Mass Minimizing of Truss and Beam Structures Subjected to Cumulative Fatigue Damage

Milan Vaško, Lenka Jakubovičová, Milan Sága, Zuzana Ságová, Ivan Kuric, Marian Handrik, and Peter Kopáš

1Department of Applied Mechanics, Faculty of Mechanical Engineering, University of Žilina, Univerzitná 1, 010 26 Žilina, Slovakia
2Department of Automation and Production Systems, Faculty of Mechanical Engineering, University of Žilina, Univerzitná 1, 010 26 Žilina, Slovakia

Abstract. The work presents the methods and solutions of the optimizing design of the truss and beam structures subjected to the cumulative fatigue damage. Two basic approaches used in the optimization of mechanical systems are presented in the paper - the so-called direct search methods and the gradient methods. A short comparative and gradient approach was made. The aim was to minimize the weight of the truss or beam structure with restrictions affecting the prescribed fatigue life. The computational model assumed only quasi-static loading with different number of cycles.

1 Introduction

The aim of the structural optimization is to find the technical realizable suggestion. It is the best solution of all possible projects. The optimal result gives the extreme of the objective function with regards to defined constrain conditions. The objective function often includes prize, weight structure, deformation, maximal stress or prescribed some eigenvalues and so on [2,5,17,21,22].

The article presents some discrete optimization methods for mass minimization of the truss and beam structures subjected to described fatigue life. The loading forces are static, with prescribed the number of cycles and loading duration for more variation loading. The damage accumulation was solved by Corten-Dolans hypothesis [1,3,4,8,18].

2 Structural Mass Minimizing Subjected to Prescribed Fatigue Life

Let’s consider the optimizing process of the mass minimizing of the truss and frame structures subjected to prescribed fatigue life of each element. Assuming the data processing of the objective function and constrain conditions, we know following optimizing methods [2,6,7,19]:

- Direct search methods – simplex’s method and its modifications, Hooke-Jeeves method, Monte-Carlo and others. The mentioned methods are working only with
function’s values of objective function and constrain conditions. They make no use of gradient information.

- **Gradient methods** – they use the information about gradient of the objective function and constrain conditions. The most known and the simplest gradient method of all is the steepest descent method.

- **Newton’s methods** – they use the information about first and second differentiation of objective functions and constrains conditions (Fletcher-Powell, conjugate gradient method, etc.).

Let’s define the optimizing problem of the optimal mass design subjected to the prescribed fatigue life \([2,14,16,23,24]\). For a structure of multiple elements, the optimization problem of discrete variables can be stated mathematically as minimizing an objective function

\[
F(x) = \sum_{i=1}^{n} \rho_i \cdot l_i \cdot X_i \rightarrow \min, \quad \text{subjected to} \quad T_i(x) - T_{ip} \geq 0, \quad i = 1, 2, ..., m, \quad (1)
\]

where \(n\) is number of the elements, \(m\) is number of the element groups (one design variable \(X_i\)), \(T_{ip}\) is prescribed fatigue life in hours. Problem (1) consists of a linear objective function and non-linear constraints. Let’s form a new objective function with penalty function

\[
F(x) = \sum_{i=1}^{n} \rho_i \cdot l_i \cdot X_i + \sum_{j=1}^{m} \lambda_j \rightarrow \min, \quad (2)
\]

where penalty function \(\lambda\) is

\[
\lambda_i = 0 \quad \text{for} \quad T_i(x) - T_{ip} \geq 0, \quad \lambda_i = 10^k \left( k = 4 \text{ to } 9 \right) \quad \text{for} \quad T_i(x) - T_{ip} < 0. \quad (3)
\]

The objective function (2) is analyzed by the Gauss-Seidel and gradient method in discrete form. These methods were implemented into MATLAB [13].

