

# The design of following controller for autonomous vehicle based on leader-follower strategies

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**Abstract.** Aiming at the problem of following control of autonomous vehicle, the following controller is designed based on Leader-follower strategies. First, the kinematic modeling is done. Next, Leader-follower model is used to describe the following structure and  $L-\phi$  method is used to build error system. Then, The speed controller for the follower is designed to achieve the objectives. Finally, the simulation is done by Matlab, the results show that the controller is effective.

**Keywords:** Autonomous vehicle, Leader-follower strategies, Error system, Speed controller.

## 1 Introduction

With the rapid development of economy and science and technology, the number of vehicles is increasing, resulting in a series of problems such as traffic congestion, traffic accidents, environmental pollution and so on. How to make the vehicle run in a certain formation has become an urgent problem to be solved, among which the most basic is vehicle following control. In recent years, multi mobile robot technology has been widely used in industry, agriculture, military and other fields. Through cooperation, multiple mobile robots can complete tasks that are difficult for a single robot to complete<sup>[1]</sup>. Therefore, the cooperative method of multi robot system can be used to coordinate the behavior of vehicles and solve the problem of vehicle following control.

Vehicle following control is generally divided into lateral control<sup>[2]</sup>, longitudinal control<sup>[3]</sup> and horizontal and longitudinal integrated control<sup>[4]</sup>. The research on multi robot formation problem mainly includes the following methods: Leader follower method<sup>[5]</sup>, behavior based method<sup>[6]</sup>, virtual structure method<sup>[7]</sup>, artificial potential field method<sup>[8]</sup>, etc. In 1934, Heinrich Freiherr, a German scholar, first proposed the leader follower strategy and described the leader follower model in detail in his published works<sup>[9]</sup>. The leader follower method was first used in robot formation control in reference<sup>[10]</sup>. There are two main control forms of leader follower method<sup>[11]</sup>:  $L-\phi$  control and  $L-L$  control. The purpose of the  $L-\phi$  control is to make the relative distance and angle between the follower and the pilot reach the given value; The  $L-L$  control mainly solves the problem of relative

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position control of three robots, When the relative distance between the follower and the pilot reaches the given value, the control goal is achieved.

In view of the above problems, the author uses the leader follower method to solve the following control problem in autonomous vehicle formation control. Firstly, the "bicycle" model is used for kinematic modeling of autonomous vehicle, and then a group of leader follower relationships in the team are modeled by  $L-\phi$  control, and the following control error system is obtained. The speed controller of following vehicle is designed, and it is proved that the following vehicle can stably follow the pilot vehicle. Finally, two groups of simulation experiments with different initial conditions are carried out with MATLAB to verify the effectiveness of the designed controller.

## 2 Model description

### 2.1 Kinematics model of autonomous vehicle

The autonomous vehicle is driven by the rear wheel and steered by the front wheel. The mass distribution on the left and right sides is uniform. The "bicycle" model is used for kinematic modeling, as shown in Figure 1. Where G is the centroid of the autonomous vehicle, and in the global coordinate system, its coordinate is  $(x, y)$ ;  $l$  is the distance between the front and rear axles,  $a$ ,  $b$  is the distance from the center of mass to the front axle and the rear axle respectively,  $\theta$  is the front wheel steering angle,  $\alpha$  is heading angle,  $v_u$ ,  $v_w$  represent the longitudinal speed and transverse speed of the autonomous vehicle respectively.

The relationship between front wheel, rear wheel and centroid of autonomous vehicle can be expressed as

$$\begin{cases} x_1 = x - b \cos \alpha \\ y_1 = y - b \sin \alpha \end{cases}, \begin{cases} x_2 = x + a \cos \alpha \\ y_2 = y + b \sin \alpha \end{cases} \quad (1)$$

Autonomous vehicle belongs to vehicle type mobile robot, which satisfies the following nonholonomic constraints<sup>[12]</sup>

$$\begin{cases} \dot{x}_1 \sin \alpha - \dot{y}_1 \cos \alpha = 0 \\ \dot{x}_2 \sin(\alpha + \theta) - \dot{y}_2 \cos(\alpha + \theta) = 0 \end{cases} \quad (2)$$

