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Mathematical methods of predicting determining factors characteristics of composites based on generalized durability models

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Abstract. The possibility of creating effective methods for predicting the defining characteristics of composites in refined variational formulations has been investigated. The methodology being developed is based on methods related to the theoretical justification of the admissibility of schemes for narrowing the set of acceptable options, in which the desired globally optimal solution continues to be found on the resulting set of smaller dimensions. Comparative computational experiments were carried out on the basis of the developed methods based on the application of necessary and sufficient extremum conditions. The conducted computational experiments have shown the effectiveness of the developed approach associated with the gradual complication of forecasting models and the corresponding sequential refinement of the optimal parameters of the model.

1 Introduction

Currently, polymer composite materials, composite structures, composite coatings are becoming increasingly common in modern fields of physics, engineering, and instrumentation, which determine scientific and technological progress [1–4]. Such areas that determine modern scientific and technological progress, and are characterized by the ever-increasing use of polymer composites, include aircraft and space technology, shipbuilding, oil and gas industry, modern technology operating in the extreme conditions of the Arctic and Subarctic zones. In connection with the ever-increasing use of polymer composites in technology, the problem of predicting the reliability, durability, and residual life of composite materials and structures made of them is acquiring significant relevance [5–8].

A constructive analysis of promising ways of increasing the potentialities associated with solving the problems of predicting the residual resource, reliability, durability of polymer composites under the influence of extreme environmental factors and operational loads is carried out. A comparative analysis of existing approaches and their possible modifications made it possible to conclude that the most promising opportunities are associated with the

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development of a methodology for effectively narrowing search areas containing globally optimal solutions.

2 Formulation of the problem

As a defining characteristic of a polymer composite, its residual resource can be considered [9–11]. Models describing the time dependence of the residual resource R under the influence of extreme climatic factors in general can be presented in the form of functional dependencies of the following form [12–14]:

$$R(u_1, u_2, \ldots, u_n; t) = R_0 + \Phi(u_1, u_2, \ldots, u_n; t),$$
$$\Delta R(u, t) = \Phi(u, t), \quad \Delta R = R - R_0. \tag{1}$$

The parameters of the optimal forecasting model $u^* = (u_1^*, u_2^*, \ldots, u_m^*)$ deliver an extreme value to the special efficiency criterion J, which is a complex multi-parameter function of the parameters of the forecasting problem. As an efficiency criterion as a complex function of the parameters of a forecasting problem of the form (1), it is advisable to choose the standard deviation of the approximation of experimental data $\tilde{R}_i$ from the theoretical values $R(u, t)$:

$$J(u_1, u_2, \ldots, u_n) = \frac{1}{m} \sum_{i=1}^{m} \left[R(u_1, u_2, \ldots, u_n; t_i) - \tilde{R}_i \right]^2. \tag{2}$$

We will assume that the experimental data adequately reflect the structure of the dependence of the change in the residual resource of the composite on the effect of extreme environmental factors, and the experimental data were obtained with insignificant errors that slightly distort the regularities of the behavior of real dependencies. Then the problem of reconstructing the parameters of models of composite materials from the influence of extreme environmental factors can be reduced to finding the absolute minimum of the quality function (2) in the extreme problem:

$$J(u^*) = \min_u J(u). \tag{3}$$

The vector of parameters $u^* = (u_1^*, u_2^*, \ldots, u_n^*)$ that deliver the absolute minimum to the efficiency indicator $J(u)$ (3) determines the optimal predicted dependence of the change in the residual life of the polymer composite under the influence of extreme environmental factors.

3 Materials and research methods

The study of the possibility of creating effective methods for predicting the defining characteristics of CM in refined variational statements based on the combined use of necessary and sufficient conditions for an extremum and methods of exhaustive enumeration has been carried out. The methodology being developed is based on the techniques associated with the theoretical substantiation of the admissibility of the schemes of narrowing the set of admissible options, in which the desired globally optimal solution continues to be found on the resulting set of lower dimension [15, 16].

On the basis of a constructive study of the structure of efficiency indicators in the investigated variational formulations of inverse problems of predicting the residual resource of composites, it was found that optimal solutions that deliver an absolute minimum to the corresponding efficiency indicators can additionally satisfy a certain system of ratios. The development of a methodology for identifying such systems of additional relations made it possible to significantly reduce the dimension of the initial variational formulations of inverse problems of forecasting the residual resource of composites and, accordingly, significantly narrow the initial set of admissible variants of forecasting models M of the form

$$M = \Big\{R(u;t)\colon u = (u_1, u_2, \ldots, u_n) \in U \subset E^n,\ 0 \le t \le T_{\max}\Big\} \tag{4}$$

to a narrow compact set M^*, on which the problem of constructing a globally optimal solution can be effectively solved. In these designations: $R(u, t)$ is the predicted time dependence of the residual resource of the composite; $u = (u_1, u_2, \ldots, u_n)$ is the vector of undefined parameters from the admissible set of parameters U, t—time, $[0, T_{\max}]$ is the considered time interval.