### 3 Formulation of the Mathematical Model for S-N Curve

The Wöhler’s curve is statistically evaluate experimental fatigue curve, that is obtained from amplitude of nominal stress \(\sigma_a\) with the number of oscillations \(N\) for failure of sample. The \(\sigma_a-N\) relation can be written as follows

\[
\sigma_a = \sigma_f \cdot (2N_f)^b, \quad (4)
\]

where \(\sigma_f\) is the fatigue stress coefficient, \(2N_f\) is number of cycles to failure, \(b\) is fatigue strength exponent and \(\sigma_a\) is stress amplitude to failure \([1, 8, 9, 19]\).

In the case of higher stress amplitudes the fatigue test with controlled deformation is better to realize \([3, 11, 15, 20]\). The total \(\varepsilon_a\) or only plastic \(\varepsilon_{ap}\) deformation vs. cycles to failure is usually presented as a well-known Manson-Coffin curve, which mathematic formulation is following

\[
\varepsilon_a = \varepsilon_{ae} + \varepsilon_{ap} = \frac{\sigma_f}{E} \left(2N_f\right)^b + \varepsilon_f \cdot \left(2N_f\right)^c, \quad (5)
\]
of discrete variables can be stated mathematically as minimizing an objective function for fatigue life \([2,14,16,23,24]\). For a structure of multiple elements, the optimization problem

\[
\text{Let’s define the optimizing problem of the optimal mass design subjected to the prescribed}
\]

is usually presented as a well-known Manson-Coffin curve, which mathematical formulation is following

\[
\frac{\sigma_a}{\sigma_{c}} = \left(\frac{\sigma_{M}}{R_f}\right)^k + \frac{1 - \left(\frac{\sigma_{M}}{R_f}\right)^k}{\left(\frac{\sigma_{M}}{R_f}\right)^k}.
\]

In the case of higher stress amplitudes the fatigue test with controlled deformation is better, where \(\sigma_a\) and \(\sigma_m\) are computed or experimental obtained stress amplitude and mean stress, \(\sigma_{c}\) and \(\sigma_{M}\) are limit stress amplitude and mean stress from Haigh diagram, or its modified version (Goodman, Soderberg, Geber). Therefore it can be written as follows

\[
\sigma_a = \sigma_c \cdot \left[1 - \left(\frac{\sigma_m}{R_f}\right)^k\right].
\]

If \(k = 1\) a \(R_f = R_m\), Soderberg’s model is used, if \(k = 2\) and \(R_f = R_m\), Goodman’s model is used and if \(k = 2\) and \(R_f = R_m\), Geber’s model is used.

The \(\sigma_{c}\) is conventional fatigue limit. Using equation (7) and (8) we assumed

\[
S = \frac{\sigma_{A}}{\sigma_a} = \frac{\sigma_{c}}{\sigma_a} \cdot \left[1 - \left(\frac{\sigma_m}{R_f}\right)^k\right].
\]

If we suggest that in the regime of the giga-cycle fatigue loading the value of fatigue limit decrease \(k_s\) times, so safety factor can be written as

\[
S = \frac{\sigma_{A}}{\sigma_a} = \frac{\sigma_{c}}{\sigma_a} \cdot \left[1 - \left(\frac{\sigma_m}{R_f}\right)^k\right] = k_s \cdot S.
\]

From experimental measuring in the giga-cycle region of loading, it can be obtained polynomial approximation of S-N curve (Fig. 1). A mathematical formulation of multi-stage S-N curve for \(i^{th}\) part will be

\[
2N_i \cdot \sigma_i^{b_i} = 2N_{i-1} \cdot \sigma_{i-1}^{b_i},
\]

where \(b_i\) exponent can be written as follows

\[
b_i = \frac{\log \left(\frac{\sigma_{i-1}}{\sigma_i}\right)}{\log \left(\frac{2N_i}{2N_{i-1}}\right)},
\]

\[
D_i = \frac{1}{2N_f}.
\]

