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Calculation of fatigue fracture in structural elements using a multimode damage model

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Abstract. The problem of nucleation and development for the fatigue fracture process in element of aircraft structure is solved using previously developed cyclic damage model. The model contains the evolution equation for the damage function depending on stress state and amplitudes of the cyclic loads. On this basis, calculations of the initiation and development of crack-like fracture zones are carried out for an aircraft structure element of a disk with a blade in a flight loading cycle (HCF mode). The damage function was calculated based on the elastic solution for the element of structure loaded by centrifugal forces in disk and blade and aerodynamical pressure on blade. An alternative cyclic loading regime associated with high-frequency vibrations of the blades was investigated. Fatigue fracture in this process corresponds to the VHCF mode. Simulation of the combined cyclic loading mode was also carried out. In all these modes, the real time of the nucleation and development of quasi-cracks was estimated up to reaching the observed disk surface.

1 Introductions

The developing of a new actual models and effective numerical methods for prediction the fatigue failure of structural elements is very important problem for different industries. In the case of aircraft industry, it was reported that more than 90% in service fractures are due to fatigue. It is important to note that the stress state of the structural elements are often combines different loading modes and regimes. In the case of engine applications, it is usually considering the low amplitude high frequency vibrations and high amplitude low frequency loadings or its combination. The high frequency vibrations are due to instability in air flow, not-perfect coaxially of shafts, design errors and other reasons. The low frequency loadings are usually associated with centrifugal force, changes in engine operating regime. For a long time, it was assumed that low amplitude vibrations are not significantly effects on fatigue life of the element if its natural frequency far from the resonant frequency of structure. However, modern advances in fatigue science have shown the destructive potential of such loadings in the very high cycle range. It was reported that material degradation and fatigue failure in structural materials can be observed well below the classical fatigue limit. In the recent years a series of in-service accidents in aircraft industry due to high frequency vibrations have

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shown the importance of the problem and the limitation of used design protocols. To improve the situation and increase a safety of fatigue designs it is necessary to develop models that are considering modern achievements in material science and fatigue.

Most of the actual models of fatigue life prediction are based on hypothesis of static stress field during the loading process. But the real material behavior and in service conditions are different. During the cyclic loading the mechanical properties of material are permanently degrading. Depending on the load amplitude this degradation can be volumetric or localized. Nonetheless, this degradation is affecting the stress state and can lead to premature crack initiation compared to predictions of model that are based on static stress state hypothesis. The developing of progressive fatigue damage accumulation theory and corresponding models is very perspective scientific field with good industrial application perspectives. This paper is focused on developing of a such model with progressive degradation of material and its application to real structures. The example of spatial damage zone calculation in the joint area of turbojet engine disk and blade under cyclic loading are presented. The lifetime of structure was estimated with taken considering as high amplitude low frequency loadings, as well low amplitude high frequency vibrations. The outstanding results of numerical simulations shows a good agreement with the in-service fatigue fractures.

2 Multimode two-criterion model of cyclic damage

Let us formulate a multimode two-criterion model of cyclic damage, first proposed, and developed in [1–3]. A fatigue fracture criterion corresponding to the left branch of the bimodal fatigue curve in classical LCF–HCF mode has the following form:

$$\sigma_{eq} = \sigma_u + \sigma_L N^{-\beta_L}, \quad \sigma_L = 10^{3\beta_L}(\sigma_B - \sigma_u). \tag{1}$$

In these formulas σ_B is the ultimate tensile strength of the material, σ_u is the classic fatigue limit of the material during a fully reverse cycle (stress ratio $R = -1$, β_L is power index of the left branch of the bimodal fatigue curve.

The fatigue fracture criterion corresponding to the right branch of the bimodal fatigue curve (VHCF mode) is assumed in the following form:

$$\sigma_{eq} = \tilde{\sigma}_u + \sigma_V N^{-\beta_V}, \quad \sigma_V = 10^{8\beta_V}(\sigma_u - \tilde{\sigma}_u). \tag{2}$$

Here $\tilde{\sigma}_u$—fatigue limit of the material under symmetric loading cycle in the VHCF regime, β_V is power index of the right branch of the bimodal fatigue curve.

