# Approximate methods of H-infinity control of nonlinear dynamic systems output

*Andrei* Panteleev[1] and *Aleksandra* Yakovleva[1,*]

[1]Moscow Aviation Institute (National Research University), 4, Volokolamskoe shosse, Moscow, 125993, Russia

**Abstract.** The problem of finding $H_\infty$-regulators in the output control problem is considered. Two approaches for approximate synthesis of the closed loop systems are proposed. The paper considers the problem of finding the H-infinity control of a nonlinear continuous dynamical system output. The system is linear in control and perturbation, with a finite time of the system operation. Methods for finding H-infinity control are used to solve problems of synthesis of controllers, particularly for solving the problem of synthesis of controllers in conditions of incomplete state information. In such problems, it is difficult to find the structure of the regulator and its parameters. Sufficient $H_\infty$-control conditions are formulated and proved. Two approximate methods for finding the H-infinity controller of closed loop nonlinear continuous systems were proposed. To illustrate the application of the methods, the problem of stabilization of the ZD559-Lynx helicopter, whose flight takes place during the imposed time period of the system operation, was considered.

## 1 Introduction

Problems and methods for finding $H_\infty$-control laws are considered to be one of the most relevant problems of designing complex aerospace systems. The developing algorithms and software needed to find matrix gains of optimal controllers allow getting the desired quality of control over the object in conditions where the mathematical model and external perturbations contain indeterminacies. It's one of the main reasons why methods of finding $H_\infty$-control have become so popular [1–6]. For many applied problems it is required to synthesize a controller in conditions of incomplete information about the state vector of the system. This article proposes applying methods like the ones described in [7, 8] for finding optimal regulator of a nonlinear system with a finite time of the system operation [7, 9]. As a result of the work, sufficient conditions of control of nonlinear dynamical system, which are required to design output feedback controller, will be formulated. To do so, we have to prove the sufficient conditions theorem, find controller in conditions of incomplete information and propose a solving method, which could be universal for various control problems and provide the desired quality of transient processes as well as the asymptotic stability of the system.

The ZD559-Lynx helicopter stabilization problem [10], which illustrates the effectiveness of the proposed approaches for solving applied control problems, is solved.

[*]e-mail: ayakovleva982@gmail.com

# 2 Methodology

## 2.1 Statement of the problem

The mathematical model of the plant is

$$\dot{x}(t) = a\big(t, x(t)\big) + B_1(t)\,w(t) + B_2(t)\,u(t), \quad x(0) = 0, \tag{1}$$

and model of a measuring system is

$$y(t) = C(t)\,x(t), \tag{2}$$

where $x \in R^n$ is a state vector, $u \in R^q$ is a control input, $w \in R^p$ is a disturbance, $y \in R^m$ is a measured output, $t \in T = [0, t_1]$ is a continuous time, $t_1$ is a given positive value. Continuous function $a(t, x)$ and matrices $B_1(t) \in R^{n\times p}$, $B_2(t) \in R^{n\times q}$, $C(t) \in R^{m\times n}$ are given. It is assumed that $w(.) \in L_2[0, \infty)$, $u(.) \in L_2[0, \infty)$; $m \le n$, $\operatorname{rg} C = m\ \forall t \in T$; the system (1), (2) is fully controllable and observable.

Let's denote $\|z(t)\|^2 = f^0(t, y(t)) + u^T(t)\,Q(t)\,u(t)$ as performance output, where $f^0(t, y)$ is a given continuous function, $Q(t)$ is a positive definite symmetric matrix $q \times q$; $\|F(t_1)\|^2 = F(y(t_1))$ as performance output at the final moment of the control process.