If we suggest that maximum value of stress amplitude in critical places will be lower than real endurance strength, the next step of computational analysis will be define a safety factor, which mathematic formulation is following

\[
S = \frac{\sigma_{A}}{\sigma_a}, \text{ or } S = \frac{\sigma_{M}}{\sigma_m},
\]

where \(\sigma_a\) and \(\sigma_m\) are computed or experimental obtained stress amplitude and mean stress, \(\sigma_{A}\) and \(\sigma_{M}\) are limit stress amplitude and mean stress from Haigh diagram, or its modified version (Goodman, Soderberg, Geber). Therefore it can be written as follows

\[
\sigma_a = \sigma_c \cdot \left[1 - \left(\frac{\sigma_m}{R_f}\right)^k\right].
\]

If \(k = 1\) a \(R_f = R_m\), Soderberg’s model is used, if \(k = 1\) and \(R_f = R_m\), Goodman’s model is used and if \(k = 2\) and \(R_f = R_m\), Geber’s model is used.

The \(\sigma_{c}\) is conventional fatigue limit. Using equation (7) and (8) we assumed

\[
S = \frac{\sigma_{A}}{\sigma_a} = \frac{\sigma_{c}}{\sigma_a} \cdot \left[1 - \left(\frac{\sigma_m}{R_f}\right)^k\right].
\]

If we suggest that in the regime of the giga-cycle fatigue loading the value of fatigue limit decrease \(k_s\) times, so safety factor can be written as

\[
S_s = \frac{\sigma_{A}}{\sigma_a} = \frac{\sigma_{c}}{\sigma_a} \cdot \left[1 - \left(\frac{\sigma_m}{R_f}\right)^k\right] = k_s \cdot S.
\]

From experimental measuring in the giga-cycle region of loading, it can be obtained polynomial approximation of S-N curve (Fig. 1). A mathematical formulation of multi-stage S-N curve for \(i^{th}\) part will be

\[
2N_i \cdot \sigma_i^{b_i} = 2N_{i-1} \cdot \sigma_{i-1}^{b_i},
\]

where \(b_i\) exponent can be written as follows

\[
b_i = \frac{\log \left(\frac{\sigma_{i-1}}{\sigma_i}\right)}{\log \left(\frac{2N_i}{2N_{i-1}}\right)},
\]

\[
D_i = \frac{1}{2N_f}.
\]
Fig. 1. Polynomial approximation of S-N curve.

Correction of the total damage with the respect to mean stress $\sigma_m$ is given by

$$D = \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} d_{ij} = \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} \frac{1}{2N_i \cdot \sigma_m^h} \cdot \sum_{j=1}^{n_x} \frac{\sigma_{\sigma_{ij}}}{\left(1 - \frac{\sigma_{\sigma_{ij}}}{R_f}\right)^k}, \quad \sigma_{i,j-1} \leq \sigma_{\sigma_{ij}} < \sigma_i$$

and final fatigue life prediction can be calculated as follows

$$T = \frac{T_r}{D_r},$$

where $T_r$ is realization time interval of simulation, $D_r$ is damage in realization time interval $T_r$.

3.1 Description of Program Toolbox

This section presents the program package for the linear static analyze and chosen discrete optimizing methods for finite element models created by MATLAB software [10,12,13]. The program STATIC.M was used for static analyzes and programs GSM.M and DISKGRAD.M were used in the optimization process (Fig. 2).

Let’s define the following computational modules:
- STATIC.M – reads the input data file, creates the stiffness matrix, solves the linear algebraic system equation and computes the elements stress,
- HEAD.M – analyses the damage accumulation and penalized objective function,
- GEOMETRY.M – is the input data file,
- TRUSS.M, BEAM.M – compute the stiffness and mass matrices for truss and beam elements,
4 Numerical Examples

