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Numerical solution of the mix quantum rabi model

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Abstract. The mixed quantum rabi model is generated based on the interaction of single photon and two photon. Based on the mix QRM under one-qubit, this paper extended it to two-qubit Hamiltonian and solve it, then analyzed the variation law of different parameter variables and the symmetry of matrix.
Keywords. mixed QRM, numerical solution, two-model.

1 Introduction

As a classical example of quantum optics, the QRM describes the simplest but indispensable interaction between two level system and single mode boson cavity. This model has been reactivated and applied in recent years with the development and application of lots solid state, including the quantum dots, trapped ions and circuit quantum electrodynamics (QED). In these devices, strong and even super-strong coupling has been achieved, and even deep strong coupling is being investigated.

The presence of single-photon and double-photon QRM is necessary for the (1+1) - dimensional black hole. Existing literature only discusses the mixed QRM of single mode. In this paper, it will be extended to double mode, solved and discussed, and its symmetry will be analyzed.

The simple basic form of QRM is as follows

$$H = \omega a^{+} a + \frac{1}{2}\Omega\sigma_{z} + g(a^{+}\sigma_{-} + a\sigma_{+}) \tag{1}$$

Where Ω is frequency of the light field, $a^{\dagger}, a$ are the creation(annihilation) operators of the two-level atom with frequency ω respectively, g is the coupling constant of the interaction between the quantum bit and the light field, σ_{+} and σ_{-} are the rise and fall operators of the atom respectively.

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2 Numerical solution of the one model mix QRM

Here we study how to generalize QRM and make it exhibit linear and nonlinear coupling between the cavities and the qubits. the mixed QRM including both Single and double photons, the Hamiltonian can be written as follows

$$H = \omega a^{\dagger}a + \Delta\sigma_x + \sigma_z g_1(a^{\dagger} + a) + \sigma_z g_2[(a^{\dagger})^2 + a^2] \tag{2}$$

Based on spin states, the Hamiltonian (2) can be expanded by the Pauli operator into the matrix form with $\omega = 1$ as follows

$$H = \begin{pmatrix} a^{\dagger}a + g_1\left(a^{\dagger} + a\right) + g_2\left[\left(a^{\dagger}\right)^2 + a^2\right] & \Delta \\ \Delta & a^{\dagger}a - g_1\left(a^{\dagger} + a\right) - g_2\left[\left(a^{\dagger}\right)^2 + a^2\right] \end{pmatrix} \tag{3}$$

Action of $\langle m|$ and $|n\rangle$ on Hamiltonian H yields the following result

$$\langle m|H|n\rangle = \begin{pmatrix} H_{11} & \delta_{m,n}\Delta \\ \delta_{m,n}\Delta & H_{22} \end{pmatrix} \tag{4}$$

with

$$\begin{aligned} &H_{11} = n\delta_{m,n} + g_1A + g_2B, H_{22} = n\delta_{m,n} - g_1A - g_2B \\ &A = \sqrt{n+1}\delta_{m,n+1} + \sqrt{n}\delta_{m,n-1}, B = \sqrt{(n+1)(n+2)}\delta_{m,n+2} + \sqrt{n(n-1)}\delta_{m,n-2} \end{aligned} \tag{5}$$

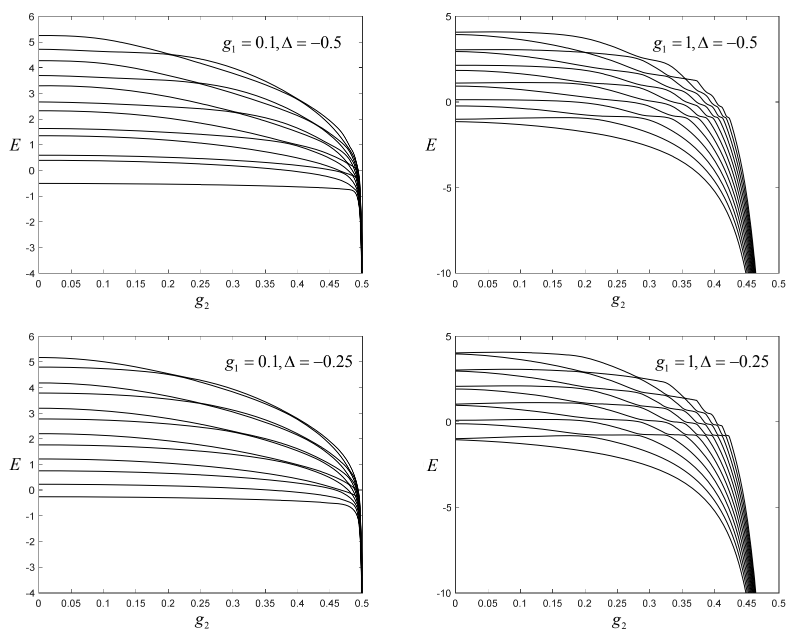

Fig. 1. Energy spectra of g_2.

