

Nonlinear error control of five-axis machining singular region

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Abstract. Aiming at the singular problem of large nonlinear error caused by the extreme change of rotation in five axis NC machining of complex surface. An optimization algorithm of tool path in singular region is proposed. Taking a-c double turntable five axis linkage NC machine tool as an example, firstly, the mathematical models of tool local milling ability, machining bandwidth and tool axis inclination are established. Based on the two constraint models, the mechanism of singularity and the size of singular region are analyzed, and it is proposed to adjust the side angle with the tool contact as the rotation center to avoid the singular region. The modified tool axis vector that does not meet the accuracy is interpolated recursively according to the interpolation principle. This method solves the problem of large nonlinear error caused by the extreme rotation of the rotating axis in the singular region on the basis of meeting the requirement of no curvature interference. At the same time, the processing bandwidth before and after modification is guaranteed to remain unchanged. Simulation results show that the algorithm can effectively detect the singular region and improve the machining quality in the singular region.

1. Introduction

The five-axis machine tool increases two degrees of freedom in the three-axis machine tool, which makes the tool can free cut the workpiece with no attitude, greatly increasing the surface removal rate of the workpiece material and improving the processing quality of the workpiece. However, due to the introduction of rotation axis, it brings special processing problems. Singularity (singular region) is one of the most important problems. In the singular region, the rotation axis will produce discontinuous speed rotation, will produce large nonlinear errors, and even interference, which will cause great damage to the workpiece itself and the machine tool components. Therefore, it is very important to detect the singularity and process the tool position data for improving the machining accuracy of workpiece and protecting the machine tool.

Affouard used polynomial interpolation to modify the tool path in the singular region, but the interpolation algorithm was complex and required a lot of calculation [1]. The method of Munlin optimized the rotation axis in the singular region, but failed to fundamentally solve the problem [2]. Wang Feng combined the Jacobian matrix with the variation of each axis to identify the singular region by conditional number, but it is difficult to determine the conditional number accurately in a quantitative way [3-5]. Yu Xiaosui proposed to solve the singular problem by adding one more rotating axis, but the machine tool mechanism was complicated and its use was very limited [6].

Based on the above issues, in local can establish cutting tools and milling and processing bandwidth and knife shaft Angle of singular area on the basis of mathematical model of cutter axis vector is optimized, and then five axis machine tool of the modified files for precision detection, did not meet the requirements of precision cutter location data for recursive interpolation, until meet the accuracy requirement.

2. Local millability conditions for five-axis machining

In tool path planning of 5-axis machining, global interference, local interference and tool bottom interference must be taken into account. For local interference, it is necessary to establish the local coordinate system at the tool contact point, and establish the geometric information function relation between the tool and the workpiece in the local coordinate system, and judge whether the local interference occurs by the difference of the two functions. The specific local coordinate system establishment method is as follows: Take the cutting contact as the coordinate origin, the cutting direction F as X-axis, the normal vector of the curved surface cutting contact as z-axis, and use the right hand rule to determine the Y-axis direction. In this coordinate system, the cutter axis vector is represented by the heel deflection Angle, namely $T(\lambda, \omega)$.

Take ring cutter as the example in Fig 1.1 below, establish tool surface function $z=f(x,y)$ and workpiece surface function $z=h(x,y)$, then the difference function is equal to

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$z = f(x,y) - z = h(x,y)$, when the Dupont indication line graph of the difference function at the cutter contact is elliptical, no local interference occurs, and the cutter meets the local millability at the cutting contact. k_{max} and k_{min} represent the maximum and minimum principal curvatures of the cutter surface at the cutting contact, L_{max} and L_{min} represent the maximum and minimum principal curvatures of the workpiece surface at the cutting contact, θ is the included Angle of the maximum principal directions of the two surfaces, and h is the residual height. Then, the Dupont indicator line of the difference function is.

$$\begin{aligned} & (L_{max} \cos^2 \theta + k_{min} \sin^2 \theta - L_{max})x^2 \\ & + 2(k_{max} - k_{min}) \sin \theta \cos \theta xy + \\ & (k_{max} \sin^2 \theta + k_{min} \cos^2 \theta - L_{min})y^2 = 2h \end{aligned} \quad (1)$$