4.1 Example No. 1

Let’s design cross sections $A_i$ of the simple frame structure excited by forces $F_1, F_2$ (Tab. 1). The geometry of the structure is presented in Fig. 3. The used cross sections are presented in Table 2. The prescribed fatigue life will consider $T_p = 2 \cdot 10^5$ [hours].
The material properties are: Young’s modulus $E = 2.1 \cdot 10^5\text{MPa}$, Poisson’s constant $\mu = 0.3$, material density $\rho = 7850 \text{kg} \cdot \text{m}^{-3}$, tensile strength $R_m = 370 \text{MPa}$, elastic limit $R_e = 247 \text{MPa}$, fatigue limit $\sigma_c = 58 \text{MPa}$, exponent of Wöhler’s curve $m = 5.2$, point of Wöhler’s curve $N_a = 180 \text{MPa}$, $\sigma_{A_{\text{max}}} = 40 \text{MPa}$. The start point was the “point” with numbers $A = [18, 18]$.

Table 1. The loading history.

<table>
<thead>
<tr>
<th>Loading case</th>
<th>Number of cycles</th>
<th>Time interval [s]</th>
<th>Node</th>
<th>Direction</th>
<th>Value [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>10</td>
<td>4</td>
<td>x</td>
<td>-20000</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>10</td>
<td>8</td>
<td>y</td>
<td>10000</td>
</tr>
</tbody>
</table>

Table 2. Applied I-sections.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>42</td>
<td>757</td>
<td>777000</td>
<td>62800</td>
<td>8570</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>50</td>
<td>1060</td>
<td>1700000</td>
<td>122000</td>
<td>15900</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
<td>58</td>
<td>1420</td>
<td>3270000</td>
<td>214000</td>
<td>27000</td>
</tr>
<tr>
<td>4</td>
<td>140</td>
<td>66</td>
<td>1820</td>
<td>5720000</td>
<td>351000</td>
<td>43300</td>
</tr>
<tr>
<td>5</td>
<td>160</td>
<td>74</td>
<td>2280</td>
<td>9340000</td>
<td>546000</td>
<td>65800</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>82</td>
<td>2790</td>
<td>14400000</td>
<td>812000</td>
<td>96200</td>
</tr>
<tr>
<td>7</td>
<td>200</td>
<td>90</td>
<td>3340</td>
<td>21400000</td>
<td>1160000</td>
<td>136000</td>
</tr>
<tr>
<td>8</td>
<td>220</td>
<td>98</td>
<td>3950</td>
<td>30500000</td>
<td>1620000</td>
<td>187000</td>
</tr>
<tr>
<td>9</td>
<td>240</td>
<td>106</td>
<td>4610</td>
<td>42400000</td>
<td>2200000</td>
<td>251000</td>
</tr>
<tr>
<td>10</td>
<td>260</td>
<td>113</td>
<td>5330</td>
<td>57300000</td>
<td>2870000</td>
<td>336000</td>
</tr>
<tr>
<td>11</td>
<td>280</td>
<td>119</td>
<td>6100</td>
<td>75800000</td>
<td>3630000</td>
<td>444000</td>
</tr>
<tr>
<td>12</td>
<td>300</td>
<td>125</td>
<td>6900</td>
<td>97900000</td>
<td>4490000</td>
<td>569000</td>
</tr>
<tr>
<td>13</td>
<td>320</td>
<td>131</td>
<td>7770</td>
<td>125000000</td>
<td>5540000</td>
<td>727000</td>
</tr>
<tr>
<td>14</td>
<td>340</td>
<td>137</td>
<td>8670</td>
<td>157000000</td>
<td>6720000</td>
<td>905000</td>
</tr>
<tr>
<td>15</td>
<td>360</td>
<td>143</td>
<td>9700</td>
<td>196000000</td>
<td>8170000</td>
<td>1150000</td>
</tr>
<tr>
<td>16</td>
<td>380</td>
<td>149</td>
<td>10700</td>
<td>240000000</td>
<td>9720000</td>
<td>1390000</td>
</tr>
<tr>
<td>17</td>
<td>400</td>
<td>155</td>
<td>11800</td>
<td>291000000</td>
<td>11400000</td>
<td>1700000</td>
</tr>
<tr>
<td>18</td>
<td>450</td>
<td>170</td>
<td>14700</td>
<td>570000000</td>
<td>16900000</td>
<td>2660000</td>
</tr>
<tr>
<td>19</td>
<td>500</td>
<td>185</td>
<td>17900</td>
<td>683000000</td>
<td>24000000</td>
<td>3990000</td>
</tr>
</tbody>
</table>

