

# The Algebraic Model of the Dry Ice Extrusion Process in a Die with a Conical-Cylindrical Channel

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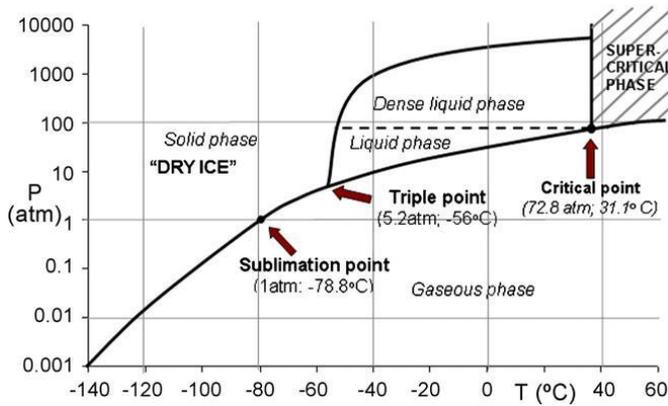
**Abstract.** The article presents the results of works on formulating an algebraic model of crystallized CO<sub>2</sub> extrusion process in a convergent circular-symmetrical channel. The paper presents the method of derivation of the model as well as preliminary comparison of the influence of variance of the 3 geometrical parameters describing the shape of the forming channel. Based on the results of the performed analysis, conclusions are formulated to determine the basis and future direction of study to the development of single-channel dies allowing to extrude dry ice with effective compacting stress values.

## 1 Introduction

An important consideration for the manufacturing process is the utilization of by-product materials. One of such materials is carbon dioxide formed in large quantities e.g. during the manufacturing of nitrogen fertilizers [9]. The recovered carbon dioxide in gaseous form is compressed and liquefied. The liquid CO<sub>2</sub> is kept in enclosed containers at temperature -18 °C and pressure approx. 20 bar [9, 11].

In for example order to utilize liquid carbon dioxide, it undergoes adiabatic expansion to atmospheric pressure. As a result of a sudden change in the energy state of the material, a phase transition occurs and the material crystallizes. The temperature of solid carbon dioxide is approx. -78,5 °C and sublimation in normal conditions [13, 17, 22, 40]. The characteristic of such phase transition is presented on Fig. 1.

Due to the indicated peculiar characteristics, it is often referred to by its common name “dry ice”. [3, 8, 40]. It is used, among others, in material transportation [2, 49] and surface cleaning [4, 12, 21, 23, 28, 34] and disinfection [32, 40]. However, in the process of crystallization of liquid CO<sub>2</sub>, a fragmented material is obtained [24], which results in a short sublimation time and low efficiency of its used in e.g. refrigeration processes [9]. Therefore, in order to extend the sublimation time of the material as well as to improve its use efficiency, it is compacted and delivered e.g. in form of pelle [17, 18].



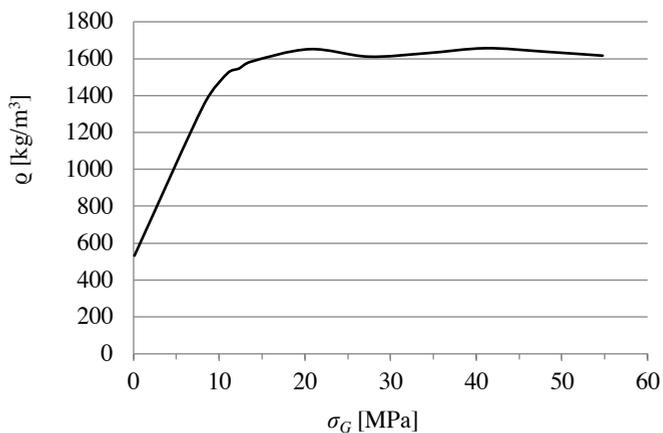
**Fig. 1.** CO<sub>2</sub> phase transition profile [1].