Conducted constructive analysis of the investigated variational formulations of inverse problems of forecasting the residual resource made it possible to establish that the necessary conditions for an extremum for efficiency indicators for a number of parameters simultaneously coincide with sufficient conditions. Let us denote such parameters by $u_{j_1}, u_{j_2}, \ldots, u_{j_q}$. The necessary and sufficient conditions for an extremum for a given selected system of parameters $u_{j_1}, u_{j_2}, \ldots, u_{j_q}$ can be taken as a system of functional relationships g_j $(j = 1, 2, \ldots, q)$, which can serve as the basis for the subsequent significant reduction in the dimension of the original forecasting problem and simplification of its structure.

Based on the application of the modern mathematical apparatus of constructive analysis of the system of relations, to which the necessary and sufficient conditions for an extremum lead, the selected system of q parameters $u_{j_1}, u_{j_2}, \ldots, u_{j_q}$ can be expressed in terms of the remaining system of $r = n - q$ parameters u_l $(l = 1, 2, \ldots, n;\ l \neq j_i, i = 1, 2, \ldots, q)$ multiparameter forecasting model:

$$u_{j_i} = \varphi_i\Big(u_1, u_2, \ldots, u_{j_1-1}, u_{j_1+1}, \ldots, u_{j_q-1}, u_{j_q+1}, \ldots, u_{n-1}, u_n\Big), \quad i = 1, 2, \ldots, q. \tag{5}$$

As a result, the developed approach made it possible to obtain a variational problem of a much smaller dimension and, accordingly, a simpler structure compared to the original variational problem, for which, taking into account, the established features of its structure, effective methods of nonlocal optimization were developed.

A statement is formulated that substantiates the approach associated with an effective reduction in the dimension of the original variational problem of predicting the residual life of composites.

Statement. Let the functions $g_i(u_1, u_2, \ldots, u_n)$, $(i = 1, 2, \ldots, q)$ be continuous in their arguments, there exist and are continuous partial derivatives $\partial g_i/\partial u_j$, $(i = 1, 2, \ldots, q;$ $j = 1, 2, \ldots, n)$; $u^* = (u_1^*, u_2^*, \ldots, u_n^*)$ is the optimal solution to the original forecasting problem, and the condition

$$\begin{gathered} rang\Big[\nabla g_1(u^*), \nabla g_2(u^*), \ldots, \nabla g_q(u^*)\Big] = q, \\ \nabla g_i(u^*) = \Big(\partial g_i(u^*)/\partial u_1, \partial g_i(u^*)/\partial u_2, \ldots, \partial g_i(u^*)/\partial u_n\Big), \\ (i = 1, 2, \ldots, q). \end{gathered} \tag{6}$$

Then, $\tilde{u}_{j_l}$ $(l = q + 1, q + 2, \ldots, n)$ if there is an optimal solution of the generated auxiliary variational problem of dimension $r = n - q$, then the optimal solution of the original forecasting problem of dimension n is determined by the relations:

$$\begin{gathered} u_{j_i}^* = \varphi_i\left(\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_{j_1-1}, \tilde{u}_{j_1+1}, \ldots, \tilde{u}_{j_q-1}, \tilde{u}_{j_q+1}, \ldots, \tilde{u}_{n-1}, \tilde{u}_n\right), \\ (i = 1, 2, \ldots, q), \\ u_l^* = \tilde{u}_l, \quad l \neq j_i, \quad i = 1, 2, \ldots, q. \end{gathered} \tag{7}$$

The system of functional relations g_j $(j = 1, 2, \dots, q)$, appearing in the formulation of Statement 1, reflects the necessary and sufficient conditions for an extremum in the forecasting problem under consideration.

According to the developed methods, based on the use of necessary and sufficient extremum conditions for effective constructive transformation of variational statements of inverse problems of predicting the residual resource of composites to variational problems of lower dimension, comparative computational experiments were carried out, which showed the effectiveness of the developed approach associated with the gradual complication of forecasting models and the corresponding sequential refinement optimal parameters of the model.

Computational experiments have shown the high efficiency of the developed approach for constructing optimal forecasting models of optimal complexity based on immersing the original forecasting problem into a parametric family of problems, in which the parameter is the number of uncertain model parameters.