Expressions for equivalent stresses σ_{eq} are determined by the microdamage mechanism and are associated with two well-known criteria of multiaxial fatigue fracture [4–6]. The stress-based Smith–Watson–Topper (SWT) criterion is associated with the development of microcracks of normal opening [4, 5]. The Carpinteri–Spagnoli–Vantadori (CSV) criterion is associated with the development of shear microcracks [6].

The process of gradual cyclic material failure is described by a damage function $0 \le \psi(N) \le 1$ [6, 7]. When $\psi = 1$ a material particle (physically small volume) is completely destroyed. Its Lamé modules become equal to zero. The damage function ψ is described by the kinetic equation:

$$\frac{\partial \psi}{\partial N} = \frac{B\psi^{\gamma}}{(1 - \psi^{1-\gamma})}, \tag{3}$$

where $0 < \gamma < 1$ is the model parameter that determines the rate of fatigue damage development.

The coefficient B is determined by the procedure that is clearly associated with the selected criterion for multiaxial fatigue failure of one type or another. The expressions for the coefficient B in various modes were obtained in [1, 2].

For the LCF–HCF at $\sigma_u + \Delta\sigma_u < \sigma_{eq} < \sigma_B$, $\Delta\sigma = 10^{-5\beta}(\sigma_B - \sigma_u)$

$$B = B_L = 10^{-3}[\langle\sigma_{eq} - \sigma_u\rangle/(\sigma_B - \sigma_u)]^{1/\beta_L}/(1-\gamma)/2 \tag{4}$$

and for VHCF at $\tilde{\sigma}_u < \sigma_{eq} \le \sigma_u + \Delta\sigma_u$

$$B = B_V = 10^{-8}[\langle\sigma_{eq} - \tilde{\sigma}_u\rangle/(\sigma_B - \tilde{\sigma}_u)]^{1/\beta_V}/(1-\gamma)/2. \tag{5}$$

When $\sigma_{eq} < \tilde{\sigma}_u$ the fatigue failure does not occur, when $\sigma_{eq} > \sigma_B$ the failure comes instantly.

The degradation of a material is described by a decrease in its elastic Lamé moduli λ and μ to zero with an increase of damage ψ to the value $\psi = 1$. The following nonlinear law of decreasing Lamé moduli is adopted:

- for $\psi \le \psi_*$, $\lambda(\psi) = \lambda_0(1 - k\psi)$, $\mu(\psi) = \mu_0(1 - k\psi)$;
- for $\psi_* \le \psi \le 1$, $\lambda = 0$, $\mu = 0$.

The parameter $0 < k < 1/2$ characterizes a relatively weak change in the moduli until the critical level $\psi_* \lesssim 1$ of damage is reached, after which a state of complete fracture occurs with the loss of the ability of the elastic resistance of the material to any loads.

Let us describe the selected criteria for two mechanisms of fatigue failure of a material under cyclic loading.

The Smith–Watson–Topper criterion of multiaxial fatigue failure in the LCF–HCF mode with the development of normal opening micro-cracks (stress-based SWT) [4, 5] is corresponding to the left branch of the bimodal fatigue curve and has the following form:

$$\sqrt{\langle\sigma_{1\,\max}\rangle\Delta\sigma_1/2} = \sigma_u + \sigma_L N^{-\beta_L}, \tag{6}$$

where σ_1 is the largest principal stress, $\Delta\sigma_1$ is the spread of the largest principal stress per cycle, $\Delta\sigma_1/2$ is its amplitude. According to the chosen criterion only tensile stresses lead to failure, so it has the value $\langle\sigma_{1\,\max}\rangle = \sigma_{1\,\max}$. We assume that the right branch of SN-curve for the VHCF regime is also describing by the similar equation (2).

