Digital realization method of FFT and its application in adjusting velocity damping coefficient of inertial navigation system

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Abstract. Aiming at the velocity damping feedback system commonly used in inertial navigation, this paper proposes the principle of decomposing the output attitude of inertial system based on FFT algorithm and gives the principle of spectrum adjustment ratio. The effectiveness of the algorithm is verified from two aspects: data simulation and actual system solution. In addition, in the design and implementation of FFT algorithm, the butterfly computation of operator symmetry is applied to effectively reduce the complexity of the algorithm, and the dual core chip technology is applied to reduce the impact of the complex algorithm on the system timing. The engineering applicability of the algorithm is demonstrated.

1. Introduction

The fast Fourier transform algorithm (FFT) was proposed in 1965. Based on the discrete Fourier statistical method, it effectively reduces the complexity of the solution cycle, making it possible to solve it in real time in the low-frequency processor system. The inertial system usually relies on the external velocity damping feedback to suppress the velocity oscillation trend. The damping coefficient absolutely determines the damping effect. When the external reference velocity error is small, the strong damping coefficient has better inhibition ability, making the velocity error converge quickly. When the external reference velocity error is large, the high damping coefficient will introduce the external reference velocity error. Therefore, real-time analysis of inertial navigation system attitude data spectrum to adjust the damping ratio can be used to improve the navigation accuracy of inertial navigation system.

2. Analysis of FFT algorithm principle

2.1 DFT transformation

For any continuous function *f(t)*, the frequency *ω* at any frequency can be obtained by Fourier transform Spectral density function *F*(*ω*) as follows.

$$f(t) = \int_{-\infty}^{+\infty} \frac{1}{2\pi} \left(\int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t} dt\right) e^{j\omega t} d\omega \tag{1}$$

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t} dt \tag{2}$$

For any discrete function, Dirac sampling function should be introduced to analyze its Fourier decomposition. Dirac function has the following definitions.

$$\int_{-\infty}^{+\infty} \delta\,(t) dt = 1 \text{ and if } t \neq 0 \;\; \delta(t) = 0 \tag{3}$$

According to its definition, we can get the following formula.

$$\int_{-\infty}^{+\infty} \delta\,(t - t_0) f(t) dt = f(t_0) \tag{4}$$

According to this formula, Dirac sampling function can assist in sampling single time value of continuous function The discrete function of any continuous function *x(t)* can be obtained by continuous sampling with period T_s.

$$x_s(t) = \sum_{n=-\infty}^{+\infty} x(t)\, \delta(t - nT_s) \tag{5}$$

The discrete spectral density of equation $x_s(t)$ can be obtained by Fourier transform as follows.

$$x_s(\omega) = \int_{-\infty}^{+\infty} \left(\sum_{n=-\infty}^{+\infty} x(t)\, \delta(t - nT_s)\right) e^{-j\omega t} dt \tag{6}$$

$$x_s(\omega) = \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(x(t) \delta(t - nT_s)\right) e^{-j\omega t} dt \tag{7}$$

$$x_s(\omega) == \sum_{n=-\infty}^{+\infty} x(nT_s)\, e^{-j\omega nT_s} \tag{8}$$

For the actual system, the analysis of spectral density should consider the completion of sampling point data in a limited time, so the sampling function $x_s(t)$ and spectral density $x_s(\omega)$.

$$x_s(t) = \sum_{n=0}^{N-1} x(t)\, \delta(t - nT_s) x_s(\omega) = \sum_{n=0}^{N-1} x(nT_s)\, e^{-j\omega nT_s} \tag{9}$$

In order to make the expression of spectral density more concise, it is simplified to the following formula.

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$$X[k]=\sum_{n=0}^{N-1}x[n]\,e^{-j\frac{2\pi}{N}kn} \quad \text{where } X[k]=x_s(\omega), x[n]=x(nT_s) \quad (10)$$

It can be seen from this formula that if you want to extract the spectral density of all frequency points within a sampling time, $2n^2$ operations are required. Systems with high real-time requirements such as inertial systems The operation of $O(N^2)$ complexity class may interfere with the solution speed and the real sequence set by the system Therefore, consider using $e^{j\theta}$Operator has periodicity to reduce the amount of computation.

2.2 FFT transform

Euler formula is as following.

$$e^{j\theta}=cos\theta+jsin\theta \quad (11)$$

According to this, $e^{j\theta}$ Operator has periodicity.