Deriving formula (1) and substituting it into equation (2)

$$\begin{cases} \dot{x} \sin \alpha - \dot{y} \cos \alpha + b \dot{\alpha} = 0 \\ \dot{x} \sin(\alpha + \theta) - \dot{y} \cos(\alpha + \theta) + a \dot{\alpha} \cos \alpha = 0 \end{cases} \quad (3)$$

The longitudinal motion direction of the autonomous vehicle is taken as the U-axis and the transverse direction as the W-axis to establish the coordinate system, Let the longitudinal velocity of the autonomous vehicle be  $v_u$ , the lateral velocity be  $v_w$ , The velocity of the autonomous vehicle along the x and y directions of the global coordinate system can be expressed as

$$\begin{cases} \dot{x} = v_u \cos \alpha - v_w \sin \alpha \\ \dot{y} = v_u \sin \alpha + v_w \cos \alpha \end{cases} \quad (4)$$

By substituting formula (4) into formula (3), we can get

$$v_w = b \dot{\alpha} = \frac{b}{l} \cdot v_u \cdot \tan \theta \quad (5)$$

## 2.2 Leader-follower structure

The pilot vehicle is defined as  $V_l$ , and the following vehicle is defined as  $V_f$ , They constitute a typical leader follower structure in a fleet, as shown in Figure 2. The centroid coordinates of the pilot vehicle and the following vehicle are  $(x_l, y_l)$  and  $(x_f, y_f)$ ;  $L$  is the distance between the centroid of the pilot vehicle and that of the following vehicle;  $\phi$  is the angle between  $L$  and the axle of the pilot vehicle;  $\theta_l$  and  $\theta_f$  are the front wheel deflection angles of the pilot car and the following vehicle respectively;  $\alpha_l$  and  $\alpha_f$  are respectively the heading angles of the pilot vehicle and the following vehicle.

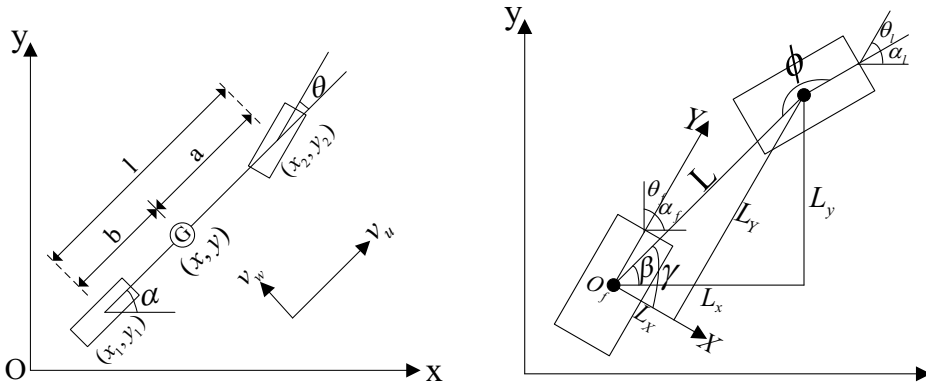


Fig. 1. Kinematic model.

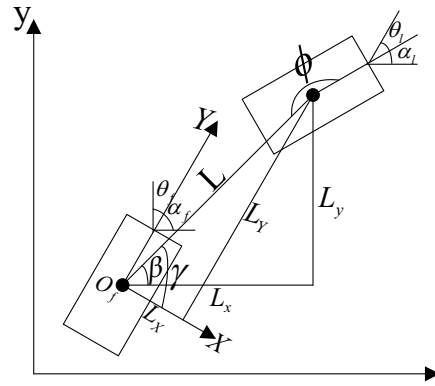


Fig. 2. Leader-follower structure.

