Numerical solution of the mix quantum rabi model

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Abstract. The mixed quantum rabi model is generated based on the interaction of single photon and two photon. Based on the mix QRM under one-qubit, this paper extended it to two-qubit Hamiltonian and solve it, then analyzed the variation law of different parameter variables and the symmetry of matrix.

Keywords. mixed QRM, numerical solution, two-model.

1 Introduction

As a classical example of quantum optics, the QRM describes the simplest but indispensable interaction between two level system and single mode boson cavity. This model has been reactivated and applied in recent years with the development and application of lots solid state, including the quantum dots, trapped ions and circuit quantum electrodynamics (QED). In these devices, strong and even super-strong coupling has been achieved, and even deep strong coupling is being investigated.

The presence of single-photon and double-photon QRM is necessary for the (1+1) - dimensional black hole. Existing literature only discusses the mixed QRM of single mode. In this paper, it will be extended to double mode, solved and discussed, and its symmetry will be analyzed.

The simple basic form of QRM is as follows

\[ H = \omega a^\dagger a + \frac{1}{2} \Omega \sigma_z + g(a^\dagger \sigma_%2 + a \sigma_+) \]  \hspace{1cm} (1)

Where \( \Omega \) is frequency of the light field, \( a^\dagger, a \) are the creation(annihilation) operators of the two-level atom with frequency \( \omega \) respectively, \( g \) is the coupling constant of the interaction between the quantum bit and the light field, \( \sigma_z \) and \( \sigma_- \) are the rise and fall operators of the atom respectively.
2 Numerical solution of the one model mix QRM

Here we study how to generalize QRM and make it exhibit linear and nonlinear coupling between the cavities and the qubits. The mixed QRM including both single and double photons, the Hamiltonian can be written as follows

\[ H = \omega a^\dagger a + \Delta \sigma_z + \sigma_z g_1 (a^\dagger + a) + \sigma_z g_2 [(a^\dagger)^2 + a^2] \]  

(2)

Based on spin states, the Hamiltonian (2) can be expanded by the Pauli operator into the matrix form with $\omega = 1$ as follows

\[
H = \begin{pmatrix}
\Delta & \Delta \\
\Delta & a^\dagger a - g_1 (a^\dagger + a) - g_2 [(a^\dagger)^2 + a^2]
\end{pmatrix}
\]  

(3)

Action of $\langle m |$ and $| n \rangle$ on Hamiltonian $H$ yields the following result

\[
\langle m | H | n \rangle = \begin{pmatrix}
H_{11} & \delta_{m,n} \Delta \\
\delta_{m,n} \Delta & H_{22}
\end{pmatrix}
\]

(4)

with

\[
H_{11} = n \delta_{m,n} + g_1 A + g_2 B, 
H_{22} = n \delta_{m,n} - g_1 A - g_2 B,
A = \sqrt{n+1} \delta_{m,n+1} + \sqrt{n} \delta_{m,n-1}, 
B = \sqrt{n(n+1)} \delta_{m,n+2} + \sqrt{n(n-1)} \delta_{m,n-2}
\]

(5)

Fig. 1. Energy spectra of $g_2$. 

\[ g_1 = 0.1, \Delta = -0.5 \]

\[ g_1 = 0.1, \Delta = -0.25 \]

\[ g_1 = 1, \Delta = -0.5 \]

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### 3 Numerical solution of the two-model mix QRM

In the two-model mix QRM, its Hamiltonian can be written as follows

$$H = \Delta_1 \sigma_{1z} + \Delta_2 \sigma_{2z} + \omega a^+ a + g_1 \sigma_{1z} [a + a^+] + g_2 \sigma_{2z} [a + a^+]$$

$$+ \lambda_1 \sigma_{1z} [a^2 + (a^+)^2] + \lambda_2 \sigma_{2z} [a^2 + (a^+)^2]$$

(6)

Where, $\omega$ is the frequency of the light field, $2\Delta_1, 2\Delta_2$ is the energy level difference of the two qubits, $g_1, g_2$ is coupling constant of the two qubits interacting with the light field, and $\sigma_{1z}$ and $\sigma_{2z}$ are the Pauli operators of the two qubits.

#### 3.1 Numerical solution calculation principle

Based on spin states, the Hamiltonian (6) can be expanded by the Pauli operator into the matrix form in units of $\omega = 1$ as follows

$$H = A(2 \times 2)$$

$$+ \begin{pmatrix}
\Delta_1 & 0 \\
0 & \Delta_1 \\
\Delta_1 & 0 \\
0 & \Delta_1 \\
\end{pmatrix}$$

(7)

With

$$A = \begin{pmatrix}
a^+ a + g_1 [a + a^+] + g_2 [a + a^+] \\
+ \lambda_1 [a^2 + (a^+)^2] + \lambda_2 [a^2 + (a^+)^2] \\
a^+ a + g_1 [a + a^+] - g_2 [a + a^+] \\
+ \lambda_1 [a^2 + (a^+)^2] - \lambda_2 [a^2 + (a^+)^2] \\
\end{pmatrix}$$

$$D = \begin{pmatrix}
a^+ a - g_1 [a + a^+] + g_2 [a + a^+] \\
- \lambda_1 [a^2 + (a^+)^2] + \lambda_2 [a^2 + (a^+)^2] \\
a^+ a - g_1 [a + a^+] - g_2 [a + a^+] \\
- \lambda_1 [a^2 + (a^+)^2] - \lambda_2 [a^2 + (a^+)^2] \\
\end{pmatrix}$$