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$$e^{-j\frac{2\pi}{N}2nk}=e^{-j\frac{2\pi n}{N/2}k}=e^{-j\frac{2\pi n}{N/2}(k+N/2)} \quad (12)$$

According to this characteristic, $X[k]$ obtained by DFT algorithm can be decomposed into two parts: n is odd and n is even.

$$X[k]=\sum_{n=0}^{N/2-1}x[2n]\,e^{-j\frac{2\pi}{N}2nk}+\sum_{n=0}^{N/2-1}x[2n+1]\,e^{-j\frac{2\pi}{N}(2n+1)k} \quad (13)$$

$$X[k]=\sum_{n=0}^{N/2-1}x[2n]\,W[2n,k]+e^{-j\frac{2\pi}{N}k}\sum_{n=0}^{\frac{N}{2}-1}x[2n+1]W[2n,k] \quad (14)$$

$$\text{where } W[n,k]=e^{-j\frac{2\pi}{N}nk}$$

It can be seen from this formula that by extracting the common factor, the calculation of the odd part can be transformed into the calculation of the even factor $W[2n,k]$ For the convenience of analysis, we can make the even part $X[k]$ and the odd part $O[k]$. Since $W[1,k]$ has periodicity, the following formula can be obtained.

$$X[k+N/2]=E[k]-W[1,k]O[k] \quad (15)$$

Taking n equals 8 as an example, we can get the following formula. It can be seen from this formula that in the actual calculation of the system, only the even part operator $e[k]$ of $x[1]$, $x[2]$, $x[3]$, $x[4]$ and the odd part operator $O[k]$ of $x[5]$, $x[6]$, $x[7]$, $x[8]$ need to be calculated. The operator $e[k]$ to be calculated can be further divided into odd and even parts according to the factor until only one term remains in one part According to this butterfly decomposition operation, the calculation diagram is as follows.

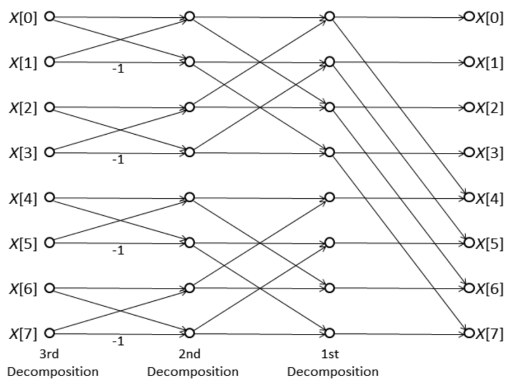

Figure 1. Schematic diagram of FFT iterative butterfly decomposition.

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3. Application of FFT algorithm in adjusting the damping coefficient of inertial navigation system

The pure inertial navigation system only calculates the speed and position information according to the system data. This undamped system without external reference information input usually leads to 84.4 minutes Schuler oscillation in various error sources and initial condition errors, which is undesirable in many cases. Therefore, the inertial system usually introduces the external reference speed information for feedback, dampens the system speed, and then suppresses the Shula oscillation, so as to obtain higher precision navigation results.

3.1 Introduction to speed damping of inertial navigation system

The mechanical arrangement equation of geographic system (n-system) navigation system with horizontal velocity damping can be expressed by the following formula.

$$\dot{r}^n=v^n-[\rho^n]r^n \quad (16)$$

$$\dot{v}^n=f^n-[\omega_{ie}{}^n+\omega_{in}{}^n]r^n-Cv_d+g^n \quad (17)$$

Where $v_d=v^n-v_r{}^n$ represents the difference between the system inertia indication speed v^n and the external reference speed $v_r{}^n$. C refers to damping feedback network. Typical damping network forms include constant velocity error feedback network, phase lead series network and phase lag lead series network. The error transfer function diagram of the typical geographical system velocity Schuler damping loop is shown in the figure below.

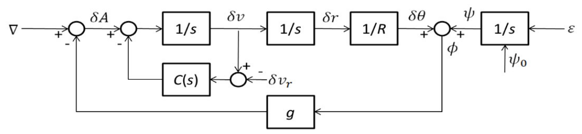

Figure 2. Block diagram of error transfer function of Schuler damping loop.