If the angle between the positive direction of  $L$  and  $x$ -axis is  $\beta$ , then the angle  $\phi$  between the central axis of the pilot vehicle and  $L$  can be expressed as

$$\phi = \beta + \pi - \alpha_l \tag{6}$$

The relative distance  $L$  between the leading vehicle and the following vehicle is decomposed into  $L_x$  in the  $x$ -axis direction and  $L_y$  in the  $y$ -axis direction.

$$L = \left( L_x^2 + L_y^2 \right)^{\frac{1}{2}} \tag{7}$$

Formula(8) is obtained by deriving formula (7).

$$\dot{L} = \frac{L_x \dot{L}_x + L_y \dot{L}_y}{L} \tag{8}$$

The position relationship among the two vehicles,  $L_x$  and  $L_y$  can be expressed as

$$\begin{cases} L_x = x_l - x_f \\ L_y = y_l - y_f \end{cases} \tag{9}$$

Formula(10) is obtained by deriving formula (9).

$$\begin{cases} \dot{L}_x = \dot{x}_l - \dot{x}_f \\ \dot{L}_y = \dot{y}_l - \dot{y}_f \end{cases} \tag{10}$$

Replace formula (4) into formula (10) to obtain formula (11).

$$\begin{cases} \dot{L}_x = v_{ul} \cos \alpha_l - v_{wf} \sin \alpha_l - v_{uf} \cos \alpha_f + v_{wf} \sin \alpha_f \\ \dot{L}_y = v_{ul} \sin \alpha_l + v_{wf} \cos \alpha_l - v_{uf} \sin \alpha_f - v_{wf} \cos \alpha_f \end{cases} \tag{11}$$

where  $v_{ul}$  and  $v_{wl}$  are the longitudinal speed and lateral speed of the pilot vehicle respectively,  $v_{uf}$  and  $v_{wf}$  are the longitudinal speed and lateral speed of the following vehicle respectively.

Replace formula (11) into formula (8) to obtain formula (12).

$$\dot{L} = v_{ul} \cos(\beta - \alpha_l) + v_{wl} \sin(\beta - \alpha_l) - v_{uf} \cos(\beta - \alpha_l) - v_{wf} \sin(\beta - \alpha_l) \quad (12)$$

Defining  $\gamma = -\frac{\pi}{2} + \phi + (\alpha_l - \alpha_f)$ , then substitute it into formula (12) to get formula (13).

$$\dot{L} = -v_{ul} \cos \phi - v_{wl} \sin \phi - v_{uf} \sin \gamma + v_{wf} \cos \gamma \quad (13)$$

Formula(14) is obtained by deriving formula (6).

$$\dot{\phi} = \frac{\dot{L}_y \cos \beta - \dot{L}_x \sin \beta}{L} - \dot{\alpha}_l \quad (14)$$

Replace formula (11) into formula (14) to obtain formula (15).

$$\dot{\phi} = \frac{v_{ul} \sin \phi - v_{wl} \cos \phi - v_{uf} \cos \gamma - v_{wf} \sin \gamma}{L} - \frac{v_{wl}}{b} \quad (15)$$

### 2.3 Vehicle following error system

On the basis of the original coordinate system, with the longitudinal speed direction of the vehicle as the X axis and the transverse speed direction as the Y axis, the auxiliary coordinate system  $XO_f Y$  is established, as shown in Fig. 2.

In the auxiliary coordinate system, the position errors of the following vehicles in X-axis, Y-axis and heading angle are established

$$\begin{cases} e_X = X_D - X_f = L_D \cos \gamma_D - L \cos \gamma \\ e_Y = Y_D - Y_f = L_D \sin \gamma_D - L \sin \gamma \\ e_\alpha = \alpha_l - \alpha_f \end{cases} \quad (16)$$

Formula (16) can be reduced to

$$\begin{cases} L_D \cos \gamma_D = e_X + L \cos \gamma \\ L_D \sin \gamma_D = e_Y + L \sin \gamma \end{cases} \quad (17)$$

Derivation of formula (16)

$$\begin{cases} \dot{e}_X = \dot{L}_D \cos \gamma_D - L_D \sin \gamma_D (\dot{\phi}_D + \dot{e}_\alpha) - \dot{L} \cos \gamma + L \sin \gamma (\dot{\phi} + \dot{e}_\alpha) \\ \dot{e}_Y = \dot{L}_D \sin \gamma_D + L_D \cos \gamma_D (\dot{\phi}_D + \dot{e}_\alpha) - \dot{L} \sin \gamma - L \cos \gamma (\dot{\phi} + \dot{e}_\alpha) \\ e_\alpha = \frac{v_{wl}}{b} - \frac{v_{wf}}{b} \end{cases} \quad (18)$$

