

Modelling of the shear resistance of self-stressed beams reinforced with FRP applying the Critical Shear Crack Theory

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Abstract This paper presents a mechanical model of the shear resistance based on Critical Shear Crack Theory (CSCT) and its application for the checking of the shear ultimate state of self-stressed elements reinforced with FRP bars. The shear force, which is transmitted through the inclined crack by aggregate interlock, residual tensile strength, dowel action and inclined chord of the compression concrete, is calculated depending on the value of the inclined crack opening, determined according to the modified law “bond-slip” for FRP bars. The reliability of the proposed approach is confirmed by comparison both with the results of our own experimental investigations and with numerous research results by various authors.

Keywords: shear resistance, modified law “bond-slip”, model of CSCT, FRP bars, beam.

Introduction

In recent construction practice, it is recommended to use fiber-reinforced polymer composites (FRP) bars as an alternative to traditional steel reinforcement of concrete elements. This is due to the fact, that this type of reinforcement has high corrosion resistance, low density and high strength in relation to steel. However, because of the low modulus of elasticity, which is comparable to the modulus of elasticity of concrete, it imposed several restrictions, especially when using such type of reinforcement in the bending elements. Additional restrictions are associated with different values of the coefficients of thermal linear expansion in the longitudinal and transverse directions [1].

Despite the increase in the production of polymer composites, their practical application remains limit, both due to the significant variability of their properties from different manufacturers, and the limited design models included in the design standards for traditional reinforced concrete structures or the special guides. Most models of shear resistance of FRP-reinforced elements introduced in codes and standards differ from the design shear resistance models for elements with traditional reinforcement only by the ratio of the elastic moduli of FRP and steel reinforcement E_f/E_s (for example, ACI 440-15 [2], ISIS-M03-01 [3], JSCE 97 [4], BISE-99 [5], CNR-DT203-2006 [6], ACI 318 [7]). At the same time, the question of whether the design models of the shear resistance of FRP-reinforced elements should be included in the general standards, or whether it is necessary to create special standards,

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remains open for discussion. In particular, in the design of concrete members reinforced with FRP bars, the effect, for example, of increased deformability of FRP bars on shear resistance mechanisms, should be established. So, according to general concepts, when using FRP-bars as a longitudinal reinforcement, cracks with a wider opening are formed in bending elements compared to elements reinforced with steel bars. In this paper, we present some propositions for modification of a mechanical model of the shear resistance of concrete elements reinforced with FRP bars, based on the Critical Shear Crack Theory (CSCT) [8]. Here, in contrast to the known model, the width of the inclined crack opening at the level reinforcement level is determined using the law “bond-slip” for FRP bars included in *fib* Model Code 2010 [9]. The results of verification of the modified calculation model of shear resistance model against the background of our own experimental data showed satisfactory agreement and made it possible to develop a failure criterion for FRP-reinforced elements without shear reinforcement.

1 Some basic assumptions of Critical Shear Crack Theory (CSCT)

The critical shear crack theory (CSCT) proposed by Muttoni et al. [8] relies on a mechanical approach that takes into account the contribution of all possible of the shear-transfer actions. A detailed description of the CSCT, and also the contribution of the shear-transfer actions, are presented in [10-13]. This theory based on the assumption that the shear resistance of reinforced concrete elements without stirrups and the deformation capacity are related by the failure criterion, while lower values of the shear resistance are associated with a high deformation capacity, and therefore a wider crack opening. This assumption means that the width of the critical shear crack opening is proportional to the product of the longitudinal strains of reinforcement in the control section by the effective depth of the element $w \infty \varepsilon_s \cdot d$. According to the research results presented in [13], cracks at a short distance can develop at the level of longitudinal reinforcement, after which they incorporate, and the sum of all cracks related to the critical shear crack gives an approximate linear profile of the horizontal component of the crack opening width (see Figure 1).

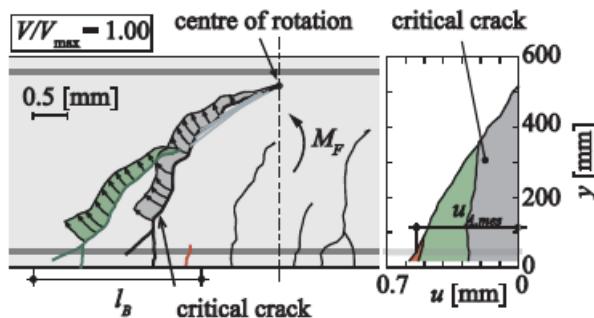


Fig. 1. Crack kinematics, center of rotation and horizontal opening u_A of the cracks tributary to the opening of the critical shear crack in a region of length l_B [13]

It should be noted that this form of notation $\varepsilon_s \cdot d$ assumes a uniform distribution of strains ε_s along the length l_b .

This tributary length is practically constant at peak load [13] and can be approximated by the expression:

$$l_b = d - c, \quad (1)$$

where c is the depth of the compression zone of concrete.