It is required to ensure (if possible) the fulfillment of the inequality

$$\frac{\int_0^{t_1} \|z(t)\|^2 dt + \|F(t_1)\|^2}{\int_0^{t_1} \|w(t)\|^2_{P(t)} dt} = \frac{\int_0^{t_1} [f^0(t, y(t)) + u^T(t)Q(t)\,u(t)]\,dt + F(y(t_1))}{\int_0^{t_1} \|w(t)\|^2_{P(t)} dt} =$$

$$= \frac{\int_0^{t_1} [f^0(t, C(t)\,x(t)) + u^T(t)\,Q(t)\,u(t)]\,dt + F(C(t_1)\,x(t_1))}{\int_0^{t_1} w^T(t)\,P(t)\,w(t)\,dt} \le \gamma^2, \tag{3}$$

where $P(t) \in R^{p\times p}$ is a positive definite symmetric matrix, $\gamma > 0$ is a given non-negative value, as well as the asymptotic stability of a closed loop system. It is preferred to find the minimum value of $\gamma$ at which these properties are still valid, which can be achieved by minimizing the value of the fraction numerator while maximizing the denominator.

In other words, the cost functional must satisfy the condition

$$I(u, w) = \int_0^{t_1} \left[\|z(t)\|^2 - \gamma^2 \|w(t)\|^2_{P(t)}\right] dt + \|F(t_1)\|^2 =$$

$$= \int_0^{t_1} \left[f^0(t, C(t)\,x(t)) + u^T(t)\,Q(t)\,u(t) - \gamma^2 w^T(t)\,P(t)\,w(t)\right] dt + F(C(t_1)\,x(t_1)) \le 0,$$

which will be fulfilled when the output vector tends to zero, minimizing control costs under the worst influence of disturbances possible.

## 2.2 Synthesis of full state vector $H_\infty$-control

Let there be a function $V(t, x) \in C^{1,1}$ and the expression

$$\begin{gathered} R(t, x, u, w) = \frac{\partial V(t, x)}{\partial t} + \left(\frac{\partial V(t, x)}{\partial x}\right)^T [a(t, x) + B_1(t)\,w + B_2(t)\,u] + \\ + f^0(t, C(t)\,x(t)) + u^T Q(t)\,u - \gamma^2 w^T P(t)\,w, \\ G(t_1, x) = V(t_1, x) - F(C(t_1)\,x), \end{gathered} \tag{4}$$

where

$$\frac{\partial V(t,x)}{\partial x} = \left(\frac{\partial V(t,x)}{\partial x_1}, \ldots, \frac{\partial V(t,x)}{\partial x_n}\right)^T.$$

**Theorem.** If a function $V(t,x) \in C^{1,1}$ exists, satisfying the condition $V(t,0) = 0$ and

$$\begin{gathered} R(t,x,u^*(x),w^*(x)) = \min_u \max_w R(t,x,u,w) = 0, \\ \forall x \in R^n, \quad \forall t \in T, \\ G(t_1,x) = 0 \quad \forall x \in R^n, \end{gathered} \tag{5}$$

where

$$u^*(t,x) = -\frac{1}{2}Q^{-1}(t)B_2^T(t)\frac{\partial V(t,x)}{\partial x}, \quad w^*(t,x) = \frac{1}{2\gamma^2}P^{-1}(t)B_1^T(t)\frac{\partial V(t,x)}{\partial x}, \tag{6}$$

and function $V(t,x) \in C^{1,1}$ satisfies the equation

$$\begin{gathered} \frac{\partial V(t,x)}{\partial t} + \left(\frac{\partial V(t,x)}{\partial x}\right)^T a(t,x) + \frac{1}{2}\left(\frac{\partial V(t,x)}{\partial x}\right)^T B_2(t)Q^{-1}(t)B^T(t)\frac{\partial V(t,x)}{\partial x} - \\ - \frac{1}{2}\left(\frac{\partial V(t,x)}{\partial x}\right)^T B_1(t)P^{-1}(t)B_1^T(t)\frac{\partial V(t,x)}{\partial x} + f^0(t, C(t)x) = 0, \\ V(t_1,x) = F(C(t_1)x), \end{gathered} \tag{7}$$

then the inequality (3) is true.

