Prediction of the macroscopic behavior of multi-layered elastic composites

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Abstract. In this work, elastic fiber reinforced composite materials consisting of several layers are considered. Each layer is assumed to be a unidirectional fiber reinforced composite. The interface between layers is assumed to be perfect, which means the displacement and the traction through interfaces are continuous. To predict the overall behavior of the considered composite a two step homogenization procedure is considered. In the first step, effective properties of fiber reinforced layers are predicted based on the Mori-Tanaka micromechanical model and in the second one, a general matrix method is constructed to obtain the macroscopic properties of multi-layered composites. Analytical expressions of effective properties, for reinforced composites constituted with arbitrary N layers, are derived. The combination of the Mori-Tanaka method with the general matrix method allowed obtaining the effective properties of reinforced elastic multi-layered composites.

1 Introduction

Composites materials are widely used in many engineering applications and industrial fields due to their unique properties compared to normal materials: light weight, resistance to damage, thermal resistance etc. The choice of the constituents' materials and their geometries allows one to deign composites with specific properties. Several methods are developed to compute effective properties taking into account the microscopic behavior and the morphological description of components.

Multilayered composites are very used especially in space craft production and many researchers work to develop mathematical models to analyze and predict the behavior of multilayered composites. Kim [1] developed a matrix method to predict the effective behavior of anisotropic multilayered composites. Kim and Rokhlin [2] determined elastic constants of anisotropic multilayered composites using acoustic microscopy measurements**.** Wang et al. [3] extended the matrix method to investigate the macroscopic behavior of multi-layered functionally graded multiferroic composites.

In this paper, a two steps homogenization method is used to estimate the effective behavior of multilayered composites where each layer is reinforced with short aligned fibers. First, the elastic moduli are determined for the multilayered composite without considering reinforcements. Then, the Mori-Tanaka mean field method is used to estimate the effective behavior of each reinforced lamina (layer). The obtained elastic moduli are injected into the derived expressions of multilayered composites effective properties.

2 Macroscopic properties of multi-layered elastic composites

The linear elastic behavior of a homogeneous multi-layered material is described by

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{1}$$

where σ_{ij} and ε_{kl} are constituents of the stress and strain tensors and C_{ijkl} are the elastic constants.

The following vector notations are considered:

$$\sigma = \begin{pmatrix} \sigma_n \\ \sigma_t \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_n \\ \varepsilon_t \end{pmatrix}, \quad C = \begin{pmatrix} C_{nn} & C_{nt} \\ C_{tn} & C_{nn} \end{pmatrix} \tag{2}$$

with subscripts n and t refer respectively to out of plane and in-plane stresses and strains. These quantities are explicitly expressed by

$$\sigma_n = \begin{pmatrix} \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \end{pmatrix}, \sigma_t = \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{pmatrix}, \varepsilon_n = \begin{pmatrix} \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \end{pmatrix}, \varepsilon_t = \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{pmatrix} \tag{3}$$

Equation (1) is reformulated in the following matrix vector-matrix form:

$$\sigma_n = C_{nn}\varepsilon_n + C_{nt}\varepsilon_t, \quad \sigma_t = C_{tn}\varepsilon_n + C_{tt}\varepsilon_t \tag{4}$$

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where C_{nn}, C_{nt}, C_{tn} and C_{tt} are the generalized Voigt matrices given by

$$C_{nn}=\begin{bmatrix} C_{33} & C_{34} & C_{35} \\ C_{34} & C_{44} & C_{45} \\ C_{35} & C_{45} & C_{55} \end{bmatrix},\ C_{nt}=\begin{bmatrix} C_{13} & C_{23} & C_{36} \\ C_{14} & C_{24} & C_{46} \\ C_{15} & C_{25} & C_{56} \end{bmatrix}$$
$$C_{tn}=\begin{bmatrix} C_{13} & C_{14} & C_{15} \\ C_{23} & C_{24} & C_{25} \\ C_{36} & C_{46} & C_{56} \end{bmatrix},\ C_{tt}=\begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{bmatrix} \quad (5)$$

with

$$C_{nn}=C_{nn}^{T},\ C_{tn}=C_{nt}^{T},\ C_{tt}=C_{tt}^{T} \quad (6)$$

where T denotes matrix transpose.