When

$$\begin{aligned} & k_{max} + k_{min} - L_{max} - L_{min} > 0 \\ & -(k_{max} - L_{max})(k_{min} - L_{min}) \\ & + \sin^2 \theta (k_{max} - k_{min})(L_{max} - L_{min}) > 0 \end{aligned} \quad (2)$$

There is no local interference in the tool contact. Based on the knowledge of differential geometry, the maximum principal curvature of the cutter surface at the cutting contact is:

$$k_{max} = \frac{1}{a} \quad (3)$$

The minimum principal curvature is:

$$\lambda = \arcsin \frac{\beta}{1 - \alpha L_{max}} \quad (4)$$

When the minimum curvature of the tool surface is equal to the maximum principal curvature of the workpiece surface, the material removal rate is the highest and the machining efficiency is the best. Which $k_{min} = L_{max}$, then

$$\frac{\sin \lambda}{\beta + \alpha \sin \lambda} = L_{max} \quad (5)$$

According to the above non-interference conditions, local interference will not occur regardless of the sideslip Angle. In order to facilitate the calculation of subsequent processing bandwidth, let

$$\begin{aligned} a &= L_{max} \cos^2 \theta + k_{min} \sin^2 \theta - L_{max} \\ b &= (k_{max} - k_{min}) \sin \theta \cos \theta \\ c &= k_{max} \sin^2 \theta + k_{min} \cos^2 \theta \end{aligned} \quad (6)$$

If the included Angle between the cutting direction and the cartesian coordinate system axis of the tangent plane is, then the processing bandwidth is:

$$W = 2 \sqrt{\frac{2h(a \cos^2 \phi + 2b \sin \phi \cos \phi + c \sin^2 \phi)}{-b^2 + ac}} \quad (7)$$

On the basis of the above calculation, the relation between machining bandwidth W and cutter inclination Angle and heel can be obtained.

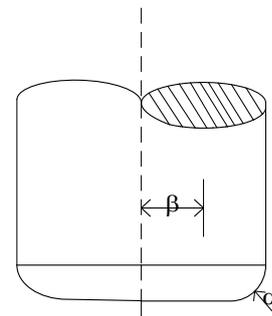


Fig.1 Schematic diagram of ring knife

3. Singularity mechanism analysis of five-axis machining

3.1 Singularity problems in five-axis machining

When i and j in the tool axis vector are both 0, that is, the tool axis vector is $(0 \ 0 \ 1)$, after the post-processing of the machine tool, the degree of freedom of the C-axis is lost, and the tool point at this time is defined as singular point. The corresponding cutter axis is singular axis. In fact, when the included Angle between the tool vector and the singular axis is less than a specific value, the C-axis Angle between the two adjacent tool points may change dramatically, resulting in extreme nonlinear errors and resulting in singular problems. The region swept by the tool axis vector that is less than the Angle is called singular region, as shown in Fig 2 below.

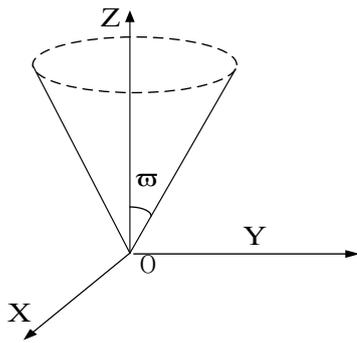


Fig.2 Singular area

3.2 The method of calculating the size of singular region

When the cutter axis vector is (0,0,1), it is the singular point. When the included Angle between the cutter axis vector and the singular axis is very small, the swept conical region is the singular region, as shown in the figure above. Assuming that the cutting depth of the tool is H and the radius of the bottom circle of the vertebral body is R, the bottom circle of the singular cone can be calculated as follows:

$$\left\{ \begin{array}{l} \|T\| = i^2 + j^2 + k^2 = 1 \\ \tan \lambda = \sqrt{\frac{i^2 + j^2}{k^2}} \\ k = \sqrt{\frac{1}{1 + \tan^2 \lambda}} \\ R = H \tan \lambda \end{array} \right. \quad (8)$$

K value can be traversed through the calendar, so that the corresponding knife point can be judged. The calculated R value represents the size of the singular circle projected by the singular cone.