The Gauss-Seidel Method solution results: $A_{\text{opt}} = [6, 8]$. 
The material properties are:

- Young's modulus $E = 2.1 \times 10^5$ MPa,
- Poisson's constant $\nu = 0.3$,
- material density $\rho = 7850$ kg·m$^{-3}$,
- tensile strength $R_m = 370$ MPa,
- elastic limit $R_e = 247$ MPa,
- fatigue limit $\sigma_c = 58$ MPa,
- exponent of Wöhler's curve $m = 5.2$,
- point of Wöhler's curve $N_a = 180$ MPa,
- $\sigma_{A_{\text{max}}} = 40$ MPa.

The start point was the "point" with numbers $A = [18, 18]$.

Table 1. The loading history.

<table>
<thead>
<tr>
<th>Loading case</th>
<th>Number of cycles</th>
<th>Time interval [s]</th>
<th>Node</th>
<th>Direction</th>
<th>Value [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>10</td>
<td>4</td>
<td>x</td>
<td>$-20000$</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>10</td>
<td>8</td>
<td>y</td>
<td>$10000$</td>
</tr>
</tbody>
</table>

Table 2. Applied I-sections.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>42</td>
<td>757</td>
<td>777000</td>
<td>62800</td>
<td>8570</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>50</td>
<td>1060</td>
<td>1700000</td>
<td>122000</td>
<td>15900</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
<td>58</td>
<td>1420</td>
<td>3270000</td>
<td>214000</td>
<td>27000</td>
</tr>
<tr>
<td>4</td>
<td>140</td>
<td>66</td>
<td>1820</td>
<td>5720000</td>
<td>351000</td>
<td>43300</td>
</tr>
<tr>
<td>5</td>
<td>160</td>
<td>74</td>
<td>2280</td>
<td>9340000</td>
<td>546000</td>
<td>65800</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>82</td>
<td>2790</td>
<td>14400000</td>
<td>812000</td>
<td>96200</td>
</tr>
<tr>
<td>7</td>
<td>200</td>
<td>90</td>
<td>3340</td>
<td>21400000</td>
<td>1160000</td>
<td>136000</td>
</tr>
<tr>
<td>8</td>
<td>220</td>
<td>98</td>
<td>3950</td>
<td>30500000</td>
<td>1620000</td>
<td>187000</td>
</tr>
<tr>
<td>9</td>
<td>240</td>
<td>106</td>
<td>4610</td>
<td>42400000</td>
<td>2200000</td>
<td>251000</td>
</tr>
<tr>
<td>10</td>
<td>260</td>
<td>113</td>
<td>5330</td>
<td>57300000</td>
<td>2870000</td>
<td>336000</td>
</tr>
<tr>
<td>11</td>
<td>280</td>
<td>119</td>
<td>6100</td>
<td>75800000</td>
<td>3630000</td>
<td>444000</td>
</tr>
<tr>
<td>12</td>
<td>300</td>
<td>125</td>
<td>6900</td>
<td>97900000</td>
<td>4490000</td>
<td>569000</td>
</tr>
<tr>
<td>13</td>
<td>320</td>
<td>131</td>
<td>7770</td>
<td>125000000</td>
<td>5540000</td>
<td>727000</td>
</tr>
<tr>
<td>14</td>
<td>340</td>
<td>137</td>
<td>8670</td>
<td>157000000</td>
<td>6720000</td>
<td>905000</td>
</tr>
<tr>
<td>15</td>
<td>360</td>
<td>143</td>
<td>9700</td>
<td>196000000</td>
<td>8170000</td>
<td>1150000</td>
</tr>
<tr>
<td>16</td>
<td>380</td>
<td>149</td>
<td>10700</td>
<td>240000000</td>
<td>9720000</td>
<td>1390000</td>
</tr>
<tr>
<td>17</td>
<td>400</td>
<td>155</td>
<td>11800</td>
<td>291000000</td>
<td>11400000</td>
<td>1700000</td>
</tr>
<tr>
<td>18</td>
<td>450</td>
<td>170</td>
<td>14700</td>
<td>570000000</td>
<td>16900000</td>
<td>2660000</td>
</tr>
<tr>
<td>19</td>
<td>500</td>
<td>185</td>
<td>17900</td>
<td>683000000</td>
<td>24000000</td>
<td>3990000</td>
</tr>
</tbody>
</table>