Substitute formula (17) in formula (18) and simplify it

$$\begin{cases} \dot{e}_X = \dot{L}_D \cos \gamma_D - L_D \sin \gamma_D \dot{\phi}_D - \frac{1}{b} v_{wl} L_D \sin \gamma_D + \frac{1}{b} e_Y v_{wf} + v_{ul} \sin e_\alpha + v_{wl} \cos e_\alpha - v_{wf} \\ \dot{e}_Y = \dot{L}_D \sin \gamma_D + L_D \cos \gamma_D \dot{\phi}_D + \frac{1}{b} v_{wl} L_D \cos \gamma_D - \frac{1}{b} e_X v_{wf} - v_{ul} \cos e_\alpha + v_{wl} \sin e_\alpha + v_{wf} \\ \dot{e}_\alpha = \frac{v_{wl}}{b} - \frac{v_{wf}}{b} \end{cases} \quad (19)$$

### 3 Controller design

Through the analysis of formula (19), we can see that The position error and angle error of the following vehicle at the next moment not only depends on the position and angle error of the following vehicle and the leader vehicle, but also depends on the speed and angular velocity of the following vehicle and the relative angle between the following vehicle and the leader vehicle. In order to stabilize the motion system composed of following vehicle and pilot vehicle, the velocity input of following vehicle is given as follows:

$$\begin{aligned} v_{uf} &= -k_1 e_y - \dot{L}_D \sin \gamma_D - L_D \cos \gamma_D \dot{\phi}_D - \frac{1}{b} v_{wl} L_D \cos \gamma_D + v_{ul} \cos e_\alpha - v_{wl} \sin e_\alpha \\ v_{wf} &= k_2 e_x + \dot{L}_D \cos \gamma_D - L_D \sin \gamma_D \dot{\phi}_D - \frac{1}{b} v_{wl} L_D \sin \gamma_D + v_{ul} \sin e_\alpha + v_{wl} \cos e_\alpha + W \end{aligned} \quad (20)$$

where  $k_1 > 0$ ,  $k_2 > 0$ ,  $W$  is a supplement to the lateral velocity  $v_{wf}$  of the following vehicle.

The Lyapunov function of the system is selected as follows:

$$V = \frac{e_x^2 + e_y^2}{2} + k_3 (1 - \cos e_\alpha) \quad (21)$$

where  $k_3 > 0$ , so  $V \geq 0$ , if and only if  $e_x = e_y = e_\alpha = 0$ ,  $V = 0$ .

Formula(22) is obtained by deriving formula (21).

$$\dot{V} = e_x \dot{e}_x + e_y \dot{e}_y + k_3 \dot{e}_\alpha \sin e_\alpha \quad (22)$$

Replace formula (19) into formula (22) to obtain formula (23).

$$\begin{aligned} \dot{V} &= e_x \left( \dot{L}_D \cos \gamma_D - L_D \sin \gamma_D \dot{\phi}_D - \frac{1}{b} v_{wl} L_D \sin \gamma_D + \frac{1}{b} e_y v_{wf} + v_{ul} \sin e_\alpha + v_{wl} \cos e_\alpha - v_{wf} \right) \\ &+ e_y \left( \dot{L}_D \sin \gamma_D + L_D \cos \gamma_D \dot{\phi}_D + \frac{1}{b} v_{wl} L_D \cos \gamma_D - \frac{1}{b} e_x v_{wf} - v_{ul} \cos e_\alpha + v_{wl} \sin e_\alpha + v_{wf} \right) \\ &+ \frac{k_3}{b} \sin e_\alpha (v_{wl} - v_{wf}) \end{aligned} \quad (23)$$

By substituting formula (20) into formula (23), formula 24 is obtained

$$\dot{V} = -k_1 e_y^2 - k_2 e_x^2 - e_x W + \frac{k_3}{b} \sin e_\alpha \begin{pmatrix} v_{wl} - k_2 e_x + \frac{1}{b} v_{wl} L_D \sin \gamma_D - v_{ul} \sin e_\alpha \\ -v_{wl} \cos e_\alpha - \dot{L}_D \cos \gamma_D + L_D \sin \gamma_D \dot{\phi}_D - W \end{pmatrix} \quad (24)$$