Based on [14], it was proposed to calculate the horizontal component of the inclined crack opening width by the following expression:

$$u_A = \varepsilon_s \cdot l_b, \quad (2)$$

Taking into account that the width of the inclined crack opening is the main basic variable in the model of shear strength CSCT and the value l_b is obtained experimentally [13], the question arises about the applicability of expression (2) for the case of concrete elements reinforced with FRP bars.

In this regard, it was proposed to make adjustments to the original CSCT model, related to the determination of the crack opening width, based on the “*bond-slip*” law for FRP bars.

2 Inclined shear crack opening depending on the development length along the longitudinal reinforcement

According to the proposed model, the width of inclined crack opening in elements reinforced with FRP bars, depending on the force (stress) in the longitudinal reinforcement, is determined from the following expression:

$$w_A = u_A = 2 \cdot s(l_b), \quad (3)$$

where $s(l_b)$ – end slip depending on the development length.

The value of the end slip of FRP bar is determined by the modified “*bond-slip*” law proposed by Eligehausen et al [15] (see Figure 2) and the procedure for determining the development length of FRP bars proposed by Cosenza et al [16].

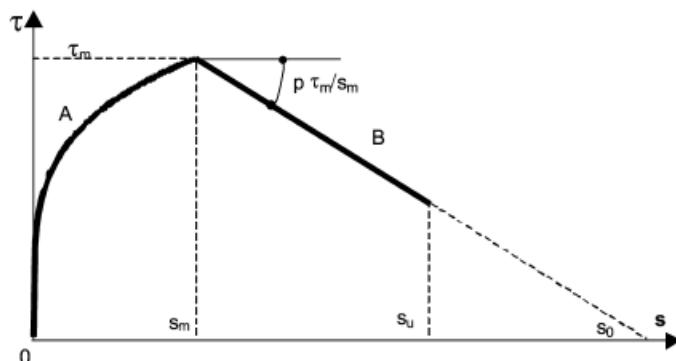


Fig. 2. Modified “ τ - s ” law [15]

The analytical solutions proposed in [16] make it possible to obtain slip, normal stresses, and bond stresses distributions along the reinforcing bar, and also make it possible to determine the development length.

The procedure for calculating the crack opening width starts with determining the tensile stresses σ_s in the reinforcing bars in the design section of the element according to the following simplified expression:

$$\sigma_s = \left(\frac{M_{Ed}}{z} + \frac{V_{Ed}}{2} \cdot \cot \theta \right) \cdot \frac{1}{A_s}, \quad (4)$$

where M_{Ed} – bending moment at the design section;

$z = 0,9 \cdot d$;

V_{Ed} – shear force at the design section;

θ – angle of inclination of the shear crack to the longitudinal axis;

A_s – cross-sectional area of bars.

There are two design cases depending on the values of tensile stresses: *case 1* – if $0 < \sigma_s \leq \sigma_1$ (ascending branch of the “ τ - s ” diagram (see Figure 2), where σ_1 are tensile stresses in reinforcing bars corresponding to slip s_m (peak point of the “ τ - s ”); *case 2* – if $\sigma_1 < \sigma_s \leq \sigma_2$ (softening branch of the “ τ - s ” diagram (see Figure 2), where σ_2 are tensile stresses in reinforcing bars corresponding to slip s_0 (see Figure 2).

The values of tensile stresses σ_1 and σ_2 , according to [16], can be determined by expressions (5) and (6), respectively.

$$\sigma_1 = \sigma(s_m) = \sqrt{\frac{8 \cdot E_f}{\varnothing} \cdot \frac{\tau_m \cdot s_m}{(1+\alpha)}}, \quad (5)$$

where E_f – elastic modulus of the FRP bar;

\varnothing – bar diameter;

τ_m – maximum bond strength;

s_m – slip at peak bond strength;

α – coefficient which describes the ascending branch “ τ - s ” law [15].

$$\sigma_2 = \sigma(s_0) = \sqrt{\frac{8 \cdot E_f}{\varnothing} \cdot A_{\tau,\max}}, \quad (6)$$

$$\text{where } A_{\tau,\max} = \frac{\tau_m \cdot s_m}{1+\alpha} + \frac{\tau_m \cdot s_m}{p}, \quad (7)$$

p – coefficient which defines the softening branch “ τ - s ” law [15].

For case 1 ($0 < \sigma_s \leq \sigma_1$)

The value of the slip corresponding to the stresses in the reinforcing bar $0 < \sigma_s \leq \sigma_1$ is determined by the following expression, according to [16]:

$$s(l_b) = \left[\frac{2 \cdot \tau_m}{E_f \cdot \varnothing \cdot s_m^\alpha} \cdot \frac{(1-\alpha)^2}{(1+\alpha)} \right]^{1/(1-\alpha)} \cdot l_b^{2/(1-\alpha)} \leq s_m, \quad (8)$$

where l_b – the development length which is determined by the expression (9):

$$l_b = l_0 \cdot \left(\frac{\sigma_1}{\sigma_s} \right)^{2-\alpha/(1+\alpha)} \cdot \left[\frac{1+\alpha}{1-\alpha} \right], \quad (9)$$

$$l_0 = l_0(\sigma_s) = \frac{\emptyset \cdot \sigma_s}{4 \cdot \tau_m}, \quad (10)$$

Further, the width of the inclined crack opening is determined by expression (3).