The Gauss-Seidel Method solution results: $A_{\text{opt}} = [6, 8]$.

Fig. 4. Design history of optimizing variables (left), design history of objective function-structural mass (right).

The Simple Gradient Method solution results: $A_{\text{opt}} = [6, 8]$.

Fig. 5. Design history of optimizing variables (left), design history of objective function-structural mass (right).

The results are presented in Table 3.

Table 3. Results evaluation.

<table>
<thead>
<tr>
<th></th>
<th>Structural mass [kg]</th>
<th>Minimum fatigue life of members with $A_1$ [hours]</th>
<th>Minimum fatigue life of members with $A_2$ [hours]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before optimization</td>
<td>978</td>
<td>$6.4 \times 10^9$</td>
<td>$7.6 \times 10^8$</td>
</tr>
<tr>
<td>After optimization</td>
<td>226</td>
<td>$4.4 \times 10^5$</td>
<td>$3.1 \times 10^5$</td>
</tr>
</tbody>
</table>
4.2 Example No. 2

Let’s minimize the mass of truss structure on Fig. 6 subjected to the prescribed minimum fatigue life. Consider the prescribed fatigue life $T_p = 2 \cdot 10^5$ hours. The structural loading is presented in Table 4.

The material properties are: Young’s modulus $E = 2.1 \cdot 10^5$ MPa, Poisson’s constant $\mu = 0.3$, material density $\rho = 7850$ kg·m$^{-3}$, tensile strength $R_m = 370$ MPa, elastic limit $R_e = 247$ MPa, fatigue limit $\sigma_c = 58$ MPa, exponent of Wöhler’s curve $m = 5.2$, point of Wöhler’s curve $N_a = 180$ MPa, $\sigma_{A_{\text{max}}} = 40$ MPa. The start point was the “point” with numbers $A = [27, 27, 27]$.

Table 4. The loading history.

<table>
<thead>
<tr>
<th>Loading case</th>
<th>Number of cycles</th>
<th>Time interval [s]</th>
<th>Node</th>
<th>Direction</th>
<th>Value [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>10</td>
<td>34</td>
<td>z</td>
<td>-52 000</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>10</td>
<td>36</td>
<td>y</td>
<td>52 000</td>
</tr>
</tbody>
</table>

Fig. 6. Truss structure.
4.2 Example No. 2

Let's minimize the mass of truss structure on Fig. 6 subjected to the prescribed minimum fatigue life. Consider the prescribed fatigue life $T_p = 2 \times 10^5$ hours. The structural loading is presented in Table 4.

The material properties are:
- Young’s modulus $E = 2.1 \times 10^5$ MPa,
- Poisson’s constant $\mu = 0.3$,
- material density $\rho = 7850$ kg·m$^{-3}$,
- tensile strength $R_m = 370$ MPa,
- elastic limit $R_e = 247$ MPa,
- fatigue limit $\sigma_c = 58$ MPa,
- exponent of Wöhler’s curve $m = 5.2$,
- point of Wöhler’s curve $N_a = 180$ MPa,
- $\sigma_{A_{\text{max}}} = 40$ MPa.

The start point was the “point” with numbers $A = [27, 27, 27]$.

Table 4. The loading history.