*Proof.* Let the assertion conditions be satisfied. Let's find $\min_u \max_w R(t,x,u,w)$ using the necessary conditions for an extremum:

$$\begin{gathered} \frac{\partial R(t,x,u,w)}{\partial u} = B_2^T(t)\frac{\partial V(t,x)}{\partial x} - 2Q(t)u = 0, \\ \text{therefore} \quad u^*(t,x) = -\frac{1}{2}Q^{-1}(t)B_2^T(t)\frac{\partial V(t,x)}{\partial x}, \\ \frac{\partial R(t,x,u,w)}{\partial w} = B_1^T(t)\frac{\partial V(t,x)}{\partial x} - 2\gamma^2 P(t)w = 0, \\ \text{therefore} \quad w^*(t,x) = \frac{1}{2\gamma^2}P^{-1}(t)B_1^T(t)\frac{\partial V(t,x)}{\partial x}. \end{gathered}$$

Here $u^*(t,x), w^*(t,x)$ are structures for controlling the plant and disturbance (external influence).

Sufficient conditions for an unconstrained minimum with respect to $u$ are fulfilled because $\dfrac{\partial^2 R(t,x,u,w)}{\partial u^T \partial u} = 2Q(t) > 0$, as well as sufficient conditions for maximum with respect to $w$ are fulfilled because $\dfrac{\partial^2 R(t,x,u,w)}{\partial w^T \partial w} = -2\gamma^2 P(t) < 0$.

Then

$$\begin{aligned} R(t,x,u,w) = R(t,x,u^*(t,x),w^*(t,x)) - \gamma^2[w - w^*(t,x)]^T P(t)[w - w^*(t,x)] + \\ + [u - u^*(t,x)]^T Q(t)[u - u^*(t,x)]. \end{aligned}$$

Therefore

$$R(t,x,u^*(t,x),w(t,x)) \le R(t,x,u^*(t,x),w^*(t,x)) \le R(t,x,u(t,x),w^*(t,x)), \tag{8}$$

i.e. there is a saddle point.

Let the available function $V(t, x) \in C^{1,1}$ satisfy the condition $V(0, 0) = 0$ and $R(t, x, u^*(t, x), w^*(t, x)) = 0$. Since the system is completely controllable and observable, for any $u(.) \in L_2[0, \infty)$, $w(.) \in L_2[0, \infty)$ we have $y(.) \in L_2[0, \infty)$.

Along the trajectories of the dynamical system it is true that

$$\frac{\partial V(t, x(t))}{\partial t} + \left(\frac{\partial V(t, x(t))}{\partial x}\right)^T [a(t, x(t)) + B_1(t)w(t) + B_2(t)u(t)] + ||z(t)||^2 - \gamma^2 ||w(t)||^2_{P(t)} =$$

$$= \frac{dV(t, x(t))}{dt} + ||z(t)||^2 - \gamma^2 ||w(t)||^2_{P(t)} = \underbrace{R(t, x(t), u^*(t, x(t)), w^*(t, x(t)))}_{0} -$$

$$- \gamma^2 [w(t) - w^*(t, x(t))]^T P(t)[w(t) - w^*(t, x(t))] + [u(t) - u^*(t, x)]^T Q(t)[u - u^*(t, x(t))].$$

Let's consider the left hand side of inequality (8), namely $R(t, x, u^*(t, x), w(t, x)) \leq \leq \underbrace{R(t, x, u^*(t, x), w^*(t, x))}_{0}$, i.e. when $u = u^*(t, x)$:

$$\frac{dV(t, x(t))}{dt} + ||z(t)||^2 - \gamma^2 ||w(t)||^2_{P(t)} \leq 0.$$

We integrate the left and right hand parts in time from 0 to $t_1$:

$$V(t_1, x(t_1)) - V(0, x(0)) + \int_0^{t_1} ||z(t)||^2 dt - \gamma^2 \int_0^{t_1} ||w(t)||^2_{P(t)} \leq 0.$$

Since $G(t_1, x) = V(t_1, x) - F(C(t_1)x) = 0$, then $V(t_1, x(t_1)) = F(C(t_1)x(t_1))$. Since $x(0) = 0$, then $V(0, x(0)) = V(0, 0) = 0$. It follows that

$$\int_0^{t_1} ||z(t)||^2 \, dt + F(C(t_1)x(t_1)) \leq \gamma^2 \int_0^{t_1} ||w(t)||^2_{P(t)} \, dt.$$

Therefore, condition (3) is fulfilled.