For case 2 ($\sigma_l < \sigma_s \leq \sigma_2$)

In this case, the value of slip is determined from the following expression, according to [16]:

$$s(l_b) = \frac{1}{p} \cdot \left[1 + p - \cos(w \cdot (l_b - l_m)) + \sqrt{\frac{2 \cdot p}{1 + \alpha}} \cdot \sin(w \cdot (l_b - l_m)) \right] \cdot s_m, \quad (11)$$

$$\text{where } w = \frac{1}{l_m} \cdot \sqrt{\frac{2 \cdot p}{1 + \alpha}} \cdot \frac{(1 + \alpha)}{(1 - \alpha)}, \quad (12)$$

$$l_m = \frac{\sigma_1 \cdot \emptyset}{4 \cdot \tau_m} \cdot \frac{(1 + \alpha)}{(1 - \alpha)}, \quad (13)$$

$$l_b = l_m \cdot \left[1 + \sqrt{\frac{(1 - \alpha)^2}{2 \cdot p \cdot (1 + \alpha)}} \cdot \left(\arcsin \sqrt{\frac{A_{\tau_2}}{A_{\tau_{\max}}}} - \arcsin \sqrt{1 - \frac{\emptyset \cdot \sigma_s^2}{8 \cdot E_f \cdot A_{\tau_{\max}}}} \right) \right], \quad (14)$$

As in the first case, the width of the inclined crack opening is determined by expression (3).

3 Calculation procedure for the shear resistance of concrete elements reinforced with FRP bars according to the modified CSCT model

The iterative procedure for calculation of the shear resistance of a beam element includes the following steps:

- 1) Choose a location of the critical shear crack x_A ;
- 2) Take the value of the design shear force in the section A ($V_{Ed,A}$);
- 3) Calculate the angle of the critical shear crack β_{AB} as a function $\alpha_A = \frac{M_{Ed,A}}{V_{Ed,A} \cdot d}$:

$$\beta_{AB} = \frac{\pi}{4} \cdot \left(1 + \frac{\alpha_A^{1/3}}{3} \right), \quad (15)$$

4) Calculate the width of the inclined crack opening $w_A = u_A$ in the selected section A according to the proposed procedure (expressions (3)-(14)) depending on the force (stress) in the longitudinal reinforcement;

5) Calculate, as a function of the shape of the crack and its kinematics, the residual tensile strength force V_{Res} , the aggregate interlock force V_{Agg} , the dowel action V_{Dowel} , the contribution of the compression zone V_{Comp} according to the provisions of the CSCT [14], take into account the width of the inclined crack opening defined in step 4.

The shear capacity is as the sum of these contributions:

$$V_{C,R} = V_{Res} + V_{Agg} + V_{Dowel} + V_{Comp}, \quad (16)$$

6) Compare $V_{Ed,A} = V_{C,R}$. If this condition is not met, go to step 2.

This iterative procedure should be carried out for sections $x_{A,i}$ along the length of the element's span and the minimum value $V_{C,R}$ is selected. We should consider this value as the ultimate shear force.

4 Location of the critical shear crack and the contribution of the different shear-transfer actions under various loading conditions

We carried our own experimental studies to estimate the accuracy of the proposed design model and to determine the control section x_A . The experimental studies included 9 beam elements reinforced with FRP bars. Also, 7 beam elements with traditional reinforcement were tested to distinguish between the types of reinforcement. These beams were tested under various loading conditions (uniformly distributed load and point forces). Geometry, reinforcement arrangement and basic parameters of beams shown in Figure 3.