Let $Q(s) = s/(s + C(s))$, the following series Schuler loop error and error transfer formula and error transfer function block diagram can be obtained.

$$v(s) = \frac{1}{s^2+Q(s)\omega_s{}^2}\left[\left(1 - Q(s)\right)s^2\delta v_r(s) + Q(s)\left(s\nabla(s) - g\varepsilon(s)\right)\right] \quad (18)$$

$$\delta r(s) = \frac{1}{s^2+Q(s)\omega_s{}^2}\left[\left(1 - Q(s)\right)s^2\delta v_r(s) + Q(s)\left(\nabla(s) - g\psi(s)\right)\right] \quad (19)$$

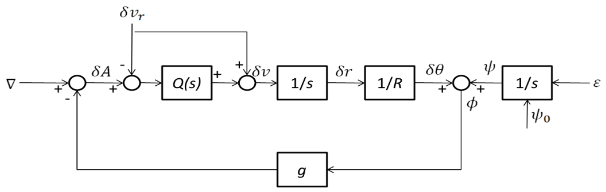

Figure 3. Error transfer function block diagram of series Schuler damping circuit.

Generally, the phase lag lead series damping network has the best performance. This paper will analyze this network. This type of damping network has the following transfer function.

$$Q(s) = ((1 + 4\xi\eta)s^2 + 2(\xi + \eta)\omega_s + \omega_s{}^2)/(s^2 + 2(\xi + \eta) + \omega_s{}^2) \quad (20)$$

Consider these two conditions, $\omega \ll \omega_s$ and $\omega_s \ll \omega$, the following table can be obtained through the derivation and simplification of the error formula.

Table 1. Error transfer function block diagram of series Schuler damping circuit.

Error source	$\omega \ll \omega_s$ Reference speed error δv_r	$\omega \ll \omega_s$ Accelerometer error ∇	$\omega \ll \omega_s$ Reference speed error δv_r	$\omega_s \ll \omega$ Accelerometer error ∇
$\delta v(j\omega)$	$\frac{2\xi\omega^3}{\omega_s{}^3}$	$\frac{\omega}{\omega_s{}^2}$	$\frac{2\xi\omega_s}{\omega}(1 - 4\mu\eta)$	$\frac{1}{\omega}$
$\delta r(j\omega)$	$\frac{2\xi\omega^2}{\omega_s{}^3}$	$\frac{1}{\omega_s{}^2}$	$\frac{2\xi\omega_s}{\omega^2}(1 - 4\mu\eta)$	$\frac{1}{\omega^2}$

Therefore, when the external reference velocity error is large, the strong damping coefficient ξ will bring higher speed position error. Common external reference velocity sources include electromagnetic log, Doppler sonar log and acoustic correlation velocity log. The electromagnetic log measures the velocity of the carrier passing through the water, and its measured value may be polluted by the distorted water flow around the carrier. The error source of Doppler sonar is the same as that of the electromagnetic log, and excessive ocean current velocity will bring high measurement error. Doppler sonar log can measure relative sea bottom velocity, but it only has effective data within 200m depth. Using this velocity source has a high limit on the bottom contact depth of underwater precision navigation. Therefore, the error of current velocity will be analyzed below.

3.2 Damping coefficient adjustment principle

In the following cases, the speed of current measurement is large. When the ship makes a large maneuvering turn at a large angle, the relative ground speed of the ship is close to zero. However, due to the large current movement caused by the turning of the ship and the installation position of the speed measuring device is not close to the turning center, the speed measurement is large. Therefore, the error of external reference speed measurement during turning is large. In addition, when encountering rough and severe sea conditions (above level 3), the current velocity is larger than the ground speed of the ship itself, and the swing error of the ship itself is also large, so it brings higher velocity measurement error. It can be seen from the above that when the ocean current velocity is large, it usually affects the transformation of the large attitude angle of the ship body, and also affects the large transformation of the output attitude angle of the ship body fixed inertial navigation. Therefore, this paper considers extracting the frequency spectrum of the inertial navigation attitude angle through the FFT algorithm, and inversely separating the external reference velocity measurement error through the frequency spectrum. When the velocity measurement error is large, a smaller damping ratio is selected to bring higher measurement accuracy. When the velocity measurement error is small, A large damping ratio is selected to suppress the stretch oscillation of the inertial system.

Due to the complex characteristics of the system to be analyzed, there is no mathematical model reference in related fields, and different ship models and different inertial navigation models are also different. Therefore, this paper gradually optimizes the damping ratio adjustment rules through off-line data simulation and physical test retest of an inertial navigation system. The adjustment principle model obtained from the final set damping ratio is as follows.