In order to keep the system stable when  $v_{ul} = 0$ , replace  $v_{ul}$  in the above formula with a number  $-k_4$  approaching 0, and  $k_4 > 0$ , Then formula (24) can be changed into

$$\dot{V} = -k_1 e_y^2 - k_2 e_x^2 - \frac{k_3 k_4}{b} \sin^2 e_\alpha - \left( e_x + \frac{k_3}{b} \sin e_\alpha \right) W + \frac{k_3}{b} \sin e_\alpha \begin{pmatrix} v_{wl} - k_2 e_x + \frac{1}{b} v_{wl} L_D \sin \gamma_D + k_4 \sin e_\alpha \\ -v_{wl} \cos e_\alpha - \dot{L}_D \cos \gamma_D + L_D \sin \gamma_D \dot{\phi}_D \end{pmatrix} \quad (25)$$

According to formula (25),  $-k_1 e_y^2 - k_2 e_x^2 - \frac{k_3 k_4}{b} \sin^2 e_\alpha \leq 0$ , let the remainder be the fourth term R, so

$$\begin{aligned} R &= - \left( e_x + \frac{k_3}{b} \sin e_\alpha \right) W + \frac{k_3}{b} \sin e_\alpha \begin{pmatrix} v_{wl} - k_2 e_x + \frac{1}{b} v_{wl} L_D \sin \gamma_D + k_4 \sin e_\alpha \\ -v_{wl} \cos e_\alpha - \dot{L}_D \cos \gamma_D + L_D \sin \gamma_D \dot{\phi}_D \end{pmatrix} \\ &\leq \left( |e_x| + \frac{k_3}{b} \right) |W| + \frac{k_3}{b} \left( 2v_{wl} + k_2 |e_x| + \frac{1}{b} v_{wl} L_D + k_4 + \dot{L}_D + L_D |\dot{\phi}_D| \right) = C \end{aligned} \quad (26)$$

In formula (26),  $e_x$  tends to 0,  $v_{wl}$  and  $L_D$  are the given value, let  $W$  be bounded, so  $R$  must be bounded, namely  $\dot{V} \leq -k_1 e_\gamma^2 - k_2 e_x^2 - \frac{k_3 k_4}{b} \sin^2 e_\alpha + C$ , the stability of the system can be ensured by adjusting the parameters in the controller.  $W$  is selected as the following form:

$$W = \frac{k_3}{b} \times \frac{2v_{wl} + k_2 |e_x| + \frac{1}{b} v_{wl} L_D + k_4 + \dot{L}_D + L_D |\dot{\phi}_D|}{|e_x| + \frac{k_3}{b}} \tag{27}$$

Thus, the controller designed for the following vehicle is

$$\begin{aligned} v_{uf} &= -k_1 e_\gamma - \dot{L}_D \sin \gamma_D - L_D \cos \gamma_D \dot{\phi}_D - \frac{1}{b} v_{wl} L_D \cos \gamma_D + v_{ul} \cos e_\alpha - v_{wl} \sin e_\alpha \\ v_{wf} &= k_2 e_x + \dot{L}_D \cos \gamma_D - L_D \sin \gamma_D \dot{\phi}_D - \frac{1}{b} v_{wl} L_D \sin \gamma_D + v_{ul} \sin e_\alpha + v_{wl} \cos e_\alpha \\ &+ \frac{k_3}{b} \times \frac{2v_{wl} + k_2 |e_x| + \frac{1}{b} v_{wl} L_D + k_4 + \dot{L}_D + L_D |\dot{\phi}_D|}{|e_x| + \frac{k_3}{b}} \end{aligned} \tag{28}$$