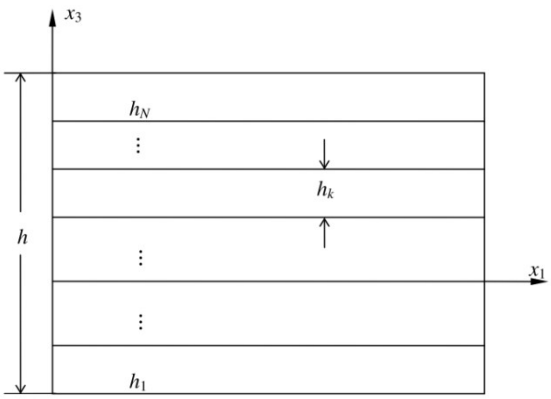

Fig. 1. Multi-layered elastic composites.

Now, an elastic multi-layered composite of N homogeneous layers of elastic materials is considered (see Fig. 1). In the following, the super-script (k) is associated to the kth layer where ($k=1,2,...,N$). Then, the constitutive behavior of the kth elastic layer, is given by

$$\sigma_n^{(k)}=C_{nn}^{(k)}\varepsilon_n^{(k)}+C_{nt}^{(k)}\varepsilon_t^{(k)},\ \sigma_t^{(k)}=C_{tn}^{(k)}\varepsilon_n^{(k)}+C_{tt}^{(k)}\varepsilon_t^{(k)} \quad (7)$$

It is assumed that the multilayered composite is subjected to a uniform state of strain, even though the real composite may be subjected to arbitrary loading. Based on this assumption, one can write:

$$\sigma_n^{(k)}=\sigma_n,\ \varepsilon_t^{(k)}=\varepsilon_t \quad for\ k=1,2,...,N \quad (8)$$

Taking the average of $\varepsilon_n^{(k)}$ over the volume of the composite and using Eqs. (7) and (8), one obtains:

$$\varepsilon_n=\sum_{k=1}^{N}f_k\varepsilon_n^{(k)}=\sum_{k=1}^{N}f_k\left(C_{nn}^{(k)}\right)^{-1}\left(\sigma_n-C_{nt}^{(k)}\right)\varepsilon_t \quad (9)$$

in which $f_k=h_k/h$ represents the volume fraction of the kth layer.

The compact form of Eq. (9), is given by

$$\sigma_n=C_{nn}^{*}\varepsilon_n+C_{nt}^{*}\varepsilon_t \quad (10)$$

with

$$C_{nn}^{*}=\left[\sum_{k=1}^{N}f_k\left(C_{nn}^{(k)}\right)^{-1}\right]^{-1} \quad (11)$$

$$C_{nt}^{*}=C_{nn}^{*}\left[\sum_{k=1}^{N}f_k\left(C_{nn}^{(k)}\right)^{-1}C_{nt}^{(k)}\right] \quad (12)$$

Averaging $\sigma_t^{(k)}$ over the volume of the composite, we then obtain

$$\sigma_t=\sum_{k=1}^{N}f_k\sigma_t^{(k)} \quad (13)$$

which can be explicitly, after some mathematical operation, written as

$$\sigma_t=\sum_{k=1}^{N}f_kC_{tn}^{(k)}\left(C_{nn}^{(k)}\right)^{-1}\sigma_n+\sum_{k=1}^{N}f_k\left[C_{tt}^{(k)}-C_{tn}^{(k)}\left(C_{nn}^{(k)}\right)^{-1}C_{nt}^{(k)}\right]\varepsilon_t \quad (14)$$

