MATEC Web of Conferences **362**, 01001 (2022) https://doi.org/10.1051/matecconf/202236201001
CMMASS 2021

Prediction of Earth polar motion in short time interval

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Abstract. The accuracy characteristics of mathematical models of the Earth pole motion in a short time intervals have been investigated. The modeling was carried out using two approaches: a numerical-analytical model of the Earth's rotational motion relative to the center of mass, based on the weighted least squares method and a neural network model for predicting the Earth pole motion. A comparison is made between the standard deviations of short-term forecasts of the Earth pole motion for 1–2 days according to the developed models and the standard deviations of forecasts published by the International Earth Rotation Service in Bulletin A. The models under consideration did not use data smoothing. The model based on neural network approach showed a higher accuracy of short-term forecasting.

1 Introduction

A lot of scientific papers are devoted to studying the Earth axial rotation irregularities and the prediction of the Earth polar motion. Recently improved methods for predicting Earth orientation parameters (EOP) have been proposed, both using classical approaches based on the least squares method, Kalman filter, spectral and wavelet analyzes, and using widely used neural networks [1, 2].

In papers [3–5], the fundamental astrometric problem of modeling the Earth's motion relative to its center of mass and constructing high-precision forecasts of the Earth pole trajectory and irregularities in the Earth axial rotation was investigated. On the basis of the developed mathematical models of the rotational-oscillatory motion of the deformable Earth relative to the center of mass [3–5] and the neural network approach, in this work a comparative analysis is carried out and the accuracy characteristics of the models of short-term (for 1–2 days) forecast of the Earth polar motion are studied in order to use them in applied problems of astrometry and navigation systems such as GLONASS/GPS.

The importance of the problem is due to the significantly increased accuracy of measurements and the lack of a rational approach when constructing models of the polar motion forecast for the intervals of various durations with the correspondingly required accuracies.

The Earth orientation parameters have an important role in navigation and control of spacecraft motion. Knowledge of the current values of the EOP (precession and nutation angles, angular coordinates of the Earth pole, dUT1 difference between UT1 and UTC) is necessary for the exact mutual transformation of the j2000 inertial coordinate system into

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the Greenwich coordinate systems WGS 84 and PZ 90-02, as well as accurate prediction of spacecraft orbits.

2 Forecast of a numerical-analytical model of the oscillatory process of the Earth pole

Numerical and analytical modeling of the Earth pole motion is based on the following structure of analytical expressions, that are convenient for numerical calculations [6, 7]:

$$x_p = c_x - a_x^c \cos N^* t + a_x^s \sin N^* t - d_x^c \cos \nu_h t - d_x^s \sin \nu_h t,$$
$$y_p = c_y + a_y^c \cos N^* t + a_y^s \sin N^* t - d_y^c \cos \nu_h t + d_y^s \sin \nu_h t.$$

Here c_x, c_y are coordinates of the mean Earth pole point; N^*, ν_h are the frequencies of the Chandler wobble and annual wobble, respectively. The values of the average Chandler and annual frequencies were selected on the basis of a spectral analysis of a number of observations of Earth pole coordinates in the time interval from 1970 to 2006. N^* is assumed to be 0.843 cycles per year, and the frequency ν_h is equal to one cycle per year. When determining the coefficients $a_x^{c,s}$, $a_y^{s,c}$, $d_x^{c,s}$, $d_y^{s,c}$ we should keep in mind the equalities $a_x^{c,s} \approx a_y^{s,c}$, $d_x^{c,s} \approx d_y^{s,c}$, that are the structural feature of the model [8]. This also means that the processes x_p and y_p are related.

For a situation that corresponds to the modern data of the International Earth Rotation Service (IERS), a short-term forecast can be given using a 5–6 parametric adaptive model, by reducing the number of processed daily measurements. This is due to a decrease in the dynamic error of approximation of the process and high measurement accuracy. The procedure for reducing the interpolation interval in the algorithmic implementation of the model (model adaptation procedure) and its impact on the forecast were evaluated in the algorithm cycle on the test interval immediately adjacent to the forecasting interval.

Table 1. Average standard deviations of the forecasts of the coordinates of the earth's pole of the developed model and the model of the IERS

Interval, day	σ_x, mas	σ_x^{IERS}, mas	σ_y, mas	σ_y^{IERS}, mas	σ_{xy}, mas	σ_{xy}^{IERS}, mas
1	0.331	0.308	0.239	0.242	0.408	0.392
2	0.793	0.488	0.396	0.362	0.886	0.608

Numerical modeling shows that the highest interpolation accuracy is usually achieved in the middle part of the interval. This property is one of the features of the least-squares method and is well known [9, 10]. In particular, theoretically it can be established in the case of polynomial filtration, i.e. a set of support functions in the form of polynomials (usually of a low degree) of the temporal smoothing parameter (for short-term forecasting). Therefore, an increase in the forecast accuracy for a short interval adjacent to the end of the interpolation interval can be achieved by introducing "weight" coefficients in the algorithm of the least-squares method and their relative increase towards the end of the interval. The filtering algorithm can contain a small number of parameters that allow to adjust the specified weighting factors and optimize the algorithm, depending on the previous results of interpolation and forecast.