$$\text{if} \sum_{n=0}^{n_{max}} k_n F(nw_0) < \xi_0$$

$$\xi = \xi_{max} - \sum_{n=0}^{n_{max}}(k_{Rn}F(nw, \theta_R) + k_{Pn}F(nw, \theta_P) + k_{Hn}F(nw, \theta_H)) \quad (21)$$

$$\text{if} \sum_{n=0}^{n_{max}} k_n F(nw_0) \geq \xi_0$$

$$\xi = 0 \tag{22}$$

In the formula, $\theta_R, \theta_P, \theta_H$ refers to the output attitude angle where $F(nw_0, \theta)$ Represents the spectral density of a specific frequency for a certain attitude angle after fast Fourier transform And k_{Rn}, k_{Pn}, k_{Hn} represents the adjustment weight of the spectral density. It is worth noting that K near the resonance frequency of the system is applied in the parameter adjustment process givek_na small weight to avoid the damping ratio caused by system resonance n_{max} represents the highest frequency band for FFT data analysis to limit the impact of high-frequency noise on damping ratio, ξ_{max} represents the maximum damping coefficient, indicating that the external information reference value at this time is fully trusted, and the system shall be damped with the highest damping ratio.

3.3 Inertial platform construction

In order to further verify the effectiveness of the algorithm, it needs to be verified in the actual inertial system. At the same time, in order to further improve the processing capacity of the system for FFT algorithm and separate the timing impact of FFT algorithm on the main program of the system, the system uses dual core DSP28379 chip to analyze the test data. The communication between dual cores adopts the form of IPC message and 28379 shared memory. The operation timing diagram of the whole system is shown in Figure 4 below.

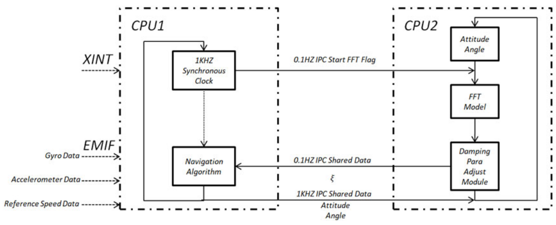

Figure 4. Dual core operation sequence of inertial system.

CPU1 will perform single-step calculation after receiving the external synchronization signal of 1000 frequency. After calculating the attitude information, CPU2 will send the attitude information to CPU2. At the same time, CPU2 will divide the frequency of 0.1 Hz in the 1000Hz frequency and send it to CPU2 as an FFT flag. After receiving the flag, CPU2 will perform FFT decomposition on the attitude data in this period and output it to the damping coefficient adjustment module. After calculating the damping ratio, CPU1 will send the real-time damping ratio back to CPU1. The navigation algorithm uses the updated damping ratio to solve the velocity and position.

4. Test data analysis

The navigation speed calculation results of a certain type of inertial equipment through dual channel FFT damping ratio adjustment algorithm and constant damping coefficient are shown in the following figure.

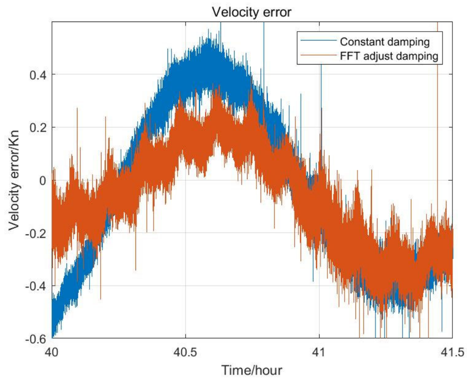

Figure 5. Navigation accuracy results under different damping conditions.

According to the test data curve, the velocity accuracy of the inertial system using FFT algorithm to adjust the damping ratio is improved by 5%, and the feasibility of using FFT algorithm to adjust the damping ratio is demonstrated.

5. Conclusion

Firstly, this paper analyzes the principle and digital implementation method of FFT algorithm, and gives the butterfly decomposition calculation diagram of FFT algorithm, which effectively reduces the computational complexity of the algorithm. On this basis, for the external reference velocity damping feedback system in inertial navigation, the quantitative expression between damping coefficient and external reference velocity is analyzed, and the adjustment principle of damping ratio is summarized through data simulation. In order to further verify the effectiveness of the algorithm, the dual core technology is used to separate the main program of the system and the FFT spectrum analysis algorithm to complete the verification on the inertial system, and higher speed accuracy is achieved. Through the above research, the conclusions are as follows.

1. The FFT algorithm will effectively reduce the complexity of spectrum analysis and calculation, and lay a foundation for its real-time operation on low dominant frequency systems.
2. Using dual core system to separate main program and FFT algorithm program segment can further reduce system pressure and reduce the possibility of complex algorithm affecting system timeliness.
3. By using FFT algorithm to analyze the output attitude of the inertial system in real time, and then analyze the complex situation of the current sea state, and reasonably planning the damping coefficient of the inertial system, the inertial system can obtain better navigation accuracy.

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