The commercially available machines for dry ice agglomeration are called pelletizers. The working systems of these machines utilize linear piston motion for dry ice compression. These machines are used to manufacture pellet with different diameters, with diameters equal to 3 and 16 mm being the most commonly observed. For diameters of 16 mm, the working system of the machine employs a single-channel die, whereas for the smaller diameter, multi-channel dies are used.

The carbon dioxide agglomerate, due to its characteristics, has found application for both indicated product diameters. It is used, among others, for food storage, water carbonization and freeze-drying of products [9, 12, 29]. Dry ice is also widely employed in abrasion cleaning of surfaces [15, 30,40].

An important parameter defining compacted dry ice quality is its density. This parameter is used for determining material quality; therefore, from the standpoint of the manufacturing process, it is necessary to obtain the maximum density equal to 1650 kg/m<sup>3</sup> utilizing the smallest possible values of compacting forces. Subject literature indicates that the threshold value of dry ice density is achieved at compaction stress equal to 14 MPa (Fig. 2.) [12].

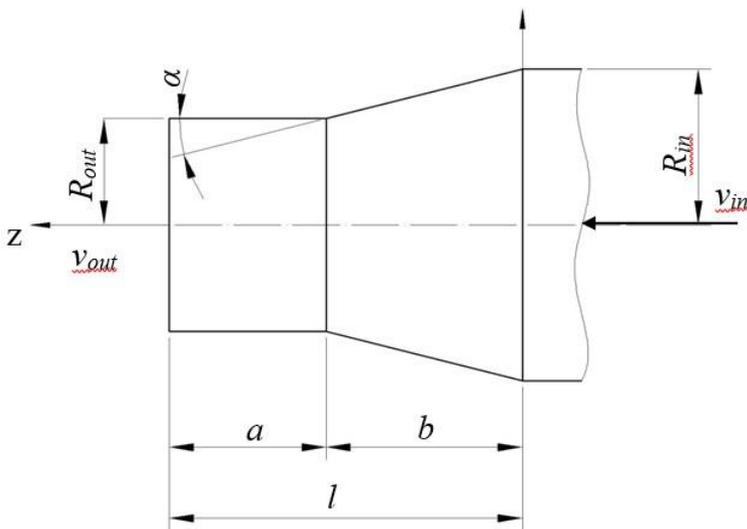
Available subject literature demonstrates a high degree of interest in works aiming to study and develop the shape of the tooling, working assemblies or process parameters used in the process in order to improve the quality of the product as well as the energy efficiency of the manufacturing process [5-7, 14, 16, 19, 20, 25-27,30, 31, 33, 35-39, 41-48].



**Fig. 2.** Variation of agglomerate density in function of threshold stress [10].

## 2 Analytical model

Fig. 3 described single-channel dies together with their geometric parameters. It shows a channel consisting of a circular-symmetrical section together with a cylindrical section.



**Fig. 3.** The geometric parameters of the convergent circular-symmetrical channel,  $\alpha$  – angle convergence of the conical section,  $a$  – cylindrical section length,  $b$  – conical section length,  $l$  – total channel length,  $R_{in}$  – inlet radius of the conical section,  $R_{out}$  – outlet radius of the cylindrical section,  $v_{in}$  – input velocity,  $v_{out}$  – output velocity.

No algebraic model was found in the available subject literature allowing to determine the limit value of compaction force acting on the dry ice  $F_{ER}$  as a function of geometric parameters of the single-channel die. Subject literature provides a mathematical model formulated on the basis of the power balance equation (refer with: Eq. 1) [19, 26]. When formulating the model, the assumption was used that the mass flux value is constant along

the entire length of the forming channel. The required power  $P_Z$  is the sum of power dissipated along the forming section  $P_D$  and frictional resistance  $P_\mu$ . Moreover, the power value  $P_Z$  is equal to the product of resistance force  $F_{ER}$  and input velocity  $v_{in}$ .