### 4 Simulation experiment

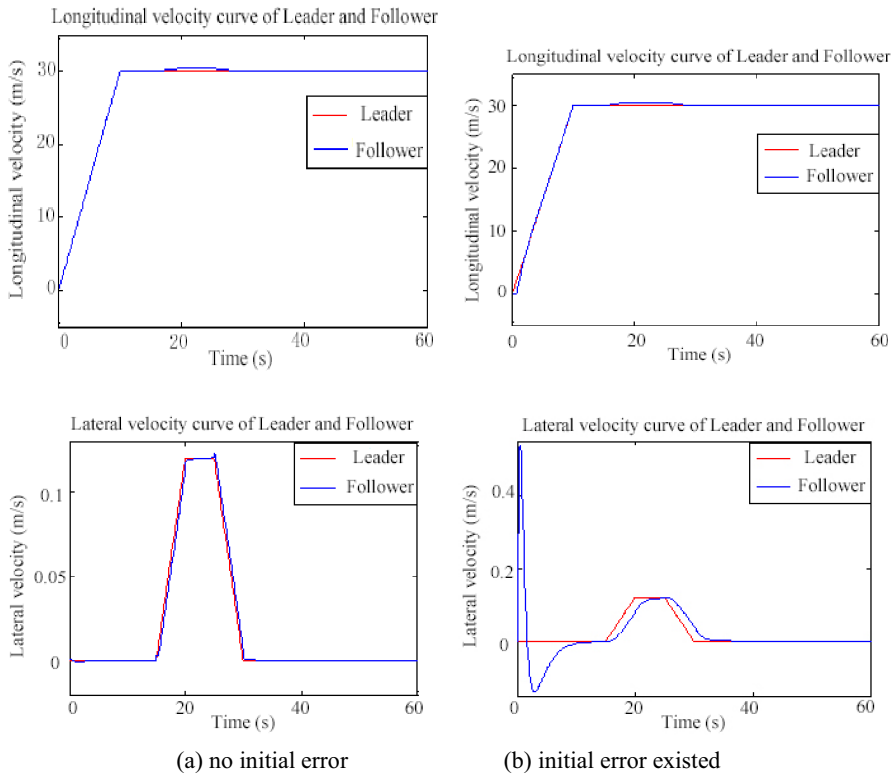
Initial position of pilot vehicle(0, 0) , axle length of following vehicle  $a = b = 2.5m$  , controller parameters  $k_1 = 1.2$  ,  $k_2 = 0.6$  ,  $k_3 = 0.01$  ,  $k_4 = 0.001$  . The speed variation of pilot vehicle is shown in

**Table 1.** Speed of the leader.

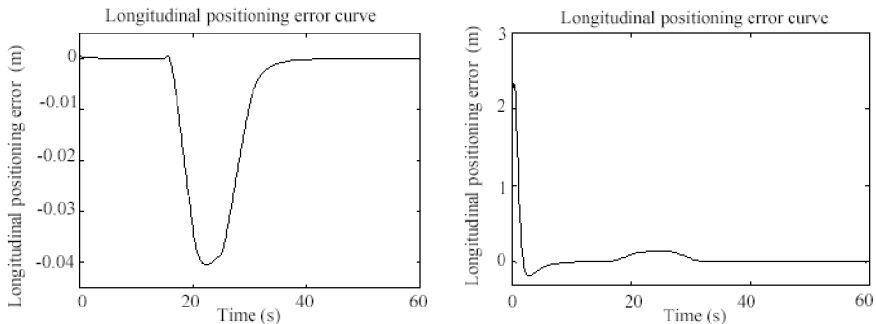
| Time(s)                                   | [0,10) | [10,15) | [15,20) | [20,25) | [25,30) | [30,60] |
|---|--------|---------|---------|---------|---------|---------|
| Longitudinal acceleration $a_{ul}(m/s^2)$ | 3      | 0       | 0       | 0       | 0       | 0       |
| Longitudinal velocity $v_{ul}(m/s)$       | 0-30   | 30      | 30      | 30      | 30      | 30      |
| Lateral acceleration $a_{wl}(m/s^2)$      | 0      | 0       | 0.024   | 0       | -0.024  | 0       |
| Lateral velocity $v_{wl}(m/s)$            | 0      | 0       | 0-0.12  | 0.12    | 0.12-0  | 0       |