We suppose that the equivalent stress has the following expression for both LCF–HCF and VHCF modes:

$$\sigma_{eq} = \sigma^n = \sqrt{\langle\sigma_{1\,\max}\rangle\Delta\sigma_1/2}. \tag{7}$$

The Carpinteri–Spagnoli–Vantadori criterion of multiaxial fatigue failure in the LCF–HCF mode, including the concept of a critical plane (stress-based CSV) [6], corresponding to the left branch of the bimodal fatigue curve, in a simplified formulation has the form:

$$\sqrt{(\langle\Delta\sigma_n\rangle/2)^2 + 3(\Delta\tau_n/2)^2} = \sigma_u + \sigma_L N^{-\beta_L}, \tag{8}$$

where $\Delta\tau_n/2$ is the amplitude of the tangential stress on the plane (critical), where it reaches its maximum value, $\Delta\sigma_n/2$ is the amplitude of the normal (tensile) stress on the critical plane, $\langle\Delta\sigma_n\rangle = \Delta\sigma_n H(\sigma_{n_{\max}})$. This criterion includes the mechanism of fatigue fracture with the formation of shear micro-cracks.

We also suppose that the equivalent stress has the following expression for both LCF–HCF and VHCF modes:

$$\sigma_{eq} = \sigma^\tau = \sqrt{(\langle\Delta\sigma_n\rangle/2)^2 + 3(\Delta\tau_n/2)^2}. \tag{9}$$

3 Numerical algorithm for fatigue damage development calculation

To integrate the equation $\partial\psi/\partial N = B\psi^\gamma/(1 - \psi^{1-\gamma})$, the damage function approximation was applied at the k-node of the computational grid for given discrete values ψ_k^t at moments N^t and sought ψ_k^{t+1} at moments N^{t+1}.

After integrating the damage equation, the new value of damage function depends on the increment of the number of cycles ΔN^t as in [2]:

$$\psi_k^{t+1} = \left(1 - \sqrt{[1 - (\psi_k^t)^{1-\gamma}]^2 - 2(1-\gamma)B\Delta N^t}\right)^{1/(1-\gamma)}, \tag{10}$$

$$\Delta N^t = \min_k \frac{1}{2}\Delta\tilde{N}_k^t, \tag{11}$$

$$\Delta\tilde{N}_k^t = \left[\psi^{1-\gamma}/(1-\gamma) - \psi^{2(1-\gamma)}/2/(1-\gamma)\right]\Big|_{\psi_k^t}^{1} \Big/ B. \tag{12}$$

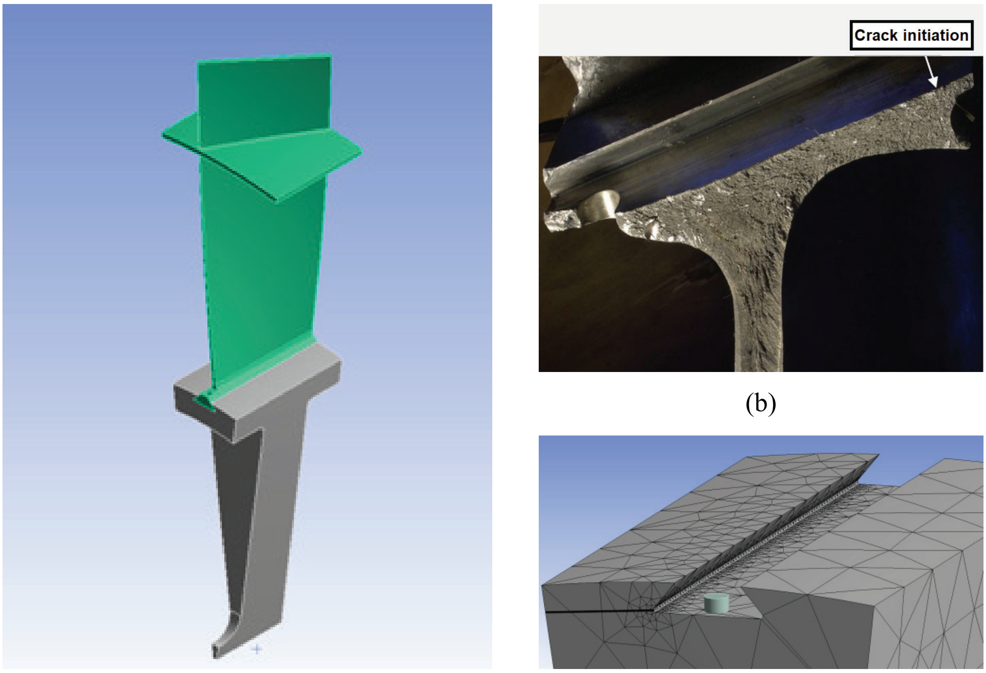
All elements are sorted out, for each of them the most damaged node is searched and according to its damage the mechanical properties of the element are adjusted for $\psi_k^t < \psi_*$:

$$\lambda(\psi_k^t) = \lambda_0(1 - k\psi_k^t), \quad \mu(\psi_k^t) = \mu_0(1 - k\psi_k^t). \tag{13}$$

Those elements that belong to nodes with damage $\psi_* \leq \psi_k^t \leq 1$ are removed from the calculation area and form a localized zones (quasi-cracks) of completely destroyed material.

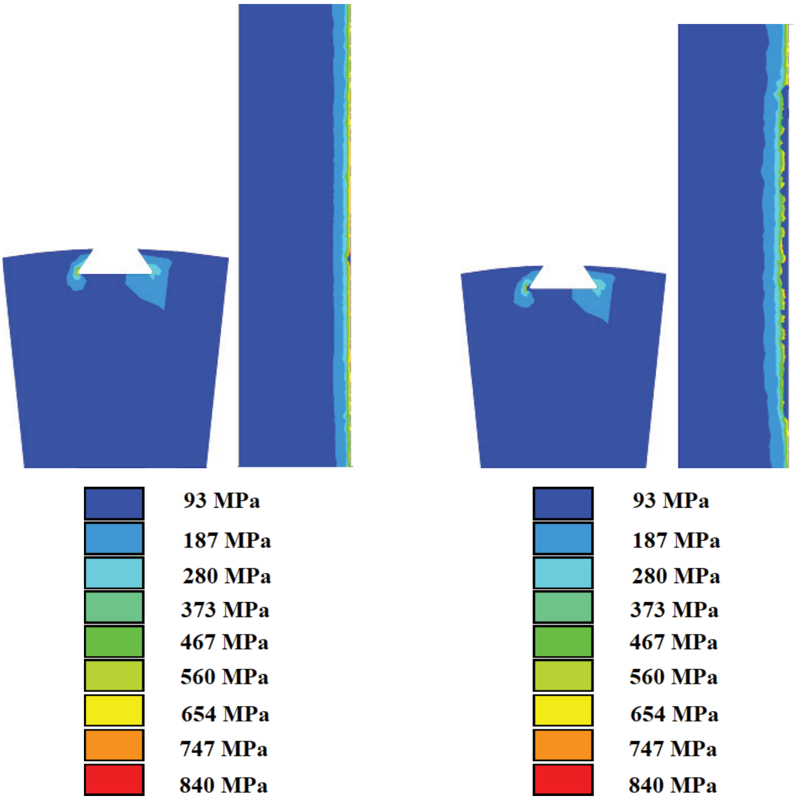
4 Calculation results for real structural element

Mathematical modelling of the experiments [8–12] using specimens of various shapes, subjected to cyclic loading in modes HCF and VHCF showed good qualitative and quantitative agreement with the geometry of real cracks and the values of durability obtained during tests.