The condensed form of σ_t is given by

$$\sigma_t=C_{tn}^{*}\varepsilon_n+C_{tt}^{*}\varepsilon_t \quad (15)$$

where

$$C_{tn}^{*}=\left[\sum_{k=1}^{N}f_k\left(C_{nn}^{(k)}\right)^{-1}C_{tn}^{(k)}\right]C_{nn}^{*} \quad (16)$$

$$C_{tt}^{*}=\sum_{k=1}^{N}f_k\left[C_{tt}^{(k)}+C_{tn}^{(k)}\left(C_{nn}^{(k)}\right)^{-1}\left(C_{nt}^{*}-C_{nt}^{(k)}\right)\right] \quad (17)$$

The effective properties of the considered multi-layered composite is fully calculated by Eq. (11-12) and (16-17). In the next section, the Mori-Tanaka approach which further allows considering reinforcements in each layer will be considered.

3 Mori-Tanaka method for multi-layered composites

The Mori-Tanaka method is very popular among the existing micromechanics methods due to its effectiveness: it is easy to implement, there is no need to iterative procedures, and is known to give good estimation of effective properties of the considered composites (see references [4-7] for more readings). The Mori-Tanaka is used here to predict the behavior of reinforced layers. Each layer is constituted of a matrix reinforced with aligned fibers (inclusions) in direction (2).

The averaged strain ε_{mn}^{I}, in the inclusion, is related to the applied macroscopic strain ε_{kl} through the Mori-Tanaka localization tensor by

$$\varepsilon_{mn}^{I}=\left[I_{mnkl}+(f_M/V_I)T_{mnij}^{II}\Delta L_{ijkl}^{I}\right]^{-1}\varepsilon_{kl} \quad (18)$$

with $\Delta L_{ijkl}^{I}=L_{ijkl}^{I}-L_{ijkl}^{M}$ and $f_M=1-f_I$, and where f_I is the inclusion volume fraction, V_I the volume of inclusion, L^I and L^M are respectively the elastic constants of the

inclusion and matrix, I is the fourth order identity tensor and T^{II} is the interaction tensor.

Denoting

$$A_{mnkl}^{MT,I} = \left[I_{mnkl} + (f_M / V_I) T_{mnij}^{II} \Delta L_{ijkl}^{I} \right]^{-1} \quad (19)$$

the effective moduli L^* of reinforced layers are obtained by

$$L^* = L^M + f_I (L^M - L^I) A^{MT,I} \quad (20)$$

The obtained effective behavior allows one to numerically predict the effective elastic properties of the reinforced layer for a wide range of reinforcement's geometry. In this work, reinforcements are considered to be fibers aligned in the x_2 direction.

4 Numerical results

Numerical results are presented for a composite constituted of three layers. The first layer and the third one are considered to be made up of an epoxy material reinforced with carbon fibers aligned in the x_2 direction. The second layer is made up of an Epoxy material. The thicknesses of layers 1, 2 and 3 are $h_1 = h_3 = 0.44\,mm$ and $h_2 = 0.28\,mm$. The elastic coefficients of the used materials are listed in Table 1. Reinforcements in the modeling is taken into account as ellipsoids of half axes a, b and c that are aligned with the global coordinates. So, to obtain fibers in the x_2 direction ellipsoids half axes are given the following values: $a = 1, b = 1000, c = 1$.

Table 1. Material properties of the used materials constituting the multi-layered composite.

Parameters	Carbon fiber	Epoxy matrix
Young's modulus (GPa)	227	3
Poisson's ratio	0.30	0.40

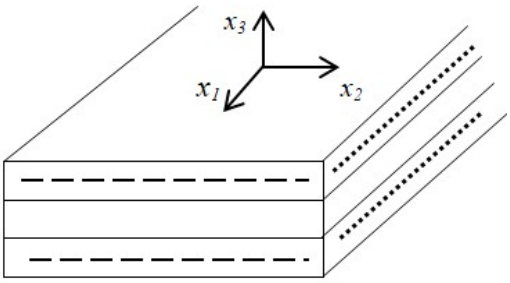

Fig. 2. Multi-layered composite where layers 1 and 3 are reinforced by short carbon fiber aligned in the x_2 direction.