<table>
<thead>
<tr>
<th>Loading case</th>
<th>Number of cycles</th>
<th>Time interval [s]</th>
<th>Node</th>
<th>Direction</th>
<th>Value [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>10</td>
<td>34</td>
<td>z</td>
<td>-52 000</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>10</td>
<td>36</td>
<td>y</td>
<td>52 000</td>
</tr>
</tbody>
</table>

Fig. 6. Truss structure.

Table 5. Applied L-sections.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>235</td>
<td>10</td>
<td>1180</td>
<td>19</td>
<td>3730</td>
</tr>
<tr>
<td>2</td>
<td>308</td>
<td>11</td>
<td>1230</td>
<td>20</td>
<td>3750</td>
</tr>
<tr>
<td>3</td>
<td>379</td>
<td>12</td>
<td>1510</td>
<td>21</td>
<td>4210</td>
</tr>
<tr>
<td>4</td>
<td>389</td>
<td>13</td>
<td>1550</td>
<td>22</td>
<td>4320</td>
</tr>
<tr>
<td>5</td>
<td>480</td>
<td>14</td>
<td>1920</td>
<td>23</td>
<td>4880</td>
</tr>
<tr>
<td>6</td>
<td>569</td>
<td>15</td>
<td>2270</td>
<td>24</td>
<td>4900</td>
</tr>
<tr>
<td>7</td>
<td>691</td>
<td>16</td>
<td>2720</td>
<td>25</td>
<td>5440</td>
</tr>
<tr>
<td>8</td>
<td>903</td>
<td>17</td>
<td>3130</td>
<td>26</td>
<td>6180</td>
</tr>
<tr>
<td>9</td>
<td>935</td>
<td>18</td>
<td>3240</td>
<td>27</td>
<td>7630</td>
</tr>
</tbody>
</table>


Fig. 7. Design history of optimizing variables (left), design history of objective function-structural mass (right).

The Simple Gradient Method solution results: $A_{\text{opt}} = [2, 2, 8]$.

Fig. 8. Design history of fatigue life.
Fig. 9. Design history of optimizing variables (left), design history of objective function-structural mass (right).

Fig. 10. Design history of fatigue life.

The results are presented in Table 6.

<table>
<thead>
<tr>
<th></th>
<th>Structural mass [kg]</th>
<th>Minimum fatigue life of members with $X_1$ [hours]</th>
<th>Minimum fatigue life of members with $X_2$ [hours]</th>
<th>Minimum fatigue life of members with $X_3$ [hours]</th>
</tr>
</thead>
<tbody>
<tr>
<td>before optimization</td>
<td>8844</td>
<td>$38.8 \cdot 10^8$</td>
<td>$12.5 \cdot 10^8$</td>
<td>$6.85 \cdot 10^{11}$</td>
</tr>
<tr>
<td>after optimization</td>
<td>4835</td>
<td>$5.4 \cdot 10^5$</td>
<td>$34.5 \cdot 10^5$</td>
<td>$8.16 \cdot 10^5$</td>
</tr>
</tbody>
</table>

5 Conclusions

The work presents the methods and solutions of the optimizing design of the truss and beam structures subjected to the damage accumulation. Considering the solution of the numerical examples, it was compared the so-called direct search methods and the gradient methods. The aim was to minimize the weight of the truss or beam structure with restrictions affecting the prescribed fatigue life. The computational model assumed only quasi-static
loading with different number of cycles. It has been shown that the direct search methods are preferable for discrete optimization problems. The discrete Gauss–Seidel method (direct search method) is suitable for problems with complicated objective function respectively constrains conditions (e.g. fatigue life) and for the structures with the less number of design variables (e.g. a few sections). The discrete gradient method (gradient method) shows less advantageous due to irregular gradient calculation and the problem of incorrect gradient follows on the penalty function definition. The first method belongs to direct search methods and it’s advantageous for application in discrete optimisation process. Therefore, we can advise it to use for mass structures minimisation with fatigue life constraints.

**Acknowledgements**

This work was supported by project KEGA No. 037ŽU-4/2018.

**References**