(a) (b) (c)

Figure 1. General view of the disk sector with blade (a), fracted disk (b) and the finite element mesh of the disk (c)



(a) (b)

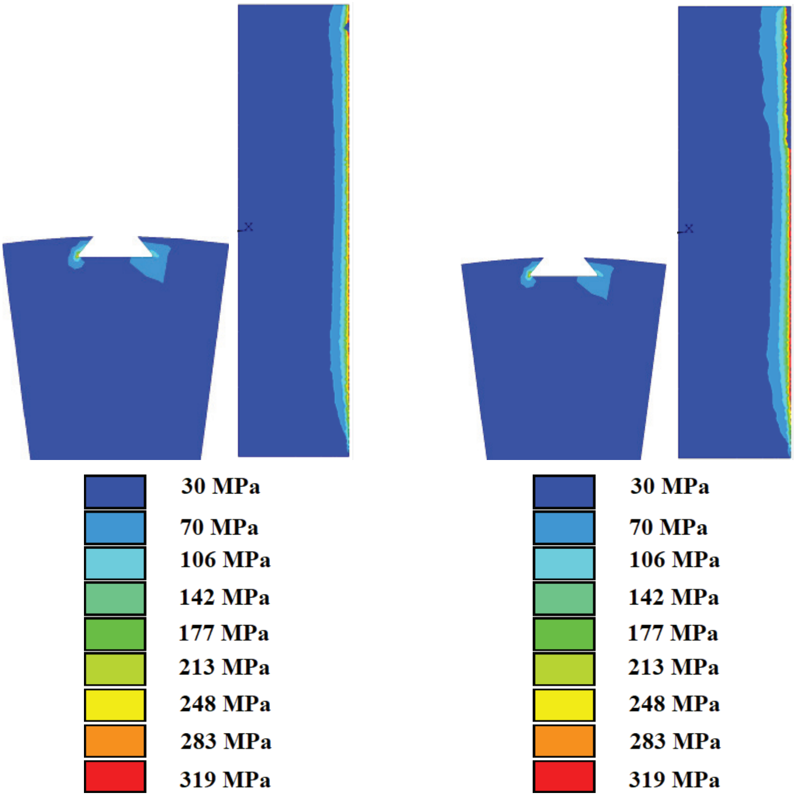
Figure 2. View in cross-section and longitudinal section at the quasi-crack initiation, $N = 5.2 \cdot 10^4$ (a); view in cross-section and longitudinal section with further growth of the quasi-crack, $N = 2.5 \cdot 10^5$ (b)

Here numerical experiments are carried out to calculate the nucleation and development of quasi-cracks in the constituent elements of aircraft structures. The turbojet engine (TJE) compressor disk with blades is loaded in the flight loading regime (HCF mode) and by high frequency vibrations of blades (VHCF mode). Previously the stress-strain state of the structural element in this cycle loadings and the time of its primary damage were determined in [13, 14] without considering the development of quasi-cracks before macro fracture.

The Ansys software was used to solve the quasi static and dynamic solid mechanics problems supplemented by a code to calculate the damage equation and changes of elastic modulus.

All numerical experiments were conducted on an element of structure such a sector of the disk with blade. On figure 1(a) shows a general view of the model, consisting of disk sector and one blade. The fracture surface of turbojet engine is presented on figure 1(b). The finite element mesh of the corresponding disk sector is presented in figure 1(c). The material used for calculation of the disk is titanium alloy with the following ultimate tensile strength and fatigue parameters $\sigma_B = 1135$ MPa, $\sigma_u = 330$ MPa, $\tilde{\sigma}_u = 330$ MPa, $\beta_L = 0.3$, $\beta_V = 0.25$. Elastic moduli are $\lambda_0 = 77$ GPa, $\mu_0 = 44$ GPa. Parameters of damage equation are $k = 0.5$, $\gamma = 0.5$, $\psi_* = 0.98$.