As a first step, the Mori-Tanaka method is used to estimate the effective properties of the first and the third layers. Figs. 3a, 4a and 5a present the predicted elastic constants with respect to the volume fraction of the epoxy material.

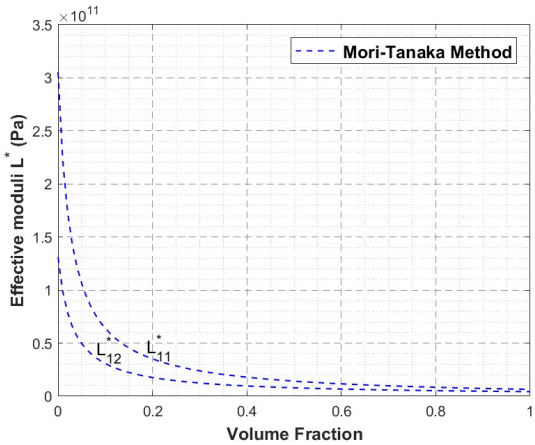

Fig. 3a. Predicted effective moduli L_{11}^* and L_{12}^*, based on the Mori-Tanaka method for the considered layered composite consisting of carbon fibers $(a = 1, b = 1000, c = 1)$ embedded in an epoxy matrix, with respect to the volume fraction of the epoxy matrix (f_M).

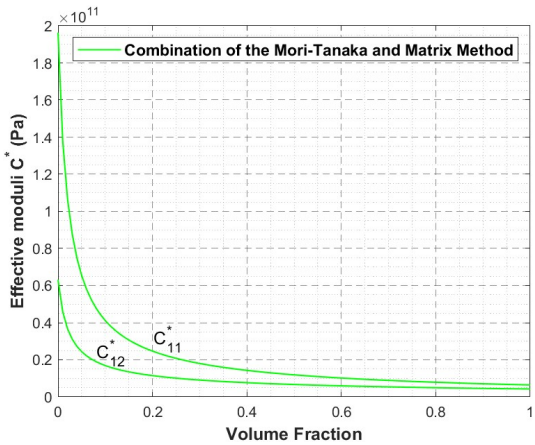

Fig. 3b. Predicted effective moduli C_{11}^* and C_{12}^* based on the combined homogenization method for the considered multi-layers composite with respect to the volume fraction of the epoxy material in layers 1 and 3.

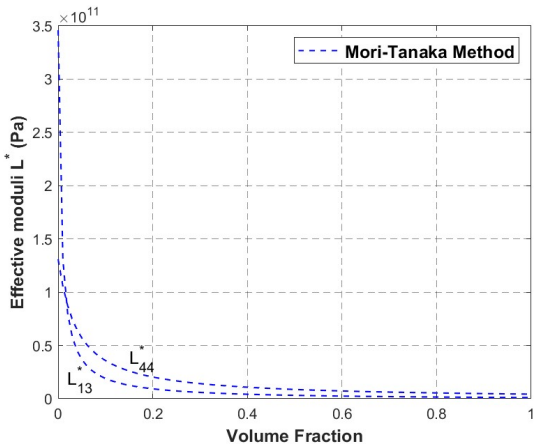

Fig. 4a. Predicted effective moduli L_{13}^* and L_{44}^*, based on the Mori-Tanaka method for the considered layer consisting of carbon fibrous $(a = 1, b = 1000, c = 1)$ embedded in an epoxy matrix, with respect to the volume fraction of the epoxy matrix (f_M).