4 Influence of the retrospection interval dimensions on the stability of the forecast accuracy assessment when applying the optimal generalized forecasting models

The study of the effect of the size of the retrospection interval $[0, T_{\min}]$, where the exposure is carried out, on the stability of the forecast accuracy assessment on the predicted time interval $[T_{\min}, T_{\max}]$ when using the optimal generalized models of forecasting the optimal complexity has been carried out.

The general case was considered when polymeric and composite materials can be simultaneously affected in various combinations by several different factors Fj associated with hardening processes, exposure to solar radiation, moisture saturation, exposure to ultraviolet radiation, exposure to extreme climatic factors, etc. A generalized model was investigated that describes the simultaneous impact of several factors. Under the assumptions that various physical factors exert an effect on a complex-built composite that is independent of the effect of other factors, and the changes caused in the composite are summarized, the studied generalized model of durability can be represented as:

$$R = R_0 + \sum_{j=1}^{p} F_j(u_{j,1}, u_{j,2}, \dots, u_{j,l_j}; t). \tag{8}$$

Each of the functions $F_j(u_{j,1}, u_{j,2}, \dots, u_{j,l_j}; t)$, $(j = 1, 2, \dots, p)$ describing the effect of the j-th factor on a complexly constructed composite was presented in the form of a series expansion in a certain system of basis functions $\psi_{kj}(\beta_{kj}; t)$, $(k = 1, 2, 3, \dots)$, which most fully characterize the features of the process of increasing material damage under the influence of extreme environmental factors.

$$F_j = \sum_{k=1}^{\infty} \alpha_{kj}(u_{j,1}, \dots, u_{j,l_j})\, \psi_{kj}[\beta_{kj}(u_{j,1}, \dots, u_{j,l_j}); t], \quad (j = 1, 2, \dots, p). \tag{9}$$

In these designations: $\alpha_{kj}(u_{j,1}, \dots, u_{j.,l_j})$, $\beta_{kj}(u_{j,1}, \dots, u_{j.,l_j})$, $(j = 1, 2, \dots, p;\ k = 0, 1, 2, \dots)$ undefined parameters of the model describing the impact of the j-th factor. The system of Arrhenius functions $\psi_{kj}(\beta_{kj}; t)$, $(k = 1, 2, 3, \dots)$ was taken as a system of basis functions.

The construction of optimal generalized models of durability of optimal complexity was carried out on the basis of the developed approach based on the principle of multiplicity of forecasting models within the framework of refined variational statements of inverse forecasting problems.

The regularities of dependence of optimal models of durability of optimal complexity $R^*_\tau(t) = R(u^*_\tau; t)$ $(0 \le \tau \le T_{\max}, 0 \le t \le T_{\max})$, built on the basis of the developed approach, based on the principle of multiplicity of forecasting models, have been investigated. Within the framework of the refined variational formulations of inverse forecasting problems, a parametric family of forecasting problems was considered, in which the parameter is the current size of the retrospection interval τ $(0 \le \tau \le T_{\max})$. u^*_τ—the optimal vector of uncertain parameters, which determines the optimal model of the optimal complexity corresponding to the parameter τ.

The important regularities of the structure of optimal models of durability of optimal complexity when changing the size of the retrospection interval have been established. Areas of stability and instability are highlighted, a qualitative characteristic of these areas is carried out, their structural features are investigated the behavior of optimal models of durability in these areas, the boundaries of these areas are estimated.

Figure 1 shows a graph of the dependence of the parametric family of optimal forecasting models of optimal complexity $R^*(\tau_r; t)$, depending on the parameter τ_r—the size of the time interval of retrospection $(0 \le tr \le T_{\max})$. The strength measured in Mpa is considered as the residual resource R of the investigated PVC type KM. The studied interval for studying the change in the residual resource of KM is $T_{\max} = 15$ years.

It was found that at the retrospection intervals the size of which τ_r is insufficient to identify the fundamental regularities of the micro- and macrostructure of the CM, which determine the nature of the change in the residual CM resource for a long period, the dependence of the parametric family of optimal models for predicting the optimal complexity $R^*(\tau_r; t)$ depending on the parameter τ_r has an unstable oscillatory character, while the amplitude fluctuations decreases as the parameter τ_r increases. At retrospection intervals, the size of which τ_r is already sufficient to highlight the fundamental regularities of the micro- and macrostructure of CM, which determine the nature of the change in the residual CM resource for a long period, the dependence of the parametric family of optimal models for predicting the optimal complexity $R^*(\tau_r; t)$ depending on the parameter τ_r acquires a monotonous stable character and smoothly approaches the real dependence residual resource as it increases τ_r (figure 1).

For the studied CM, already after seven years of observations $(\tau_r = 7)$, the dependence of the parametric family of optimal models for predicting the optimal complexity $R^*(\tau_r; t)$, depending on the parameter τ_r, acquires a monotonous stable character and does not significantly differ from the actual dependence of the residual resource.