In the first series of numerical experiments, calculations of the flight loading cycle were carried out (stress ratio is $R = 0$) The damage function was calculated based on the elastic



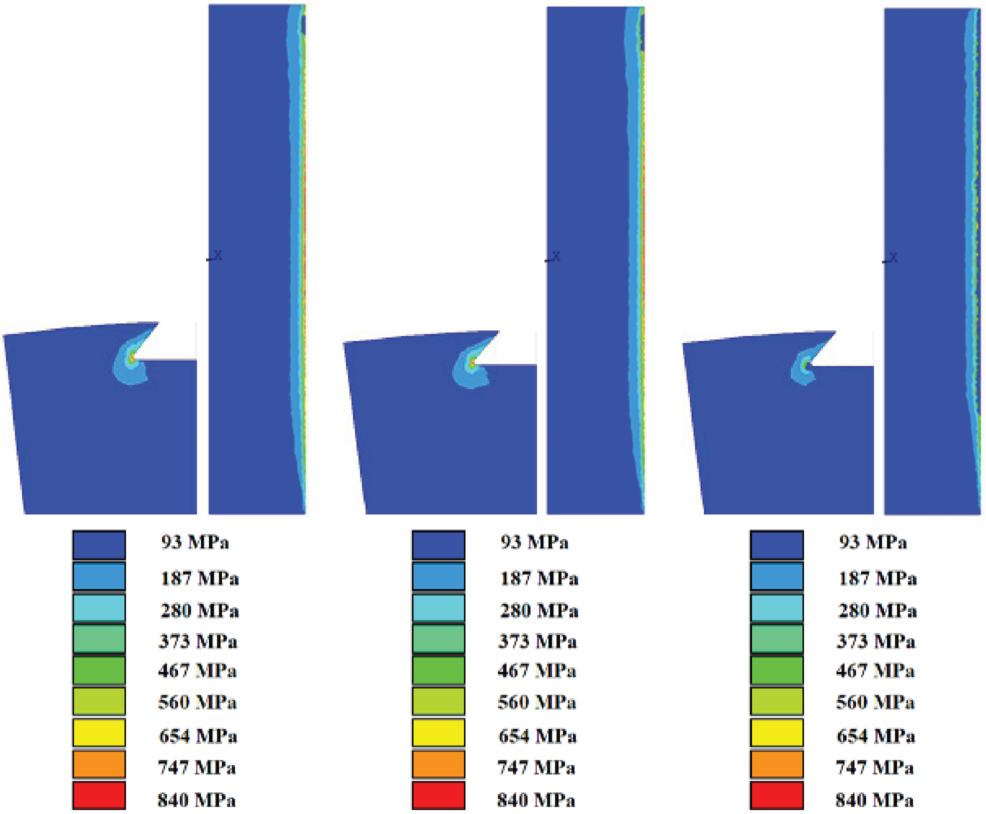
(a) (b)

Figure 3. View in cross-section and longitudinal section at the moment of quasi-crack nucleation, $N = 7.3 \cdot 10^{10}$ (a); view in cross-section and longitudinal section with further growth of the quasi-crack, $N = 1.5 \cdot 10^{11}$ (b)

solution for the element of structure loaded by centrifugal forces in disk and blade and aerodynamical pressure on blade on the hypothesis of an isolated profile. The values of the external load were chosen in the range that allows us to study crack nucleation and development for high-cycle fatigue. The angular speed of rotation of the disk has a value of$\omega = 425$ rad/s, flow speed $V = 200$ m/s.

The estimation of the distributed aerodynamic pressure on the blade, leading to its bending and additional stresses in the contact zone with the disk, was carried out by the method [15]. The contact interaction of the disk and the blade assumes the possibility of normal separation of the contact surfaces along the normal and shear with friction, the coefficient of friction $q = 0.1$.

In the second series of numerical experiments the calculation of the observed high-frequency bending-torsional vibrations of the blade caused by axial displacements of the ends of the blade shroud ring with an amplitude of ~ 0.5 mm and frequency 1000 Hz (reverse cycle, stress asymmetry ratio is $R = -1$ VHCF mode) are presented. On figure 3(a) shows the levels of equivalent stresses in cross-section and longitudinal section one for quasi-crack nucleation, number of cycles $N = 7.3{\cdot}10^{10}$, on figure 3(b) shows the levels of equivalent stresses in cross-section and longitudinal section with further growth of the quasi-crack, $N = 1.5{\cdot}10^{11}$.