3.3 Adjusting strategy of cutter axis vector avoidance in singular region

Based on the above constraints to avoid interference and the relationship between machining bandwidth and tool axis inclination Angle, a strategy was proposed to avoid the singular region by adjusting the sideslip Angle with the tool contact as the rotation center. The specific scheme is as follows: The tool axis vector at each tool contact point on the tool path is projected onto the oxy horizontal plane, and the space vector is planized. For any tool axis vector P(px, py, pz), multiply each coordinate component of the tool axis vector P by 1/ pz to obtain the processed P'(px/pz, py/pz, 1). The Z coordinate is constant, so the z coordinate can be ignored, so the processed projection tool axis vector is P'(px/pz, py/pz).

According to the range of singularity circle projected by singular cone, the cutter position data was taken to judge

whether the cutter axis vector entered the singular circle on the bottom surface of singular cone, and the cutter axis vector between the last projection of the cutter axis vector C1 before entering the singular circle and the first one after opening the singular circle C2 was detected.

According to the above constraints of local millability and the function relationship between machining bandwidth and tool axis inclination Angle, when the tool axis vector is adjusted to avoid the singular region, no interference phenomenon must be ensured for each tool contact in the singular region, and the machining bandwidth remains unchanged before and after adjustment. According to the above, when the minimum curvature of the tool surface is equal to the maximum curvature of the workpiece surface at the tool contact point, the meshing effect between the tool and the workpiece is the best, and the machining bandwidth is the maximum. At the same time, no interference occurs when the sideslip Angle is arbitrarily adjusted under the tool attitude. The machining bandwidth and sideslip Angle are symmetric according to the function of machining bandwidth and tool axis inclination Angle. According to the transformation relation of adjacent cutter axis vectors in space, the cutter axis vectors detected in the singular circle can be adjusted to the singular region, so that the adjusted cutter axis vector and the cutter axis vector before adjustment are in a straight line at the center of the singular circle, so that the singular region can be avoided by adjusting the sideslip Angle. At the same time, the adjusted distance G keeps the processing bandwidth unchanged before and after adjustment according to the relationship between the cutter axis vector and the cutter axis pose Angle, as shown in the figure below P1 to P6 are the tool points in the detected singular region.

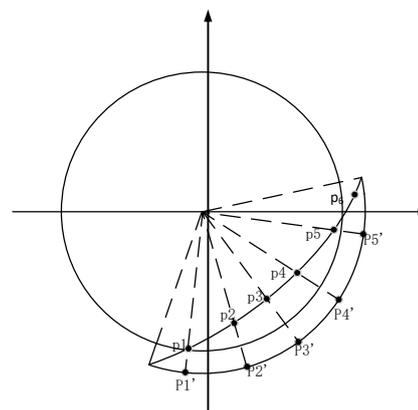


Fig.3 Schematic diagram of tool axis vector avoidance

According to the insertion point, judge whether the adjusted cutter axis vector in the singular region meets the error requirements, and perform recursive interpolation for those that do not meet the error requirements. Insert the processing point according to the Angle of the tool axis vector to determine. The basis for inserting the processing point is:

$$\left| \frac{f(s) - f(m)}{f(s)} \right| \leq \epsilon_f \quad (9)$$

$f(s)$ is the current tool axis vector, $f(m)$ is the adjacent tool axis vector, ϵ_f is the given Angle error.

4. Simulation verification and experimental data analysis

By MATLAB tool path error analysis can be properly through the singular region, the sharp increase of the error is far beyond the limit of the error value. The maximum error value is as high as 0.745mm, as shown in Fig 4 below. Meanwhile, post-processing of the compiled tool position data shows that from tool point 1 to tool point 6, although the variation range of tool axis vector is small, the corresponding change Angle of AXIS A and C is large, as shown in Fig 6. The change of Angle C is 66.7° , and that of axis A is 55.7° . This change will cause the machine tool to produce setbacks, damage the machine tool, for the workpiece processing, it is easy to occur curvature interference, and occur too large nonlinear errors, causing singular problems. Therefore, the tool axis vector should be adjusted in time in tool path planning to avoid the occurrence of strange phenomena.