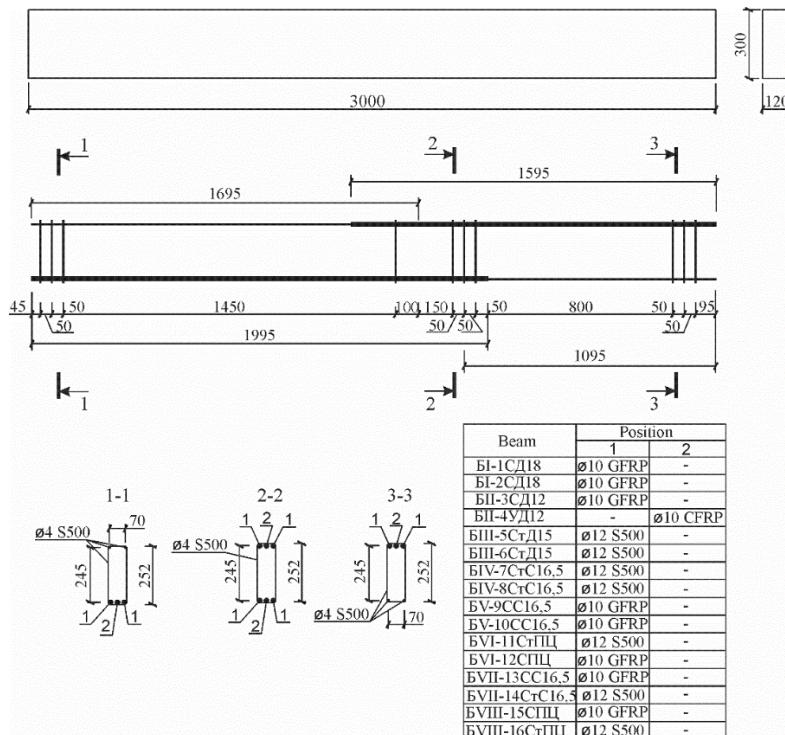


Fig. 3. Geometry, reinforcement arrangement and parameters of tested beams

The test beams were reinforced with bars GFRP Ø10 (Maperod G and AKS 10), CFRP Ø10 (Maperod C) and steel reinforcement Ø12S500. Main mechanical parameters of reinforcement are listed in table 1.

Table 1. FRP reinforcement mechanical parameters

Class	Diameter \varnothing , mm	Yield strength, f_yk , MPa	Strength, f_y , MPa	Modulus of elasticity $E_{fx} \times 10^3$, MPa	Ultimate strain, %
Maperod G	10	-	760	40,8	2
Maperod C	10	-	2000	155	1,5
AKS 10	10*	-	1244	32	2,7
S500	12	652	749	200	-
	12**	678	778	200	-

Notes:

1. «*» – this reinforcement was used in beams of the series БV, БVI-12СПЦ, БVII-13CC16,5 and БVIII-15СПЦ.
2. «**» – this reinforcement was used in beams of the series БIV, БVI-11СтПЦ, БVII-14СтC16,5 and БVIII-16СтПЦ.

In the own experimental study, Self-stressing concrete with different energy and Ordinary Portland-cement concrete were used (see Table 2).

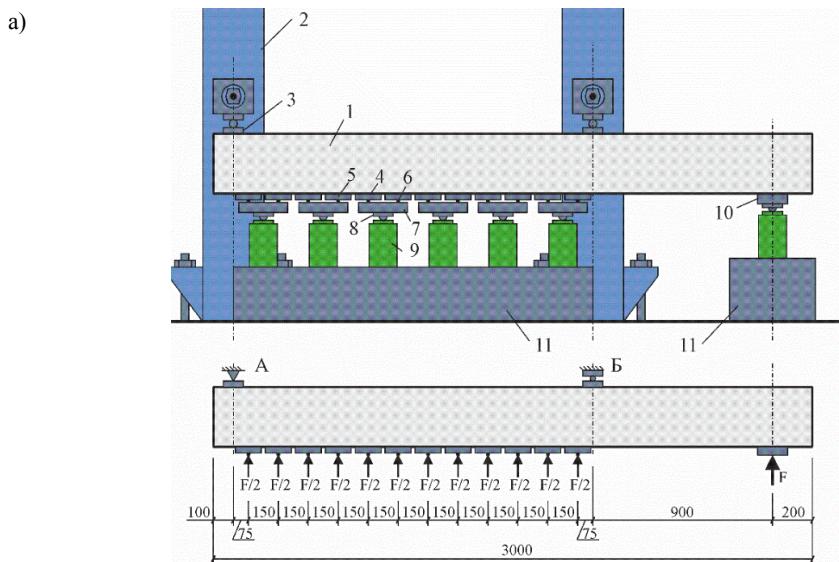
Table 2. Main characteristics of concrete at the time of static tests

Series	Beam	Mechanical concrete characteristics at the time of static testes				Expansion characteristics of concrete at the time of static tests	
		$f_{cm,cube}$, MPa	$f_{cm,prizma}$, MPa	$f_{cm,cyl}$, MPa	E_{cm} , GPa		
						$\varepsilon_{CE,f}$, %	$f_{CE,k}$, MPa
I	БI-1СД18	-	51,1	-	31,7	1,12	1,8
	БI-2СД18	-	55,1	-			1,1
II	БII-3СД12	77,1	76,5	56,1	43,3	-	-
	БII-4УД12		-	-			-
III	БIII-5СтD15	86,2	82,4	67,1	46,3	0,05	0,45
	БIII-6СтD15	86,2	82,4	67,1			0,50
IV	БIV-7СtC16,5	29,3	47,8	-	29,8	0,64	2,70
	БIV-8СtC16,5	17,4	43,5	-			2,90
V	БV-9CC16,5	58,1	52,0	34,5	32,7	0,45	1,80
	БV-10CC16,5	54,6	54,4	32,3	34,7		1,65
VI	БVI-11СtПЦ	56,1	47,1	36,1	40,5	-	-
	БVI-12СПЦ						-
VII	БVII-13CC16,5	29,2	50,2	21,9	31,2	0,42	1,90
	БVII-14СtC16,5	33,9	53,8	31,2	32,6		1,95
VIII	БVIII-15СПЦ	37,1	30,7	25,0	32,1	-	-
	БVIII-16СtПЦ						-