The proof is over.

*Remark.* If $P(t) = E$ and the energy of disturbances acting on the system is limited, i.e.

$$\int_0^{t_1} ||w(t)||^2 dt \leq 1, \quad \text{then} \quad \int_0^{t_1} ||z(t)||^2 dt + \underbrace{F(C(t_1)x(t_1))}_{F(t_1)} \leq \gamma^2.$$

### 2.3 Synthesis of Output $H_\infty$-control

If $m = n$ and the matrix $C(t)$ is non-degenerate, then the state vector could be found directly from the output vector: $x = C^{-1}(t)y$. Then output control

$$u^*(t, y) = -\frac{1}{2} Q^{-1}(t) B_2^T(t) \frac{\partial V(t, C^{-1}(t)y)}{\partial x}.$$

If $m \leq n$, $\operatorname{rg} C(t) = m$ $\forall t \in T$, one can apply two approaches. The first one is related to finding a pseudo-solution $\tilde{x}$ of system $y = C(t)x$ and its application in control law, the second—to the synthesis of the state observer that develops the estimate $\hat{x}$ of the state vector, and using estimates in control.

First approach. Find system pseudo-solution $y = C(t)x$ using a pseudo-inverse matrix $C^{\sim 1}(t) = C^T(t)[C(t)C^T(t)]^{-1}$:

$$\tilde{x} = C^{\sim 1}(t)y,$$

i.e. column $\tilde{x}$ with the smallest modulus among all columns of $x$ minimizing $|C(t)x-y|$, where the column modulus is $|x| = \sqrt{x_1^2 + \ldots + x_n^2}$. Then the output control has the form

$$u^*(t,y) = u^*(t,\tilde{x}) = -Q^{-1}(t)\,B_2^T(t)\frac{\partial V(t, C^T(t)[C(t)C^T(t)]^{-1}y)}{\partial x}. \tag{9}$$

Second approach. Synthesize an asymptotic full order observer

$$\frac{d\hat{x}}{dt} = a(t,\hat{x}(t)) + B_1(t)w(t) + B_2(t)u(t) + K(t)[y(t) - C(t)\hat{x}(t)], \quad \hat{x}(0) = x_0^*, \tag{10}$$

where $K(t) \in R^{n\times m}$ is the gain observer matrix, $x_0^*$ is a column containing the a priori information about the initial state. Matrix $K(t)$ is selected with respect to condition, providing that the error $\varepsilon(t) = x(t) - \hat{x}(t)$ asymptotically tends to zero. Then the control law has the form

$$u^*(t,y_0^t) = u^*(t,\hat{x}) = -Q^{-1}(t)\,B_2^T(t)\frac{\partial V(t,\hat{x})}{\partial x}, \tag{11}$$

where $y_0^t = \{y(\tau),\ 0 \le \tau \le t\}$ is an accumulated information about the output measurement results.

### 2.4 Algorithm of Output $H_\infty$-control

**Step 1.** Set parameter $\gamma > 0$. Find a solution to equation

$$\frac{\partial V(t,x)}{\partial t} + \left(\frac{\partial V(t,x)}{\partial x}\right)^T a(t,x) + \frac{1}{2}\left(\frac{\partial V(t,x)}{\partial x}\right)^T B_2(t)\,Q^{-1}(t)\,B^T(t)\frac{\partial V(t,x)}{\partial x} -$$
$$-\frac{1}{2}\left(\frac{\partial V(t,x)}{\partial x}\right)^T B_1(t)\,P^{-1}(t)B_1^T(t)\frac{\partial V(t,x)}{\partial x} + f^0(t,C(t)x) = 0,$$
$$V(t_1,x) = F(C(t_1)\,x)$$

at fixed $\gamma$, satisfying stability conditions of a closed loop system. Sequentially reducing $\gamma$, find the minimum value under which all conditions remain fulfilled.