$$\begin{cases} P_Z = F_{ER} \cdot v_{in} \\ P_Z = P_D + P_\mu \end{cases} \quad (1)$$

where  $P_\mu$  is the sum of frictional power at the convergent section  $P_{\mu S}$  and the cylindrical section  $P_{\mu C}$  of the circular-symmetrical channel. Therefore, by substituting to the power balance equation (refer with: Eq. 1) we arrive at,

$$F_{ER} \cdot v_{in} = P_D + P_{\mu S} + P_{\mu C} \quad (2)$$

Based on Huber's hypothesis [26], the substitute yield strength was determined as  $\sqrt{3}k_t$ . Hence, the dissipated power value  $P_D$ , as a function of the geometric and kinematic parameters of the process in an axial-symmetrical channel can be described with the below equation,

$$P_D = \sqrt{15} \cdot \tau_a \cdot v_{in} \cdot R_{in}^2 \cdot \ln \frac{R_{in}}{R_{out}} \quad (3)$$

where  $\tau_a$  is the dry ice agglomerates shear stress.

Based on the product displacement value, an algebraic formula was established to describe the variance in energy dissipation due to friction in a symmetrically convergent forming channel  $P_{\mu S}$ .

$$P_{\mu S} = \int_{S_s} \mu_T \cdot \tau_a \cdot w_k \, dS_s \quad (4)$$

where  $S_s$  is the surface area of the side of the convergent section, its value can be described with the formula below,  $w_k$  is the resultant agglomerate displacement speed and  $\mu_T$  is a friction factor,

$$S_s = \int_{S_s} 2\pi \cdot R(z) \, dS_s \quad (5)$$

After integration and transformation of the above equations (refer with: Eq. 4, Eq. 5) we arrive at the following,

$$P_{\mu S} = \mu_T \cdot \tau_a \cdot \frac{v_{in} \cdot R_{in}^2}{\cos \alpha \cdot R_{out}^2} \cdot \frac{2\pi}{\cos \alpha} \left( R_{in} \cdot b \cdot -\frac{b^2}{2} \cdot \tan \alpha \right) \quad (6)$$

Subsequently, frictional power in the cylindrical section of the forming channel was determined similarly, using the equation below,

$$P_{\mu C} = \int_{S_C} \mu_T \cdot \tau_a \cdot v_{in} \, dS_C \quad (7)$$

where  $S_C$  is the surface area of the side of the cylindrical section, which can be described with the following equation,

$$S_C = \int_{\theta=0}^{2\pi} \int_{z=0}^a R_{out} \, dzd\theta = \int_{\theta=0}^{2\pi} \int_{z=0}^a R_{in} - b \cdot \tan \alpha \, dzd\theta \quad (8)$$

After integration and transformation of the equations 7 and 8, we arrive at the following,

$$P_{\mu C} = \mu_T \cdot \tau_a \cdot v_{in} \cdot \frac{R_{out}^2}{R_{in}} \cdot a \quad (9)$$

Afterwards, the developed expressions referring to individual dissipated forces were substituted and transformed (refer with: Eq. 3, Eq. 6, Eq. 9) into the power balance equation (refer with: Eq. 2), as a result, we arrived at the formula allowing to determine the value of  $F_{ER}$  as a function of geometric and physical parameters of compacted dry ice.

$$F_{ER} = \tau_a \cdot R_{in}^2 \left( \sqrt{15} \cdot \ln \frac{R_{in}}{R_{out}} + 2\pi \cdot \mu_T \cdot \frac{1}{\cos^2 \alpha \cdot R_{out}^2} \left( R_{in} \cdot b - \frac{b^2}{2} \tan \alpha \right) + \frac{a}{R_{out}} \right) \quad (10)$$

### 3 Analysis of parameters influence

The formulated model allows to determine the value of  $F_{OP}$  as a function of  $R_{in}$ ,  $R_{out}$ ,  $\alpha$ ,  $a$ ,  $b$ . The indicated parameters are geometrically related which allows exclude the variable parameter  $\alpha$  from the rest. Given that the forming dies are installed in working channels with set diameters, for the purpose of the analysis it was assumed that the  $R_{in}$  value is equal to 18 mm and must remain constant. This serves to limit the scope of analysis of geometric parameter variance to the following ranges