3 Numerical solution of the two-model mix QRM

In the two-model mix QRM, its Hamiltonian can be written as follows

$$\begin{aligned} H &= \Delta_1\sigma_{1z} + \Delta_2\sigma_{2z} + \omega a^+ a + g_1\sigma_{1x}[a+a^+] + g_2\sigma_{2x}[a+a^+] \\ &+ \lambda_1\sigma_{1x}[a^2+\left(a^+\right)^2] + \lambda_2\sigma_{2x}[a^2+\left(a^+\right)^2] \end{aligned} \tag{6}$$

Where, ω is the frequency of the light field, $2\Delta_1, 2\Delta_2$ is the energy level difference of the two qubits, g, λ is coupling constant of the two qubits interacting with the light field, and σ_{ix} and σ_{iz} are the Pauli operators of the two qubits.

3.1 Numerical solution calculation principle

Based on spin states, the Hamiltonian (6) can be expanded by the Pauli operator into the matrix form in units of $\omega = 1$ as follows

$$H = \begin{pmatrix} A(2\times 2) & \begin{matrix} \Delta_1 & 0 \\ 0 & \Delta_1 \end{matrix} \\ \begin{matrix} \Delta_1 & 0 \\ 0 & \Delta_1 \end{matrix} & B(2\times 2) \end{pmatrix} \tag{7}$$

With

$$\begin{aligned} A &= \begin{pmatrix} \begin{matrix} a^+ a + g_1[a+a^+] + g_2[a+a^+] \\ +\lambda_1[a^2+\left(a^+\right)^2] + \lambda_2[a^2+\left(a^+\right)^2] \end{matrix} & \Delta_2 \\ \Delta_2 & \begin{matrix} a^+ a + g_1[a+a^+] - g_2[a+a^+] \\ +\lambda_1[a^2+\left(a^+\right)^2] - \lambda_2[a^2+\left(a^+\right)^2] \end{matrix} \end{pmatrix} \\ D &= \begin{pmatrix} \begin{matrix} a^+ a - g_1[a+a^+] + g_2[a+a^+] \\ -\lambda_1[a^2+\left(a^+\right)^2] + \lambda_2[a^2+\left(a^+\right)^2] \end{matrix} & \Delta_2 \\ \Delta_2 & \begin{matrix} a^+ a - g_1[a+a^+] - g_2[a+a^+] \\ -\lambda_1[a^2+\left(a^+\right)^2] - \lambda_2[a^2+\left(a^+\right)^2] \end{matrix} \end{pmatrix} \end{aligned} \tag{8}$$

If we set $g_1 + g_2 = n_1$, $\lambda_1 + \lambda_2 = n_2$, $g_1 - g_2 = m_1$, $\lambda_1 - \lambda_2 = m_2$,action of $\langle m|$ and $|n\rangle$ on Hamiltonian H yields the following result

$$\langle m|H|n\rangle = \begin{pmatrix} H'_{11} & \Delta_2\delta_{m,n} & \Delta_1\delta_{m,n} & 0 \\ \Delta_2\delta_{m,n} & H'_{22} & 0 & \Delta_1\delta_{m,n} \\ \Delta_1\delta_{m,n} & 0 & H'_{33} & \Delta_2\delta_{m,n} \\ 0 & \Delta_1\delta_{m,n} & \Delta_2\delta_{m,n} & H'_{44} \end{pmatrix} \tag{9}$$

With (5)

$$\begin{aligned}H_{11} &= n\delta_{m,n} + n_1 A + n_2 B \\ H_{22} &= n\delta_{m,n} + m_1 A + m_2 B \\ H_{33} &= n\delta_{m,n} - m_1 A - m_2 B \\ H_{44} &= n\delta_{m,n} - n_1 A - n_2 B\end{aligned} \tag{10}$$

With the increase of controllable coefficients in the calculation formula, the discussion of the results in this case will become more complicated.

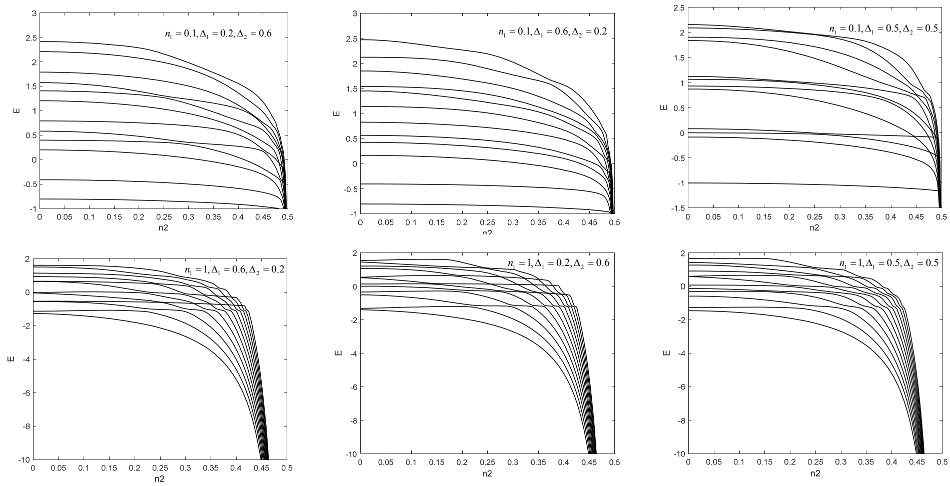

Fig. 2. Energy spectra of n_2 .