To assess the accuracy of the developed model, the time interval from 53773 to 57060 of the MJD date was chosen. The filtering procedure and tuning of the algorithm by the "weighted" least-squares method for the first forecast for the time interval from 53773 MJD date was carried out on the adjacent interpolation interval. The standard deviations of the forecast for 1–2 days from the IERS observations were referred to the date of the forecast. Forecasts were calculated with a step equal to a day. The end date corresponding to the last constructed forecast was 57000 MJD date. For comparison, the forecasts of the IERS were taken, published in "Bulletin A", the standard deviations of which were calculated in a similar way for 1–2 days. Table 1 shows the average deviations of 1–2 daily forecasts of the coordinates of the earth's pole and published forecasts of the IERS for the entire seven-year forecasting interval.

3 Neural Network approach in simulation of the Earth pole oscillations

Artificial neural network (ANN) is a mathematical model built on the principle of organization and functioning of biological neural networks—networks of nerve cells of a living organism. The ANN model is built on the basis of interconnected and interacting nodes—neurons, which form a structured network among themselves. Each of these neuron nodes can receive and process both input data and values generated by other network nodes. Based on the received input, each neuron, taking into account the weighting coefficients of the connections, forms a weighted average sum using it as an argument of the activation function. It allows to generate output values for each neural node. Such output values can serve as input data for other neural network nodes, or form the resulting values, the so-called network output.

A network based on a multilayer perceptron with end-to-end connections from the input layer to the hidden layers was used for modeling. This allows a more accurately simulation

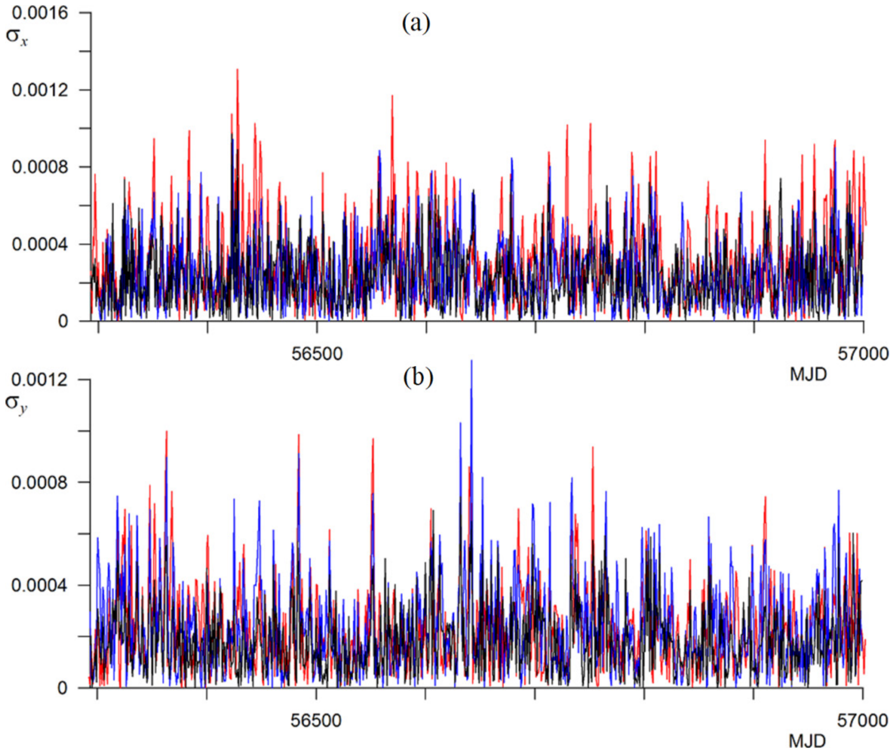

Figure 1. The absolute value of the deviation of the daily forecasts of the pole coordinates x_p (a), y_p (b) according to the developed neural network model (black), numerical-analytical model (red) and IERS forecasts (blue)