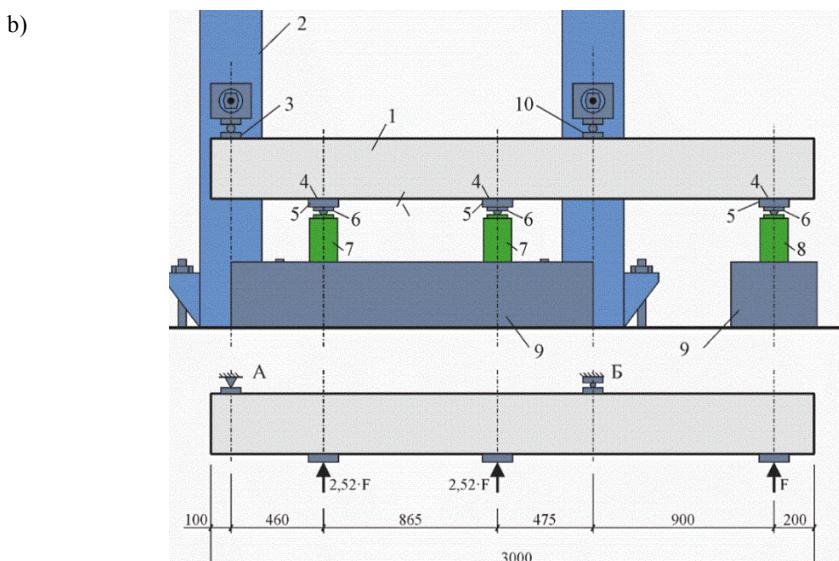
Notes:

- $f_{cm,cube}$ – cube compressive strength at 28 days;
 $f_{cm,prizma}$ – mean concrete compressive strength based on prisms (100x100x400 mm);
 $f_{cm,cyl}$ – mean concrete compressive strength based on cylinders ($\varnothing 150$, $h=300$ mm);
 $f_{CE,k}$ – self-stresses grade based on prisms (100x100x400 mm);
 $\varepsilon_{CE,f}$ – strain of free expansion based on cylinders ($\varnothing 150$, $h=300$ mm).

Beams were tested under static loading with monotonically increasing uniformly distributed load and point forces. Test setups are shown in Figure 4.



1 – experimental beam; 2 – U-shaped steel frames connected to the strong floor; 3 – 4 steel supports (100x200x30 mm); 4 – 13 rubber bearings; 5 – 12 steel plates (130x130x30 mm); 6 – 12 steel bars ($\varnothing 14$ mm); 7 – steel plates (245x155x50 mm); 8 – 7 steel hinges; 9 – 7 hydraulic jacks (250 kN); 10 – steel plates (150x150x40 mm); 11 – steel girders.



1 – experimental beam; 2 – 2 U-shaped steel frames connected to the strong floor; 3 – 4 steel supports (100x200x30 mm); 4 – 3 rubber bearings; 5 – 3 steel plates (100x200x30 mm); 6 – 3 steel hinges; 7 – 2 hydraulic jacks (250 kN); 8 – hydraulic jack (100 kN); 9 – 11 – steel girders; 10 – load cell АЦД/1С-200/4И-1 (200 кН).

Fig. 4. Test setups:

a) series БI-BVI, b) series БVII-BVIII

The forces registered during static tests, corresponding to the formation of cracks and the ultimate forces, are presented in Tables 3 and 4.

Table 3. Results of static tests under the action of a uniformly distributed load in the span of a beam

Series	Beam	P _{cr} , kN	q _{cr} , kN/m	P _u , kN	q _u , kN/m	Mode of failure
I	БI-1СД18 (without loading the cantilever)	-	29,6	-	87,3	Diagonal crack
	БI-2СД18	10,4	34,7	22,5	75,0	Diagonal crack in the span
II	БII-3СД12	10,4	34,7	24,5	81,7	Diagonal crack in the cantilever
	БII-4УД12	8,4	28,0	24,5	81,7	--//--
III	БIII-5СТД15	12,4	41,4	34,6	115,3	Bending crack on support B
	БIII-6СТД15	14,5	48,2	36,6	122,1	Diagonal crack in the span
IV	БIV-7СТС16,5	12,4	41,4	34,6	115,3	--//--
	БIV-8СТС16,5	12,4	41,4	37,9	126,3	--//--
V	БV-9СС16,5	6,4	21,3	20,2	67,5	--//--
	БV-10СС16,5	6,4	21,3	18,5	61,6	Diagonal crack in the cantilever
VI	БVI-11СТПЦ	8,4	28,0	26,5	88,5	Diagonal crack in the span
	БVI-12СТПЦ	6,4	21,3	18,2	60,6	--//--

Notes:

1. P_{cr} – point force applied in the cantilever, corresponding to cracking;
2. q_{cr} – uniformly distributed load applied in the span of a beam corresponding to cracking;
3. P_u – point force applied in the cantilever, corresponding to failure;
4. q_u – uniformly distributed load applied in the span of a beam corresponding to failure.

