meshing. In order for simultaneous meshing to be effective and not result in wedging of the gears, accurate tooth profile design and uniformity of the pitch between tooth profiles is required. For the coefficient ε_α, the following relation holds:

$$\varepsilon_\alpha = \frac{z_1}{2\pi}(tg\alpha_{a1} - tga_w) + \frac{z_2}{2\pi}(tg\alpha_{a2} - tga_w) \quad (3)$$

Theoretically, the gear is functional at any $\varepsilon_\alpha \geq 1$, but in practice at the design always should be guaranteed the condition $\varepsilon_\alpha \geq 1{,}2$. The value of ε_α is prescribed depending on the loads and the accuracy of the gear design and assembly. As an exception, $\varepsilon_\alpha \geq 1{,}1$ is allowed for high precision manufacturing.

The magnitude of the transverse contact ratio depends in direct proportion from the number of teeth of the gear wheels, in inverse proportion of the correction coefficients and more specifically of the coefficient of the total displacement and does not depend on the module.

Specific slip coefficient

The joint operation of the profiles of the contour pairs is accompanied by relative slip between them. The wear of the profiles and the friction power depend on many factors, on the magnitude and nature of the distribution of the reaction in the contour pair, on the lubrication of the profiles that determines the friction coefficient at sliding, on the materials of the profiles and the quality of their machining, etc.

The degree of influence of the kinematic and geometric factors on the intensity of the sliding and the wear of the profiles is taken into account by the specific slip coefficients of the conjugated profiles.

The specific slip at the tooth profiles can be determined simply and visually by graphing (fig. 7).

For the specific slip of the endpoints from the roots of the active profiles, the following dependencies hold:

$$\left| \begin{array}{l} \Theta_1 = -\frac{(u+1)(tg\alpha_{a2} - tga_w)}{tga_w - u(tg\alpha_{a2} - tga_w)} \\ \Theta_2 = -\frac{(u+1)(tg\alpha_{a1} - tga_w)}{utga_w - (tg\alpha_{a1} - tga_w)} \end{array} \right. \quad (4)$$

Achieving more advantageous values for the specific slip is possible by correcting the meshing.

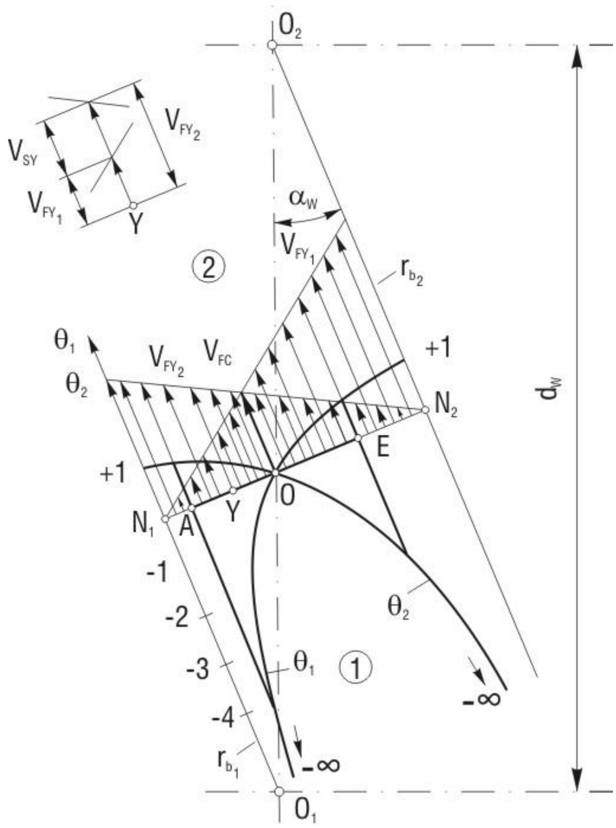


Fig. 7. Specific slip at gear profiles.

Slip speed

The two triangular diagrams depict the alteration of the speed $\boldsymbol{V_{Fy1}}$ and $\boldsymbol{V_{Fy2}}$ as the meshing point *Y* moves along the meshing line. By conducting simple and fast calculations it is possible to plot the specific slip curves θ_1 and θ_2 for the two tooth profiles at several points depending on the position of the meshing points *Y* on the meshing line.

Wear

The specific slip can also be plotted as a function of the position of the meshing point of the tooth profile (fig. 8). The figure gives a visual indication for the influence of the specific slip on the wear, which is largest where the specific slip is largest.

The issue of wear is complex and it is not characterized only by the specific slip.

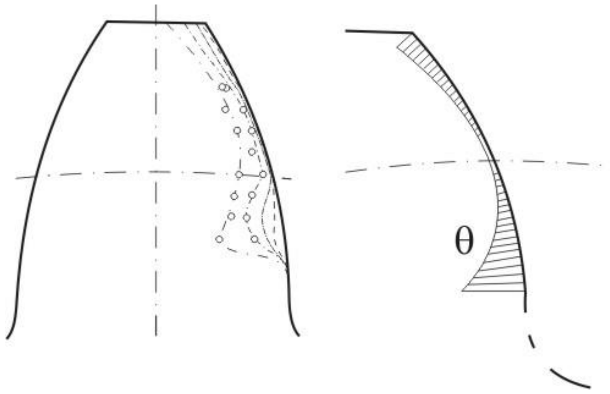


Fig. 8. Plotting the specific slip as a function of the position of the meshing point of the gear profile.