**Step 2.** Find the control of a plant of the form (9) or (11) and the law of change of the disturbing effect $w^*(t,x) = \dfrac{1}{2\gamma^2}P^{-1}(t)\,B_1^T(t)\dfrac{\partial V(t,x)}{\partial x}$.

**Step 3.** Find output control law and closed-loop system trajectories.

Case 1. Control system with arbitrary disturbances satisfying the condition $\int_0^{t_1}\|w(t)\|^2dt \le 1$:

$$\dot{x} = a(t,x(t)) + B_1(t)w(t) + B_2(t)u(t), \quad x(0) = 0, \quad y(t) = C(t)x(t),$$

first approach: $u(t,y) = u^*(t,\tilde{x}) = -Q^{-1}(t)\,B_2^T(t)\dfrac{\partial V(t, C^T(t)[C(t)C^T(t)]^{-1}y)}{\partial x}$;

second approach: $u(t,y_0^t) = u^*(t,\hat{x}) = -Q^{-1}(t)\,B_2^T(t)\dfrac{\partial V(t,\hat{x})}{\partial x}$,

$$\frac{d\hat{x}}{dt} = a(t,\hat{x}(t)) + B_1(t)w(t) + B_2(t)u(t) + K(t)[y(t) - C(t)\hat{x}(t)], \quad \hat{x}(0) = x_0^*.$$

Case 2. Control system with the worst perturbations $w(t,x) = \dfrac{1}{2\gamma^2}P^{-1}(t)\,B_1^T(t)\dfrac{\partial V(t,x)}{\partial x}$:

$$\dot{x} = a(t,x(t)) + B_1(t)w(t) + B_2(t)u(t), \quad x(0) = 0, \quad y(t) = C(t)x(t),$$

first approach: $u(t,y) = u^*(t,\tilde{x}) = -Q^{-1}(t)\,B_2^T(t)\dfrac{\partial V(t, C^T(t)[C(t)C^T(t)]^{-1}y)}{\partial x}$;

second approach: $u(t,y_0^t) = u^*(t,\hat{x}) = -Q^{-1}(t)\,B_2^T(t)\dfrac{\partial V(t,\hat{x})}{\partial x}$,

$$\frac{d\hat{x}}{dt} = a(t,\hat{x}(t)) + B_1(t)w(t) + B_2(t)u(t) + K(t)[y(t) - C(t)\hat{x}(t)], \quad \hat{x}(0) = x_0^*.$$

Verify the asymptotic stability of the closed loop system for each case.

## 3 Results and discussion

Based on the outlined algorithms, software was developed to find the parameters of synthesized controllers, to simulate processes in dynamic systems for a given period of time, to obtain information about the quality of transient processes and analyze the asymptotic stability of a closed loop system. We used the MATLAB computing environment.

Following is a model example, for which an analytical solution is obtained. Numerical experiment was carried out using the data of the model example.

### 3.1 Example

Let us consider the problem of stabilizing the ZD559-Lynx aircraft model [10, 11], where in (1)–(2) we set $n = 8$, $p = 1$, $q = m = 4$. It is important to note that $a(t,x(t)) = A(V(t))\,x(t)$ and $B_2(t) = B_2(V(t))$ in [10] are listed for the Lynx at flight speeds from hover to 140 knots in straight and level flight, i.e. trim angular velocities in fuselage axes system are equal to zero. Let us approximate these matrices. The equations of the linearized system are obtained for eight fixed values of the helicopter speed in [10] $V_i = (i-1)\,\Delta V$, $i = 1,\dots,8$, where $\Delta V = 20$ knots:

$$\dot{x}(t) = A_i x(t) + B_i u(t), \quad i = 1,\dots,8, \quad x(0) = x_0.$$

It is proposed to use an extended model

$$\dot{x}(t) = A(V)\,x(t) + B_2(V)\,u(t), \quad x(0) = x_0,$$

where $V(t)$ can be found by solving the equation $\dot{V}(t) = 140/T$, $V(0) = 0$, which is used for mode with linearly increasing speed or another equation $\dot{V}(t) = 280t/T^2$, $V(0) = 0$, which is used for mode with positive acceleration. As a result, $a(t,x(t)) = A(V)\,x$, $B_2(t) = B_2(V(t))$ can be approximated with law:

$$A(V) = \sum_{i=1}^{8} A(V_i)S_p^*(i,V) = \sum_{i=1}^{8} A(V_i)S_p^*\Big(\frac{V}{\Delta V} - (i-1)\Big),$$

$$B_2(V) = \sum_{i=1}^{8} B_2(V_i)S_p^*(i,V) = \sum_{i=1}^{8} B_2(V_i)S_p^*\Big(\frac{V}{\Delta V} - (i-1)\Big),$$

where $S_p^*(t) = \begin{cases} 2^{p-1}(1+t)^p, & t \in [-1,-1/2], \\ 1 - 2^{p-1}|t|^p, & t \in [-1/2,1/2], \\ 2^{p-1}(1-t)^p, & t \in [1/2,1], \\ 0, & t \notin [-1,1] \end{cases}$—finite function with parameter $p$.

Then the other matrix in plant model is described by the following one:

$$B_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.1 \\ 0.2 \\ 0 \\ 0 \end{pmatrix}.$$

The matrix in the equation of the measuring system has the form:

$$C = \begin{pmatrix} 0 & 0 & 0 & 57.2958 & 0 & 0 & 0 & 0 \\ 0 & 57.2958 & 0 & 0 & 0 & 0 & 0 & 0 \\ -16.26 & -0.9788 & 0.4852 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

The state vector is as follows: $x_1$—velocity along aircraft x-axis, $x_2$—velocity along aircraft z-axis, $x_3$—angular velocity along aircraft y-axis, $x_4$—pitch angle, $x_5$—velocity along aircraft y-axis, $x_6$—angular velocity along aircraft x-axis, $x_7$—angle of attack, $x_8$—angular velocity along aircraft z-axis.

### 3.2 Simulations

**Simulation Settings.** For the correct computation of the output feedback control, it is necessary to select $Q$. During a search for suitable elements, we changed their design several times. After all we selected the design matrices as in table 1, which contains parameters of numerical simulations.

In addition, we choose function given in table 1 to describe disturbance, which affects the state vector.

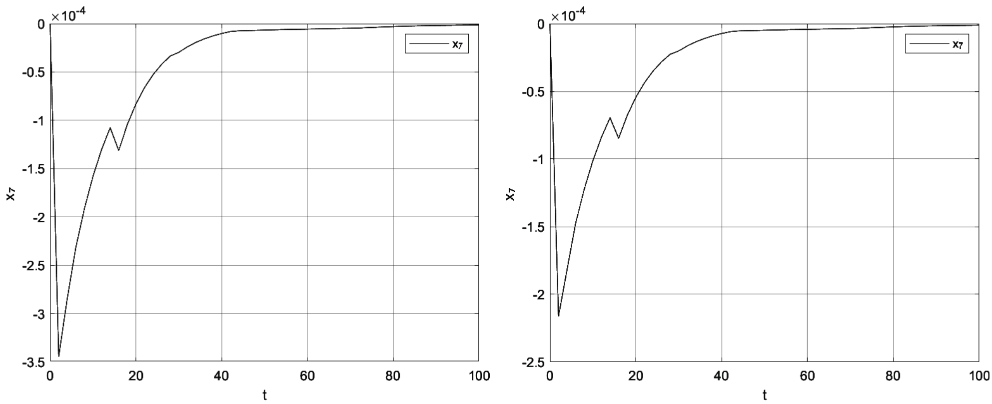
**Table 1.** Numerical simulation parameters

| Initial conditions $x$ | $Q$ | $T = [t_0; t_1]$ | Disturbance |
|---|---|---|---|
| $\overline{0}$ | $\begin{pmatrix} 0.169 & 0 & 0 & 0 \\ 0 & 0.169 & 0 & 0 \\ 0 & 0 & 0.78 & 0 \\ 0 & 0 & 0 & 0.1 \end{pmatrix}$ | [0; 100] | $w(t) = \begin{cases} 0, & t < 0 \text{ or } t > t_w, \\ 1, & 0 \le t \le t_w,\ t_w = 1. \end{cases}$ |

**Numerical Simulations.** For one of the described cases, namely the case of control system with arbitrary disturbances, we performed solution of the model example with the parameter $\gamma = 0.05$, which satisfies (3) and still guarantees the fulfillment of the asymptotic stability property of a closed loop system.