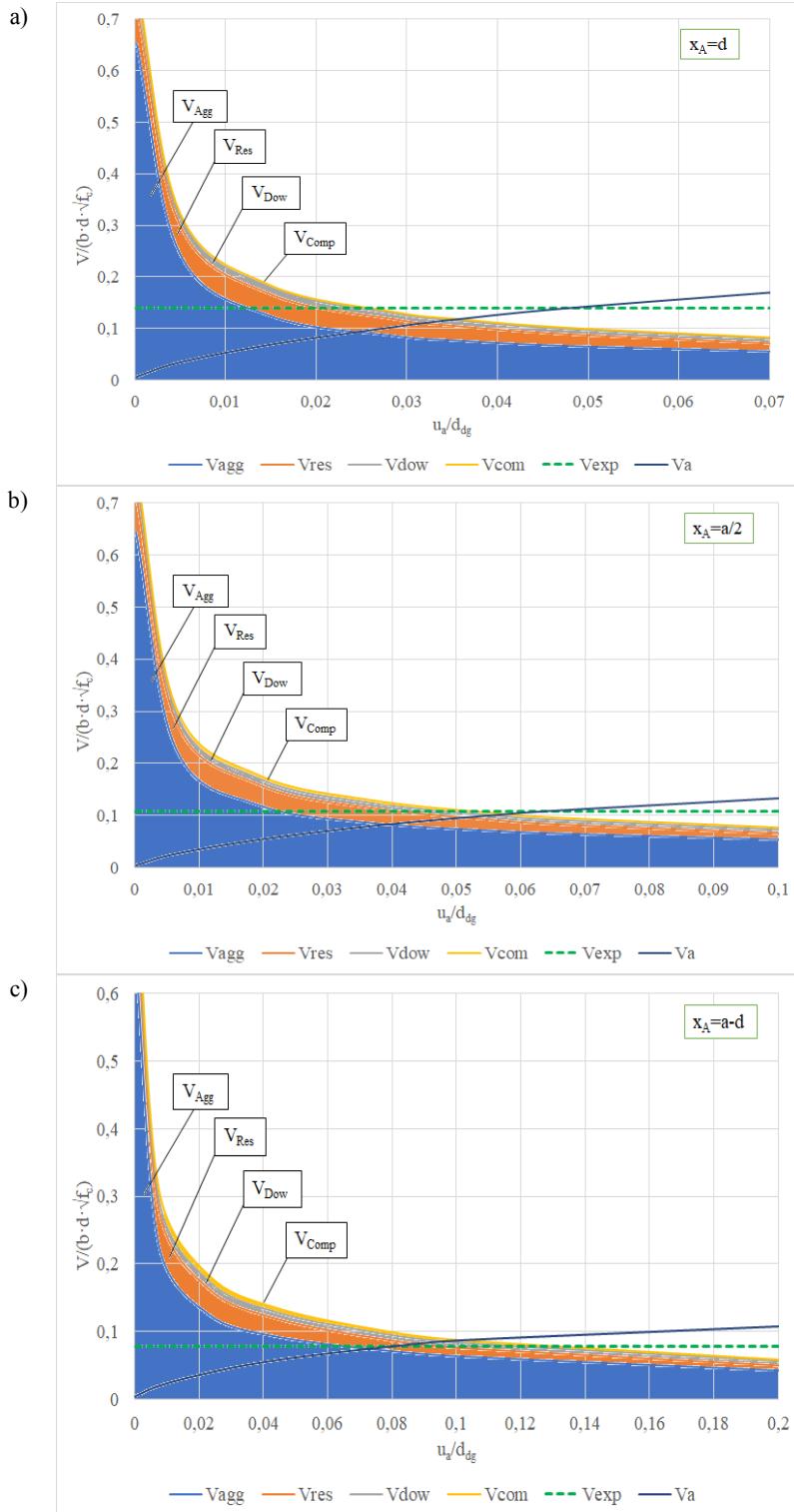
Table 4. Results of static tests under the action of point loads in the span of a beam

Series	Beam	P _{cr} , kN	P _{tot,cr} , kN	P _u , kN	P _{tot,u} , kN	Mode of failure
VII	БVII-13СС16,5	6,5	32,9	21,9	110,6	Diagonal crack in the cantilever
	БVII-14СТС16,5	14,6	73,5	34,8	175,2	Diagonal crack in the span
VIII	БVIII-15СТПЦ	4,5	22,8	14,6	73,4	--//--
	БVIII-16СТПЦ	4,5	22,7	24,9	125,5	--//--

Notes:

1. P_{cr} – point force applied in the cantilever, corresponding to cracking;
2. P_{tot,cr} – point force applied in the span of a beam, corresponding to cracking;
3. P_u – point force applied in the cantilever, corresponding to failure;
4. P_{tot,u} – point force applied in the span of a beam, corresponding to failure.