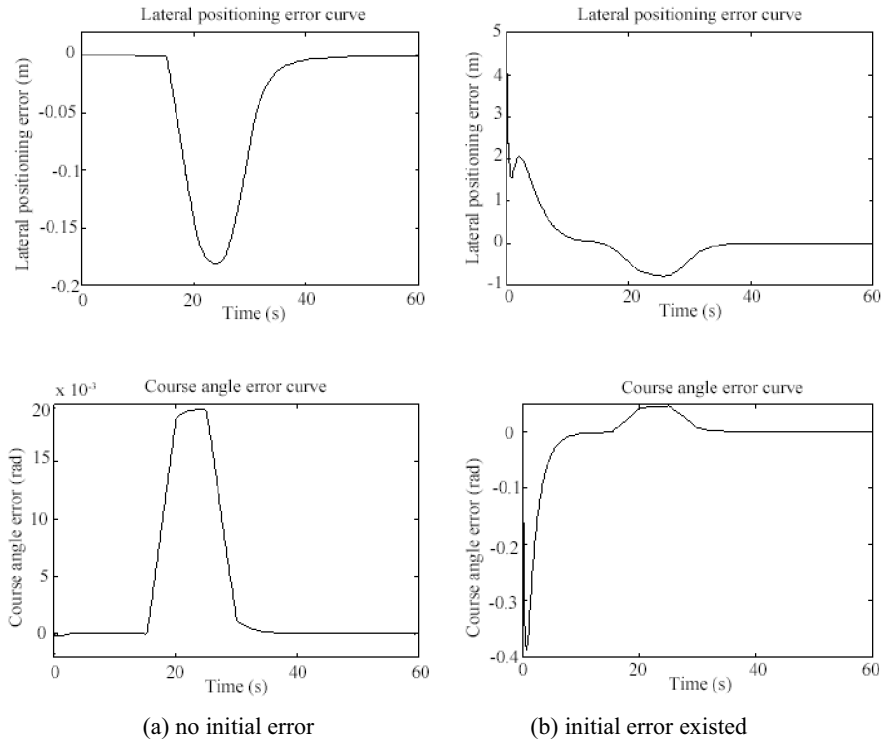
Two groups of simulation experiments were carried out, the first group (without initial error): Follower vehicle initial position(-8.66, -5) ,  $L_D = 10m$  ,  $\phi_D = \frac{7\pi}{6} rad$  , Initial error  $e_{x0} = 0$  ,  $e_{y0} = 0$  ,  $e_{\alpha0} = 0$  ; The second group (with initial error): Follower vehicle initial position(-20, -15) ,  $L_D = 20m$  ,  $\phi_D = \frac{7\pi}{6} rad$  , Initial error  $e_{x0} = 5m$  ,  $e_{y0} = 2.68m$  ,  $e_{\alpha0} = 0$  . The speed variation curve of the two vehicles is shown in Fig. 3, and the error variation curve is shown in Fig. 4.

In the case of no initial error, it can be seen from Figure 3 (a) that the speed of the following vehicle fluctuates a little at the beginning, and then changes with the pilot vehicle, and the two are basically the same. As can be seen from Figure 4 (a), the system error tends to zero after 35s, which proves that the following vehicle can stably follow the pilot vehicle; In the case of initial error, it can be seen from Figure 3 (b) that due to the existence of initial error, the following vehicle speed fluctuates greatly at the beginning, and then the fluctuation gradually weakens. After 35s, it is basically consistent with that of the pilot vehicle. As can be seen from Figure 4 (b), the system error basically approaches 0 after 35s, which proves that the designed controller can correct the error and make the following vehicle stable follow the pilot vehicle.



**Fig. 3.** The speed curve of leader and follower.





**Fig. 4.** The error curve of leader and follower.

## 5 Conclusion

The research object of this paper is autonomous vehicle, and the design of autonomous vehicle following controller based on leader follower method is studied. The "bicycle" model is used to establish the kinematics model of autonomous vehicle, and the leader follower structure is used to describe the following control problem of the autonomous vehicle. The following error system is established by  $L-\phi$  control method and the speed controller of the following vehicle is designed. The simulation experiments are carried out with MATLAB under the conditions of no initial error and with initial error. The results show that the controller involved can be used for vehicle following Follow the control.

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