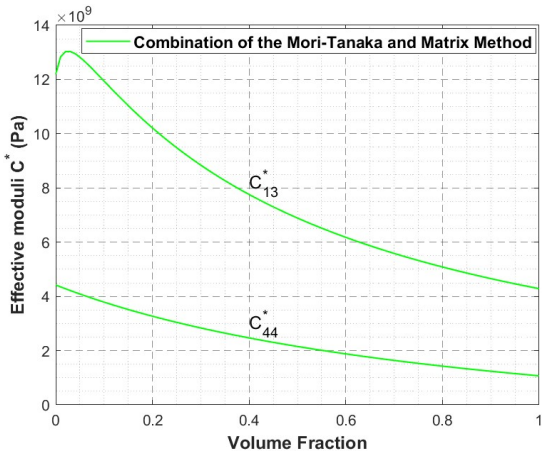

Fig. 4b. Predicted effective moduli C_{13}^{*} and C_{44}^{*} based on the combined homogenization method for the considered multi-layers composite with respect to the volume fraction of the epoxy material in layers 1 and 3.

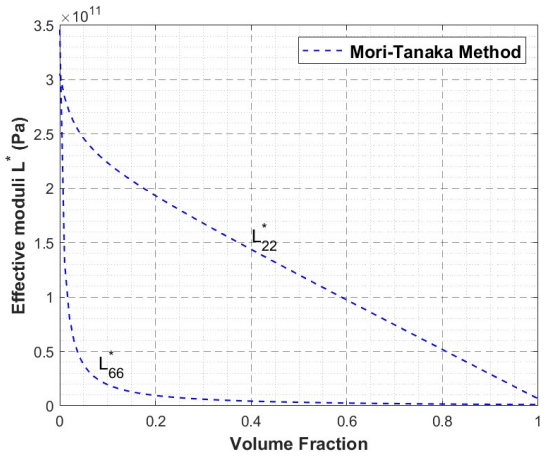

Fig. 5a. Predicted effective moduli L_{22}^{*} and L_{66}^{*}, based on the Mori-Tanaka method for the considered layer consisting of carbon fibrous $((a=1, b=1000, c=1)$ embedded in an epoxy matrix, with respect to the volume fraction of the epoxy matrix (f_M).

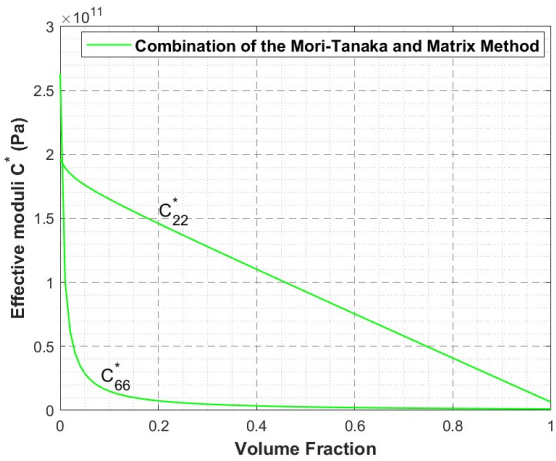

Fig. 5b. Predicted effective moduli C_{22}^{*} and C_{66}^{*} based on the combined homogenization method for the considered multi-layers composite with respect to the volume fraction of the epoxy material in layers 1 and 3.

Then, in the second step, prediction of the behavior of the considered multi-layered composite is performed based on the homogenization procedure combining the Mori-Tanaka method with the matrix one. Figs. 3b, 4b and 5b present the elastic coefficients of the multi-layered composite with respect to the volume fraction of epoxy material in layers 1 and 3. When the volume fraction equals 0 layers one and three are constituted of carbon material and when the volume fraction equals 1, layers one and three become epoxy.

5 Conclusions

In this work, a homogenization procedure combining the Mori-Tanaka mean field approach for each reinforced layer with the matrix homogenization method for multi-layered composites was developed. It allows the estimation of multi-layered reinforced composites effective properties. Numerical results were presented for a three layers reinforced composite. The first and third layers are constituted with an epoxy matrix reinforced with carbon fibers and the second layer is considered to be epoxy. The numerical results were presented as effective properties for the reinforced layers using the Mori Tanaka model and for the whole three layers reinforced composite using the combined homogenization procedure.

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