Load capacity of the gear transmission

An essential point in the design of gear transmissions and gear reducers is the correct assessment of their load capacity under specified operating conditions, geometric parameters, materials and heat treatment of the gears. In the increasing demands on drives, this issue becomes of paramount importance. The accumulated experience in the field of design, manufacture and operation of gear transmissions has led to a considerable improvement of the methodologies for their strength calculation.

The best solution for the determination of the load capacity is the development of a program version of the complete methodology, starting with its modification and based on a developed system of geometrical calculations. This provides not only a sufficiently accurate assessment of the load capacity of the gear transmission, but also an opportunity for its optimization according to different criteria and tendencies, as well as for solving design and optimization problems. All of these strength tasks are reduced to an accurate assessment of the load capacity of the gear transmission with known geometric and kinematic characteristics, materials, machining, accuracy degrees, and operating conditions.

For the obtained optimal variants, modelling of the loads and derivation of values of the stresses occurring are carried out by finite element method. The use of modern CAD/CAM systems for geometric synthesis, modelling, and strength analysis of gears has many advantages.

Stage 8 – Solving the single-criteria and the multi-criteria optimization problem

Many decision-making problems need to achieve a number of objectives such as risk reduction, reliability improvement, cost reduction, etc. [8]. When solving a single-criteria optimization problem, the optimal option is assumed to be the one in which the corresponding criterion has an extreme value, either a minimum or a maximum. The problem with single-criteria optimization is that it is usually not possible to provide alternative solutions to the problem, i.e., optimization by only one criterion is not always the best solution [9]. In this case, the optimization problem is multi-objective. In multi-criteria optimization, there is no single "optimal" solution. It is necessary to find a compromise solution, the so-called Pareto-optimal solution.

Due to differences in the measuring units and the range of variation of the values of the selected criteria, it is necessary to carry out the so-called normalization, i.e., the quantitative evaluations of the obtained variants are dimensionless, according to the following relation:

$$\lambda_i(x) = \frac{f_i(x) - f_{min}(x)}{f_{max}(x) - f_{min}(x)} \quad (5)$$

where $f_i(x)$ is the current value of the corresponding criterion, $f_{min}(x)$ is the minimal value of the corresponding criterion and $f_{max}(x)$ is the maximal value of the corresponding criterion.

After converting the physical parameters in dimensionless a generalized criterion is derived, which is expressed by target function, determined by the weight coefficients method, as follows:

$$Z(\chi_i \lambda_i) = \sum_{i=1}^{n} \chi_i \lambda_i(x) \quad (6)$$

where $\lambda_i(x)$ is the *i*-th normalized value of the corresponding criterion, and χ_i are the weight coefficients, satisfying the following conditions: $0 < \chi_i < 1$, ($i = 1, 2, ..., n$), $\sum \chi_i = 1$.

These conditions are satisfied in particular by the weight coefficients, determined by:

$$\chi_m = \frac{2(n-1+m)}{n(n+1)} \quad (7)$$

where χ_m e *m*-th weight coefficient, and *n* is the number of criteria, ranked in a priority order.

The optimization problem can be solved by applying various multi-criteria optimization methods as well as automated using available software products.

3 Conclusions

The approach presented in this paper includes all the main stages for the design and optimal selection of cylindrical gears. The possible ways of forming the involute profile of the gears, the influence of the correction on the involute shape as well as on the variation of basic parameters such as sliding speed, Specific slip coefficient, specific sliding coefficient, and load capacity of two jointly operating involutes are discussed.

The use of CAD/CAM systems allows automation of the geometric computer modelling process, graphical visualization allows rapid analysis of the resulting constrained profiles, and the change of one or more geometric parameters is automatically reflected in the model.

References

1. K. Arnaudov, I. Dimitrov, P. Jordanov, L. Lefterov, Machine elements (in Bulgarian) (1980)
2. P. Vaghela, J. Prajapati, Integrated Symmetric and Asymmetric Involute Spur Gear Modelling and Manufacturing, *Materials Today: Proceedings* **22** (2020), pp. 1911–1920
3. V. Zelený, I. Linkeová, J. Sýkora, P. Skalník, Mathematical approach to evaluate involute gear profile and helix deviations without using special gear software, Mech. Mach. Theory **135** (2019), pp. 150-164
4. F. Karpat, C. Yuce, O. Doğan, Experimental measurement and numerical validation of single tooth stiffness for involute spur gears, Measurement, **150** (2020)
5. F. Litvin, A. Fuentes, Gear Geometry and Applied Theory. Second Edition. Cambridge, University Press (2011)
6. V. Goldfarb, A. Tkachev, An Advanced Approach To Optimal Gear Design, Gear technology, January/February (2005)
7. V. Goldfarb, A. Kapelevich, A. Tkachev, New Approach to Computerized Design of Spur and Helical Gears, Gear technology, August (2008)
8. D. Savic, Single-objective vs. multiobjective optimisation for integrated decision support, *Integrated Assessment and Decision Support, Lugano, Switzerland* (2002)
9. P. Genova, D. Dahterova, M. Tsoneva, Theory of machine design (in Bulgarian) (2011)