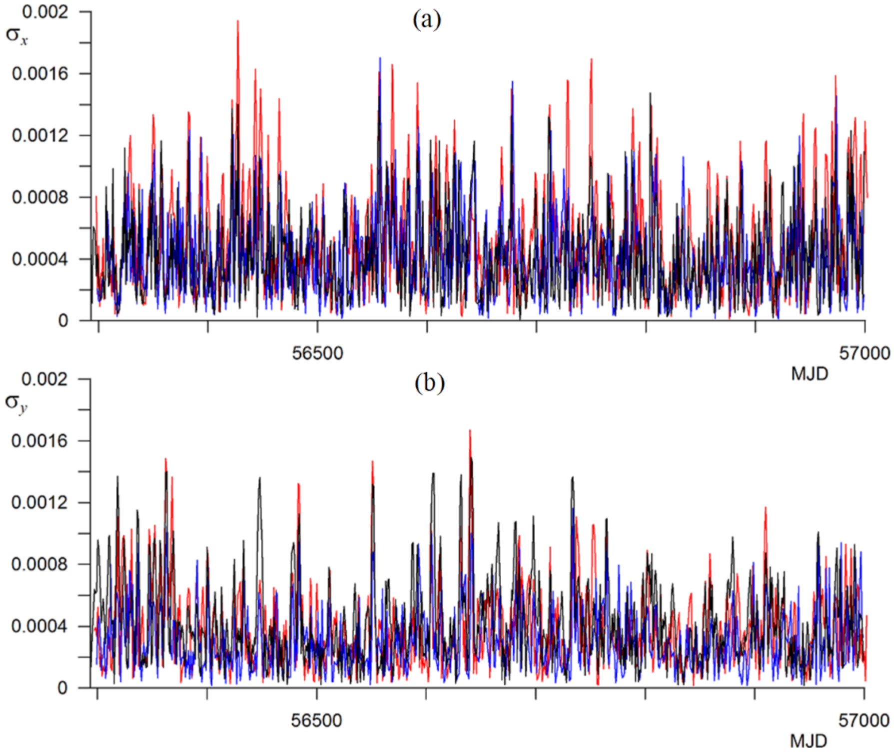

Figure 2. Standard deviations of 2-day forecasts of the pole coordinates x_p (a), y_p (b) according to the developed neural network model (black), numerical-analytical model (red) and IERS forecasts (blue)

Table 2. Average standard deviations of the forecasts of the coordinates of the earth's pole of the developed model and the model of the IERS

Interval, day	σ_x, mas	σ_x^{IERS}, mas	σ_y, mas	σ_y^{IERS}, mas	σ_{xy}, mas	σ_{xy}^{IERS}, mas
1	0.230	0.253	0.182	0.182	0.293	0.312
2	0.418	0.421	0.319	0.300	0.526	0.516

of the linear parts of functions calculated by hidden layers. In addition, a pyramidal structure of hidden layers was used. The first of which is slightly larger or comparable in the number of neurons with the number of inputs, and the subsequent ones decrease within 30–40% of the number of neurons in the previous layer.This makes it possible to achieve a more accurate and smoother result compared to the classic two-layer perceptron with one hidden layer, comparable in the number of neurons and neural connections.

As input data, the values of x_p and y_p were taken for 11 days immediately before the forecast and 31 days before this period with a step of 17 days to take into account the periodic nature of the input data (82 values in total). The result of the network operation is the forecast of the Earth pole coordinates for one and two days ahead from of the current value.

Since it is classical multilayer perceptrons with a sigmoidal activation function, the input data must be in the range from -1 to $+1$, so the data must be scaled before being processed by the network. In the case of direct use of the values of x_p and y_p, the scaling factor was about 600, which leads to the need to train the network with an accuracy of the order of 10^{-4} to achieve an accuracy comparable to polynomial modeling.

As one of the options for solving the problem of large input data scaling factors and, consequently, increased requirements for the accuracy of network training, we can use not the Earth pole coordinates themselves, but the differences between the current and previous values, which make it possible to reduce the scaling factor to 6. To restore the immediate values of the predicted coordinates, it is enough to add the results obtained to the current coordinates x_p and y_p. With such a data presentation, the features associated with high-frequency oscillations in the Earth pole coordinates become more noticeable.

The period from 1962 (the beginning of observations) to the end of 2012 (about 18 thousand values in total) was taken as a training sample. And the data for 2013 and 2014 were used as a test sample. Table 2 shows the average deviations of 1–2 daily forecasts of the coordinates of the Earth pole and published forecasts of the IERS for the test forecasting interval (2013–2014).

In figures 1 and 2 shows a comparison of the absolute values of the deviations of the first day of the forecast from the observed and standard deviations of 2-day and 3-day (respectively) forecasts of the developed neural network model (black), numerical-analytical model (red) and IERS model (blue).

4 Conclusion

The analysis of the numerical modeling results of the Earth pole oscillations based on the developed methods and algorithms, that was carried out in this work, shows the adequacy of the mathematical models of the Earth pole motion to the IERS observations and measurements data. The developed models, in terms of the approximation accuracy of the process, provide good agreement with the IERS data for short (1–2 days) time intervals. Comparative analysis with existing analogs shows that the obtained numerical results of the forecasting the Earth pole coordinates are fundamentally comparable with them.

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