Thus, it has been established that the success of the effective solution of the problems of predicting the residual life of composites largely depends on how adequately it is possible to take into account in the models of durability the fundamental regularities inherent in the processes occurring both at the micro- and at the macrolevel, which have a significant effect on the change in the residual resource of the composite. In accordance with this, an objective assessment of the minimum size of the retrospection interval τ_r, at which these fundamental regularities are already manifested in full, sufficient to solve the problems of effective forecasting, becomes important.

At the same time, it was found that the dependence of the parametric family of optimal forecasting models of optimal complexity $R^*(\tau_r; t)$ depending on the parameter τ_r characterizing the size of the retrospection interval is oscillatory. In this case, the nature of the oscillatory process largely depends on whether the fundamental regularities that characterize the processes occurring at the micro and macro levels, which have a decisive effect on the change in the residual resource of the composite, are sufficiently manifested in the studied time interval.

The given types of stable and unstable oscillatory processes, to which the parametric families of optimal models for predicting the optimal complexity $R^*(\tau_r; t)$ when the parameter τ_r

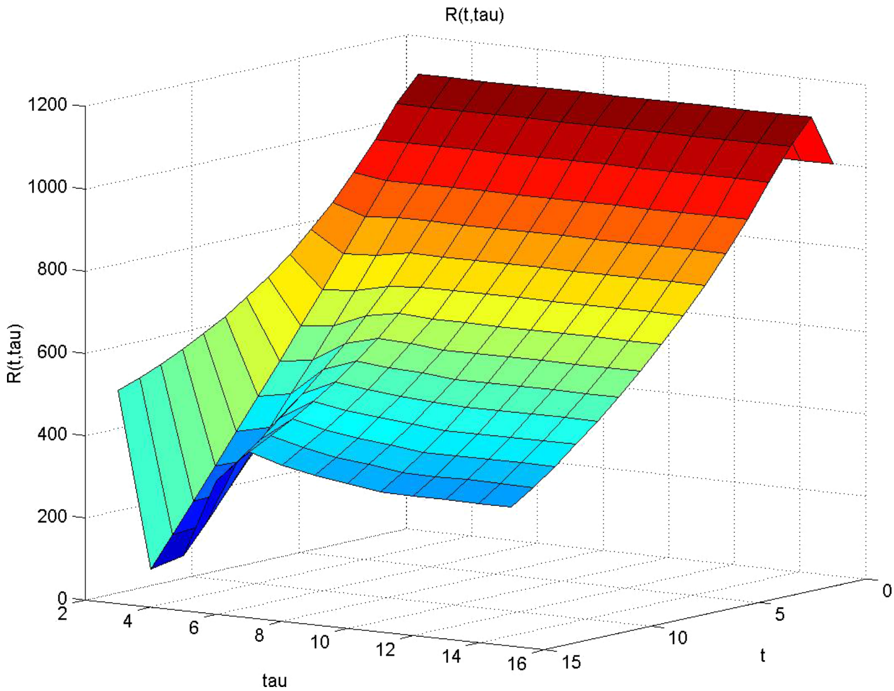

Figure 1. A graph of the dependence of the parametric family of optimal forecasting models of optimal complexity $R^{*}(\tau_r; t)$ depending on the parameter τ_r the size of the retrospection time interval $(0 \leq \tau_r \leq T_{\max})$

changes, are similar to oscillatory transient processes in stable and unstable automatic control systems (ACS).

5 Conclusion

Prospective ways of increasing the potentialities associated with solving the problems of predicting the residual resource, reliability, durability of polymer composites under the influence of extreme environmental factors are analyzed. A comparative analysis of existing approaches and their possible modifications made it possible to conclude that the most promising opportunities are associated with the development of a methodology for effectively narrowing search areas containing globally optimal solutions. The study of the influence of the size of the retrospection interval on the stability of the forecast accuracy estimate when using optimal generalized forecasting models of optimal complexity is carried out. Comparative computational experiments were carried out according to the developed methods based on the use of necessary and sufficient conditions for an extremum for an effective constructive transformation of variational statements of inverse problems of predicting the residual resource of composites to variational problems of lower dimension.

It is shown that the success of the effective solution of the problems of predicting the residual life of composites largely depends on how adequately it is possible to take into account in the models of durability the fundamental regularities inherent in the processes occurring both at the micro- and at the macro-level, and having a significant effect on the change in the residual resource of the composite. In accordance with this, an objective assessment of the minimum size of the retrospection interval at which these fundamental regularities are already manifested in full, sufficient for solving the problems of effective forecasting, becomes important.

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