(8)

If we set $g_1 + g_2 = n_1$, $\lambda_1 + \lambda_2 = n_2$, $g_1 - g_2 = m_1$, $\lambda_1 - \lambda_2 = m_2$, action of $|m\rangle$ and $|n\rangle$ on Hamiltonian H yields the following result

$$\langle m | H | n \rangle = \begin{pmatrix}
H_{11} & \Delta_2 \delta_{m,n} & \Delta_1 \delta_{m,n} & 0 \\
\Delta_1 \delta_{m,n} & H_{22} & 0 & \Delta_1 \delta_{m,n} \\
\Delta_2 \delta_{m,n} & 0 & H_{33} & \Delta_2 \delta_{m,n} \\
0 & \Delta_1 \delta_{m,n} & \Delta_2 \delta_{m,n} & H_{44} \\
\end{pmatrix}$$

(9)
With (5)

\[
\begin{align*}
H_{11} &= n\delta_{m,n} + n_1A + n_2B \\
H_{22} &= n\delta_{m,n} + m_1A + m_2B \\
H_{33} &= n\delta_{m,n} - m_1A - m_2B \\
H_{44} &= n\delta_{m,n} - n_1A - n_2B
\end{align*}
\]  

(10)

With the increase of controllable coefficients in the calculation formula, the discussion of the results in this case will become more complicated.

Fig. 2. Energy spectra of \( n_2 \).

The observation results show that when \( n_1 \) increases, the law of the two-mode case still remains the same as that of the one-mode case, but with the increase of \( n_1 \), the convergence of the energy spectrum becomes faster. Meanwhile, for \( \Delta_1 \) and \( \Delta_2 \), only a small value of \( n_1 \) can have an obvious influence on the change of the energy spectrum.

About symmetry, let’s say the wave function

\[
|a\rangle = \sum_{n=0}^{\infty} \sqrt{n!} \begin{pmatrix} a_{1,n} \\ a_{2,n} \\ a_{3,n} \\ a_{4,n} \end{pmatrix} |n\rangle, |a\rangle' = i^n |a\rangle
\]  

(11)

Based of \( H|a\rangle = E|a\rangle \) with \( \langle h \rangle \) can get
\[
\begin{pmatrix}
\left( E - m \right) a_{1,m} + n_i (m + 1) a_{2,m} - \Delta\alpha_{a_{1,m} - \Delta\alpha_{a_{2,m}}} - n_i a_{1,m+1} - n_i a_{2,m+1} \\
\left( E - m \right) a_{2,m} - m_i (m + 1) a_{3,m} - \Delta\alpha_{a_{2,m} - \Delta\alpha_{a_{3,m}}} - m_i a_{2,m+1} - m_i a_{3,m+1} \\
\left( E - m \right) a_{3,m} + m_i (m + 1) a_{4,m} - \Delta\alpha_{a_{3,m} + \Delta\alpha_{a_{4,m}}} + m_i a_{3,m+1} + m_i a_{4,m+1} \\
\left( E - m \right) a_{4,m} + n_i (m + 1) a_{4,m+1} - \Delta\alpha_{a_{4,m} + \Delta\alpha_{a_{4,m}}} + n_i a_{4,m+1} + n_i a_{4,m+2}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\left( E - m \right) a_{1,m} - n_i E' a_{1,m} - \Delta\alpha_{a_{1,m} - \Delta\alpha_{a_{1,m} - \Delta\alpha_{a_{1,m}}}} - n_i a_{1,m+1} + n_i a_{2,m} \\
\left( E - m \right) a_{2,m} - m_i E' a_{2,m} - \Delta\alpha_{a_{2,m} - \Delta\alpha_{a_{2,m} - \Delta\alpha_{a_{2,m}}}} - m_i a_{3,m+1} + m_i a_{2,m+1} \\
\left( E - m \right) a_{3,m} + m_i E' a_{3,m} - \Delta\alpha_{a_{3,m} + \Delta\alpha_{a_{3,m} - \Delta\alpha_{a_{3,m}}}} + m_i a_{2,m+1} - m_i a_{3,m+1} \\
\left( E - m \right) a_{4,m} + n_i E' a_{4,m} - \Delta\alpha_{a_{4,m} + \Delta\alpha_{a_{4,m} - \Delta\alpha_{a_{4,m}}}} + n_i a_{1,m+1} - n_i a_{4,m+1}
\end{pmatrix}
\]

\[
C = (m + 2)(m + 1), D = m + 2, E' = m + 1, F = m - 1, G = m - 2
\]

The above equation cannot satisfy the symmetry of single and double photons such as ±1 or ±i.

4 Conclusion

For the mix QRM, we extended it from single-mode mix QRM to dual-mode mix QRM, obtained its Hamiltonian and numerically solved it, discussed the influence of different parameters on the result, and the symmetry of it.

References