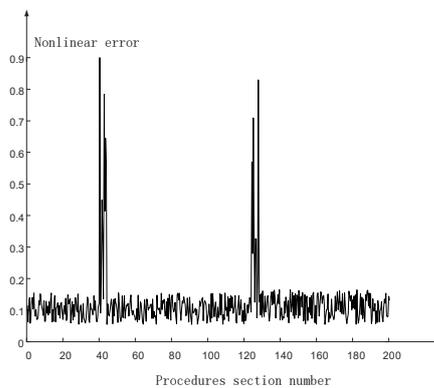


Fig.4 Before optimization

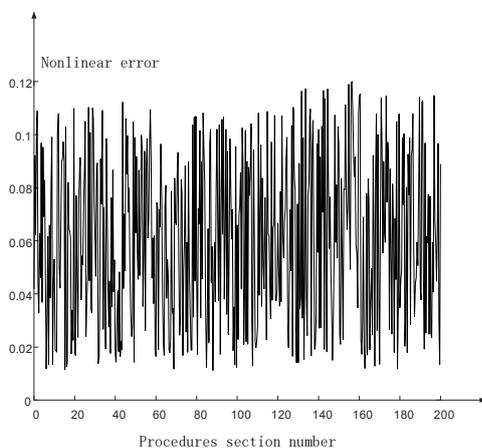


Fig.5 After optimization

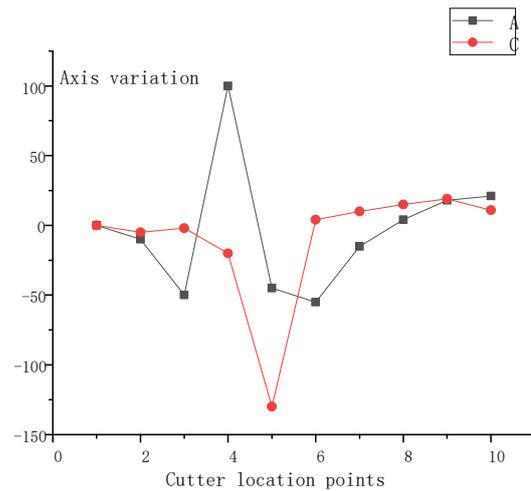


Fig.6 Schematic diagram of change of rotation axis

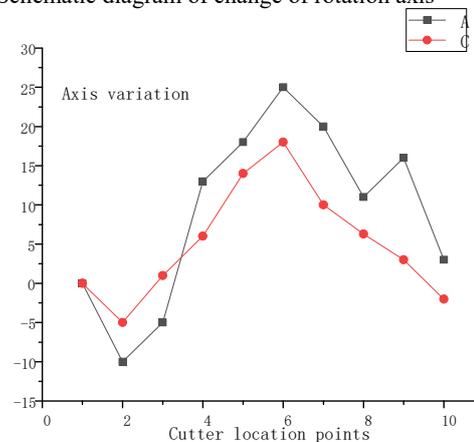


Fig.7 Schematic diagram of rotation axis change after adjustment

Based on the establishment of local milling condition of tool axis vector and the relationship between machining bandwidth and tool axis inclination Angle, the tool axis vector in the singular region is avoided. Compared with the traditional method, the efficiency of machining with the maximum machining bandwidth is greatly improved for the overall tool path. At the same time, the avoidance strategy of singular region has a great improvement over the calculation of simple dense interpolation method.

In terms of machining accuracy, the optimization method proposed in the singular region can control the nonlinear error within a reasonable range, as shown in Fig 5 below. After post-processing, it is verified that the rotation axis of the machine tool changes gently. Meet the requirements of workpiece processing quality. See Fig 7.

5. Conclusion

(1) In this paper, based on the establishment of the function relation between the local milling ability and the machining bandwidth and the tool axis inclination Angle in five-axis machining, a tool axis vector avoidance method is proposed for the problems in the singular region. After modification, error calculation and post detection are carried out.

(2) In this way, the processing bandwidth remains unchanged before and after modification, and the processing efficiency and accuracy have been improved. After the post-processing verification of machine tool each axis of the processing stability has a considerable degree of improvement.

Acknowledgments

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