According to the above presented procedure for calculation the shear resistance of concrete elements reinforced with FRP bars, the contributions of the shear-transfer actions were calculated for various sections of the БV-9СС16,5 beam, as well as the traditional reinforced beam (БIII-6СТД15) (see Figure 5). Table 6 shows the results of comparing the shear resistance calculated by the proposed procedure with the results of an experimental study for various sections.



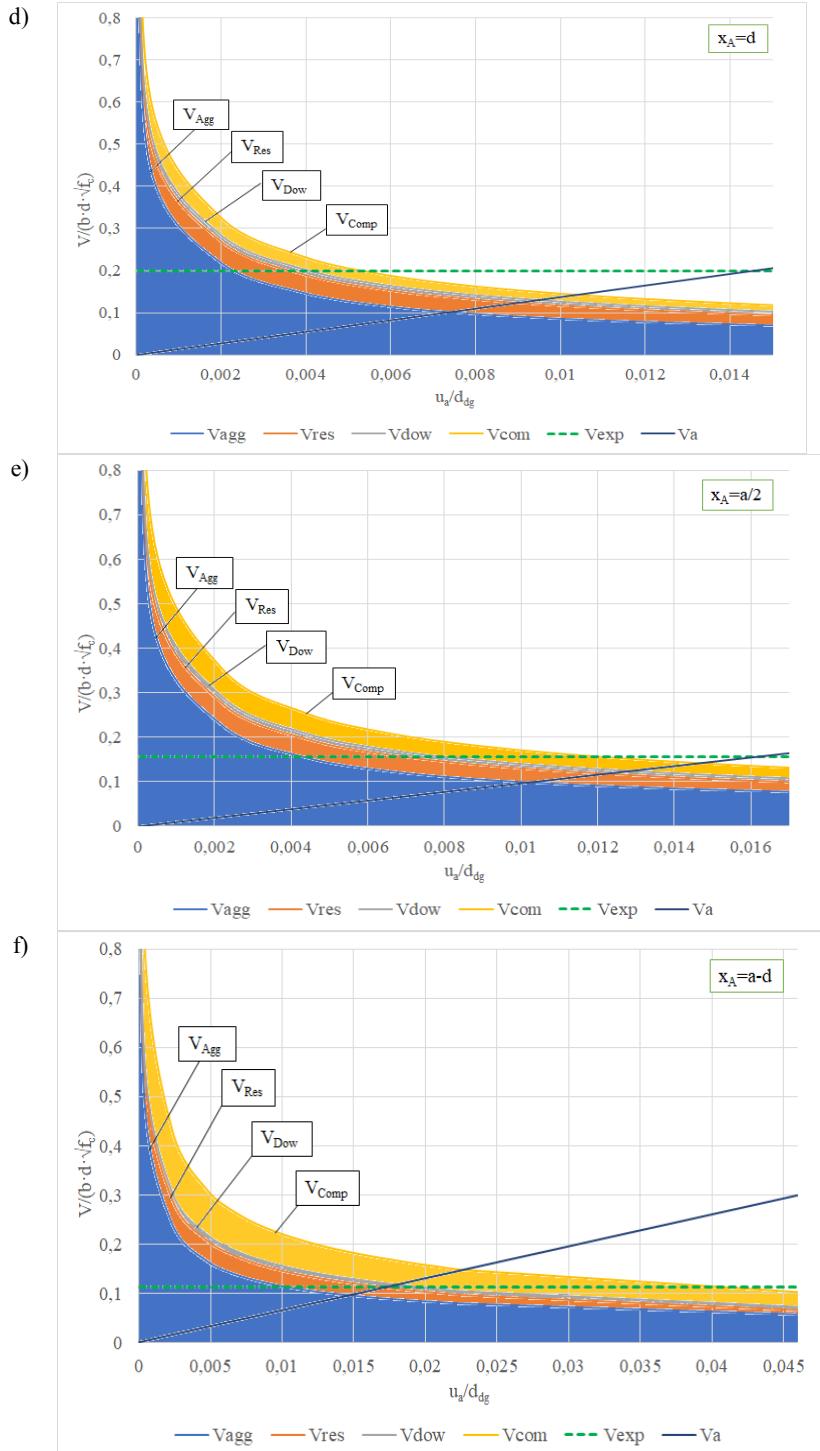


Fig. 5. Failure criterion and load-deformation curve for critical section of beam BV-9CC16,5 a) $x_A=d$, b) $x_A=a/2$, c) $x_A=a-d$ and beam БIII-6СтД15 d) $x_A=d$, e) $x_A=a/2$, f) $x_A=a-d$; shear-transfer actions: aggregate interlock, residual tensile strength of concrete, dowelling action and contribution of the compression zone

Table 5. Comparison of theoretical values of shear resistance according to the proposed procedure with experimental data for different design sections along the span of FRP-reinforced beam elements