Following are plots of the changing angle of attack for the first and second approaches (figure 1).

Analysis of plots indicates that problems of synthesis of suboptimal $H^\infty$-controller can be solved by both approaches, which were proposed in the paper. Results for the both methods on the example presented on the figure 1 for the coordinate $x_7$ look similar due to the fact that




(a) (b)

**Figure 1.** State response $x_7(t)$—angle of attack with $\gamma = 0.05$: (a)—for the first approach; (b)—for the second approach

each of the methods results in the optimal trajectory of the coordinate. Simulations give good results. A rather large influence is exerted by the elements of matrix $Q$ in the quality criteria.

The coordinate $x_7$ and other coordinates of the state vector asymptotically converge to zero, which indicates the robustness of the system and the correct selection of parameters, for which the system retains its property of stability for any given disturbances.

Generally, both of the approaches produce comparable results observed with the processes change over time, which allows to make a conclusion about them both being a viable option in further research.

## 4 Summary

The purpose of this study was to design output feedback controller using the $H^\infty$-control for nonlinear dynamical system. Based on the obtained results we can state that proposed algorithms of synthesis of controllers provide the desired quality of transient processes and asymptotic stability of closed loop systems. Using $H^\infty$-control allows to effectively neutralize the negative influence of limited disturbances on the system.

$H^\infty$-control method was successfully applied to solving the considered example. The described method successfully deals with the problem, demonstrating good results. The proposed methods could be useful in autopilot design problems for aircrafts and rotorcrafts.

Nowadays, methods such as LQG-synthesis or frequency response conditions are used to find $H^\infty$-control. Often finding the control by these methods is reduced to solving linear-matrix inequalities, so the main difference between the methods proposed in the paper and other methods is that the proposed methods allow to avoid the solution of linear matrix inequalities and are distinguished by their simplicity for application.

## References

[1] M. Green, D.J.N. Limebeer, *Linear Robast Control* (Dover Publications, 2012)

[2] J. Gadewadikar, F.L. Lewis, M. Abu-Khalaf, *Necessary and Sufficient Conditions for H-infinity Static Output–Feedback Control*, J. Guid. Control Dyn., **29(4)** (2006)

[3] S. Skogestad, I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design* (John Wiley and sons, 2005)
[4] H.W. Koobloch, A. Isidori, D. Flockerzi, *Topics in control theory*, bd. 22 (Springer, Basel, 1993)
[5] J. Doyle, B. Francis, A. Tannenbaum, *Feedback Control Theory* (Publishing Co., 1990)
[6] D.V. Balandin, M.M. Kogan, *Minimax filtering: $\gamma_0$-optimal observers and generalized $H_\infty$-optimal filters*, Autom. Remote Control, **74(4)**, 575 (2013)
[7] A.V. Panteleev, A.A. Yakovleva, *Sufficient conditions for H-infinity control on the finite time interval*, J. Phys.: Conf. Ser., **1925**, 012024 (2021)
[8] A.V. Panteleev, A.A. Yakovleva, *Approximate methods for H-infinity output feedback control*, IOP Conf. Ser., **927(1)**, 012026 (2020)
[9] M.M. Tchaikovsky, V.N. Timin, *Synthesis of anisotropic suboptimal control for linear time-varying systems on finite time horizon*, Autom. Remote Control, **78(7)**, 1203 (2017)
[10] G.D. Padfield, *Helicopter flight dynamics: the theory and application of flying qualities and simulation modeling*, 2nd ed. (Blackwell Publishing, 2007)
[11] H. Jinshuo, G. Hongbin, *Survey on flight control technology for large-scale helicopter*, Hindawi Int. J. Aerosp. Eng., **2017**, 5309403 (2017)