$$R_{out} \in (0.0001; 0.018) \quad (11)$$

$$a \in (0; 0.25) \quad (12)$$

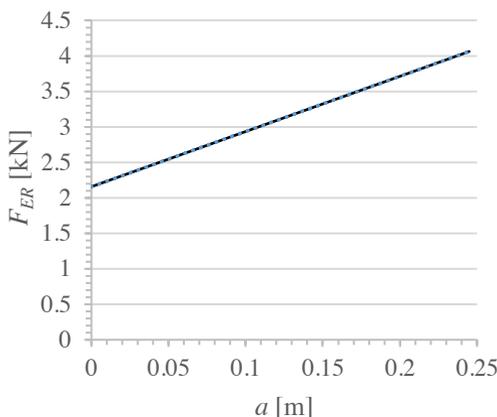
$$b \in (0; 0.017) \quad (13)$$

If the variance of a geometric parameter value did not affect the value of another parameter, the following characteristics were assumed as initial–boundary

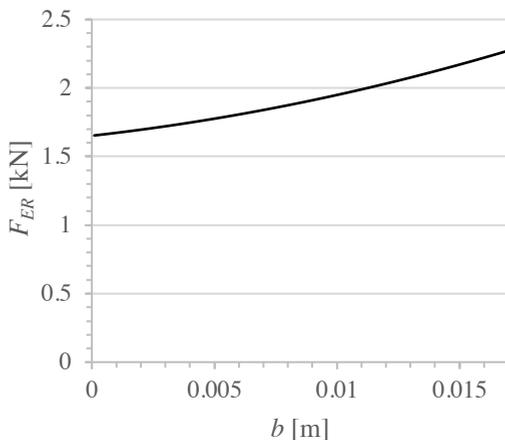
- $R_{in} = 8 \text{ mm}$ ,
- $\alpha = 10^\circ$ ,
- $a = 15 \text{ mm}$ ,
- $b = 17 \text{ mm}$ .

The results of the analyses are presented in form of three graphs describing the variance of  $F_{ER}$  value as a function of  $R_{in}$ ,  $a$ ,  $b$  (Fig. 4-6).

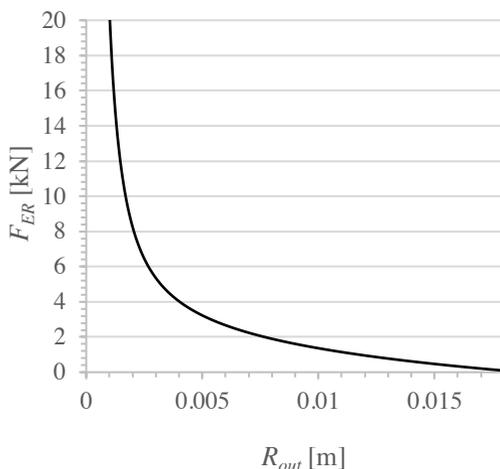
For the purpose of comparison of the influence of individual parameters on the resistance force value  $F_{ER}$ , function value variability gradient was determined as indicated in the characteristics. The values are provided in Table 1.



**Fig. 4.** Variance of  $F_{ER}$  value as a function of  $a$ .



**Fig. 5.** Variance of  $F_{ER}$  value as a function of  $b$ .



**Fig. 6.** Variance of  $F_{ER}$  value as a function of  $R_{out}$ .

**Table 1.** Model sensitivity indicators.

	$f(a)$	$f(b)$	$f(R_{out})$
$\nabla F_{ER}$	7777	36 913	$3.22753 \cdot R_{out}^{2.239}$

## 4 Conclusions

The developed algebraic model allows to estimate the value of the resistance force in the compaction and forming process of crystallized carbon dioxide. The carried out analysis of the influence of individual geometric parameter value allows to determine that:

- the use of dies with high value of  $R_{in}$  to  $R_{out}$  quotient may materially affect process efficiency; therefore, there are grounds for utilizing multi-channel dies for the compaction of dry ice with diameter below 6 mm,
- the length of the cylindrical section has a significantly larger influence than the length of the convergent section.

Further work will verify the algebraic model based on empirical data results.

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