(a) (b) (c)

Figure 4. View in cross-section and longitudinal section (nucleation), VHCF mode, $N = 7.3 \cdot 10^{10}$ (a); view in these sections with growth of the quasi-crack, HCF mode, $N = 9.3 \cdot 10^{3}$ after switching of the modes (b); development of a quasi-crack to the lateral surface of the disk, HCF mode, $N = 4.2 \cdot 10^{4}$ cycles after switching (c)

The nucleation zone is located at the far zone of the “dovetail”, arises at the number of cycles $N = 7.3 \cdot 10^{10}$ (dark spot in rectangle in figure 3(a). The quasi-crack grows, reaches the end of the disk, and becomes observable from the outside at $N = 1.5 \cdot 10^{11}$ (figure 3(b)).

In both loading modes, the growth of a quasi-crack from its nucleation to reaching the observed surface takes a significant number of cycles, which can be estimated using the proposed damage model.

In the third series of numerical experiments the combined cyclic loading of the disk is considered. At the beginning of the cyclic process, a quasi-crack nucleates according to the VHCF mode from the oscillations of the blades. Then, because of the appearance of a concentrator, it develops in flight loading cycles according to the HCF mode. Such a sequence of damage development in a real disk during exploitation is described in [16].

In figure 4(a) shows the levels of equivalent stresses in cross-section and longitudinal section at the moment of quasi-crack nucleation in VHCF mode, number of cycles $N = 7.3 \cdot 10^{10}$, in figure 4(b) shows the levels of equivalent stresses in cross-section and longitudinal section with further growth of the quasi-crack in HCF mode, $N = 9.3 \cdot 10^{3}$ after mode switching. In figure 4(c) shows the levels of equivalent stresses in cross-section and longitudinal section at moment of a quasi-crack reaching the lateral surface of the disk $N = 4.2 \cdot 10^4$ after mode switching.

It is possible to assess the real time of the nucleation and development of a quasi-crack in each of the modes.

Assuming that the average flight cycle is approximately 3 hours, in the HCF mode this time is $t_1 = 5.2 \cdot 10^4 \cdot 3\ \text{h} \sim 1.6 \cdot 10^5$ h. Time to reach the surface is $t_2 = 2.5 \cdot 10^5 \cdot 3\ \text{h} \sim 7.5 \cdot 10^5$ h. In the VHCF mode at an oscillation frequency of 1000 Hz, the time for crack initiation is $t_3 = 7.3 \cdot 10^{10} \cdot 0.001\ \text{s} \sim 2 \cdot 10^4$ h. Thus, these estimates show that the microcrack initiation is more probable in the VHCF mode. However, the minimum time before the emergence of a quasi-crack on the surface is achieved with the combined mechanism and mode switching after the initiation of the VHCF damage. This time is $t_4 = 2 \cdot 10^4\ \text{h} + 4.2 \cdot 10^4 \cdot 3\ \text{h} \sim 1.5 \cdot 10^5 < t_1$. These estimates confirm the high probability of the combined mechanism realization for fatigue fracture of the disk.

5 Conclusion

The problem of nucleation and development for the fatigue fracture process in element of aircraft structure is solved using previously developed cyclic damage model. The model contains the evolution equation for the damage function depending on stress state and amplitudes of the cyclic loads. The calculation algorithm is based on a special post proceeding of a numerical solution for solid mechanics problem. The normal crack opening and shear crack mechanisms are taken into account in this model.

On this basis, calculations of the initiation and development of crack-like fracture zones are carried out for an aircraft structure element of a disk with a blade in a flight loading cycle (HCF mode). The damage function was calculated based on the elastic solution for the element of structure loaded by centrifugal forces in disk and blade and aerodynamical pressure on blade.

An alternative cyclic loading regime associated with high-frequency vibrations of the blades was investigated. Fatigue fracture in this process corresponds to the VHCF mode. Simulation of the combined cyclic loading mode was also carried out. In all these modes, the real time of the nucleation and development of quasi-cracks was estimated up to reaching the observed disk surface. The geometry and shape of quasi-cracks in the vicinity of the contact zone of the disk and the blade are similar to those observed in real operation.

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