The observation results show that when n_1 increases, the law of the two-mode case still remains the same as that of the one-mode case, but with the increase of n_1 , the convergence of the energy spectrum becomes faster. Meanwhile, for Δ_1 and Δ_2 , only a small value of n_1 can have an obvious influence on the change of the energy spectrum.

About symmetry, let's say the wave function

$$\left|a\right\rangle = \sum_{n=0}^{\infty} \sqrt{n!} \begin{pmatrix} a_{1,n} \\ a_{2,n} \\ a_{3,n} \\ a_{4,n} \end{pmatrix} \left|n\right\rangle, \left|a\right\rangle' = i^l \left|a\right\rangle \tag{11}$$

Based of $H\left|a\right\rangle = E\left|a\right\rangle$ with $\left\langle h\right|$ can get

$$\begin{pmatrix} a_{1,m+2} \\ a_{2,m+2} \\ a_{3,m+2} \\ a_{4,m+2} \end{pmatrix} = \begin{pmatrix} \dfrac{(E-m)a_{1,m} - n_1(m+1)a_{1,m+1} - \Delta_2 a_{2,m} - \Delta_1 a_{3,m} - n_1 a_{1,m-1} - n_2 a_{1,m-2}}{n_2 C} \\ \dfrac{(E-m)a_{2,m} - m_1(m+1)a_{2,m+1} - \Delta_2 a_{1,m} - \Delta_1 a_{4,m} - m_1 a_{2,m-1} - m_2 a_{2,m-2}}{m_2 C} \\ \dfrac{(E-m)a_{3,m} + m_1(m+1)a_{3,m+1} - \Delta_2 a_{4,m} - \Delta_1 a_{1,m} + m_1 a_{3,m-1} + m_2 a_{3,m-2}}{-m_2 C} \\ \dfrac{(E-m)a_{4,m} + n_1(m+1)a_{4,m+1} - \Delta_2 a_{3,m} - \Delta_1 a_{2,m} + n_1 a_{4,m-1} + n_2 a_{4,m-2}}{-n_2 C} \end{pmatrix}$$

$$\begin{pmatrix} a_{4,D} \\ a_{3,D} \\ a_{2,D} \\ a_{1,D} \end{pmatrix} = \begin{pmatrix} \dfrac{(E-m)a_{4,m} - n_1 i^1 E' a_{4,E'} - \Delta_2 a_{3,m} - \Delta_1 a_{2,m} - n_1 i^{-1} a_{4,F} + n_2 a_{4,G}}{-n_2 C} \\ \dfrac{(E-m)a_{3,m} - m_1 i^1 E' a_{3,E'} - \Delta_2 a_{4,m} - \Delta_1 a_{1,m} - m_1 i^{-1} a_{3,F} + m_2 a_{3,G}}{-m_2 C} \\ \dfrac{(E-m)a_{2,m} + m_1 i^1 E' a_{2,E'} - \Delta_2 a_{1,m} - \Delta_1 a_{4,m} + m_1 i^{-1} a_{2,F} - m_2 a_{2,G}}{m_2 C} \\ \dfrac{(E-m)a_{1,m} + n_1 i^1 E' a_{1,E'} - \Delta_2 a_{2,m} - \Delta_1 a_{3,m} + n_1 i^{-1} a_{1,F} - n_2 a_{1,G}}{n_2 C} \end{pmatrix} \quad (12)$$

$$C = (m+2)(m+1), D = m+2, E' = m+1, F = m-1, G = m-2$$

The above equation can not satisfy the symmetry of single and double photons such as ± 1 or $\pm i$.

4 Conclusion

For the mix QRM, we extended it from single-mode mix QRM to dual-mode mix QRM, obtained its Hamiltonian and numerically solved it, discussed the influence of different parameters on the result, And the symmetry of it.

References

1. Duan, L., Xie, YF. & Chen, QH. The mixed quantum Rabi model. Sci Rep 9, 18353 (2019) 49.
2. Braak, D., Chen, Q. H., Batchelor, M. T. & Solano, E. Semi-classical and quantum Rabi models: in celebration of 80 years. J. Phys. A 49, 300301 (2016)
3. Pedernales, J. S. Dirac equation in (1+1)-dimensional curved spacetime and the multiphoton quantum Rabi model. Phys. Rev. Lett.120, 160403 (2018)
4. Qing-Hu Chen, Chen Wang, Shu He, Tao Liu, and Ke-Lin Wang.Phys. Rev. A 86, 023822
5. Duan, Liwei et al. Concise analytic solutions to the quantum Rabi model with two arbitrary qubits. Annals of Physics 355 (2015): 121-129.