Beam	Reinforcement type	Mode of failure	Section, mm	V_{theo} , kN	V_{exp} , kN	$V_{\text{exp}}/V_{\text{theo}}$
БI-1СД18 (without loading the cantilever)	GFRP	Diagonal crack in the span	d=270	29,13	49,73	1,71
			a/2=420	24,29	36,65	1,51
			a-d=570	19,67	23,56	1,20
БI-2СД18	GFRP	Diagonal crack in the span	d=270	29,46	36,02	1,22
			a/2=375	25,71	28,14	1,10
			a-d=480	22,18	20,26	0,91
БII-3СД12	GFRP	Diagonal crack in the cantilever	d=270	33,41	24,52	0,73
			a/2=450	29,13	24,52	0,84
			a-d=630	26,37	24,52	0,93
БII-4УД12	CFRP	Diagonal crack in the cantilever	d=270	32,83	24,52	0,75
			a/2=450	28,68	24,52	0,86
			a-d=630	26,23	24,52	0,94
БV-9CC16,5	GFRP	Diagonal crack in the span	d=270	27,47	32,38	1,18
			a/2=375	23,79	25,30	1,06
			a-d=480	20,32	18,22	0,90
БV-10CC16,5	GFRP	Diagonal crack in the cantilever	d=270	29,37	18,48	0,63
			a/2=450	25,56	18,48	0,72
			a-d=630	23,08	18,48	0,80
БVI-12СПЦ	GFRP	Diagonal crack in the span	d=270	26,86	29,09	1,08
			a/2=375	23,29	22,73	0,98
			a-d=480	19,83	16,36	0,83
БVII-13CC16,5	GFRP	Diagonal crack in the cantilever	d=270	28,84	21,94	0,76
			a/2=450	25,12	21,94	0,87
			a-d=630	22,73	21,94	0,97
БVIII-15СПЦ	GFRP	Diagonal crack in the span	d=270	38,43	29,72	1,29
			a/2=230	27,91	29,72	1,06

Based on the results of the comparative analysis presented in Table 6, the critical section for concrete beam elements reinforced with FRP bars subjected to uniformly distributed load in the span is at a *distance* $x_4=a/2$ (the shear span under a uniformly distributed load is determined from the expression $a=L/2$, where L is the beam span). However, it should be noted that for beam elements subjected to point loads in a span, the critical section is at a *distance* $x_4=a-d$ (for beam elements with a shear span $a/d \geq 2,5$), which, as in the case of a uniformly distributed load, differs on the position of critical sections in beam elements with traditional reinforcement with steel bars [14].

5 Failure criterion

For design purposes, instead of calculating the contributions of the shear-transfer actions by integrating stresses along the critical shear crack, there was a hyperbolic failure criterion [8].

The failure criterion was obtained against the background of experimental data [17] (see Figure 6) by approximating the exponential expression:

$$\frac{V_{R,c}}{b \cdot d \cdot \sqrt{f_c}} = k \cdot \left(\frac{w_A}{d_{dg}} \right)^{-1/2} \quad (17)$$

where $k = 0,0224$;
 $d_{dg} = 16 + d_g$;
 d_g – maximum aggregate size, mm.

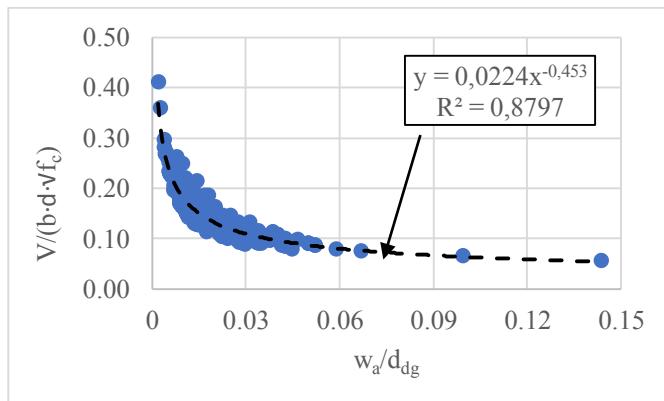


Fig. 6. Failure criterion of the CSCT+“bond-slip” against the background of the experimental database

Blue points in Figure 6 represent the intersections of the failure criterion calculated according to the proposed procedure in the control section $x_A=a-d$ for beam elements with a shear span $a/d \geq 2,5$ and the “load - crack opening width” relationship.

As can be seen from Figure 6, the obtained criterion for elements reinforced with FRP bars has practically the same analytical description as in the case of using reinforcement from steel bars. The difference in criterion (17) is the value of the coefficient $k=0,0224$ (for steel bars, according to the approximation results, $k=0,019$).

The modified shear resistance model of the CSCT shows a good agreement with the experimental data (see Figure 7). So, for beams with a shear span $a/d \geq 2,5$ and a control section $x_A=a-d$ simulation error parameters were: $b(V_{Rd,exp}/V_{R,theo}) = 0,77$ with a value of CoV of $V_x = 24,3\%$.

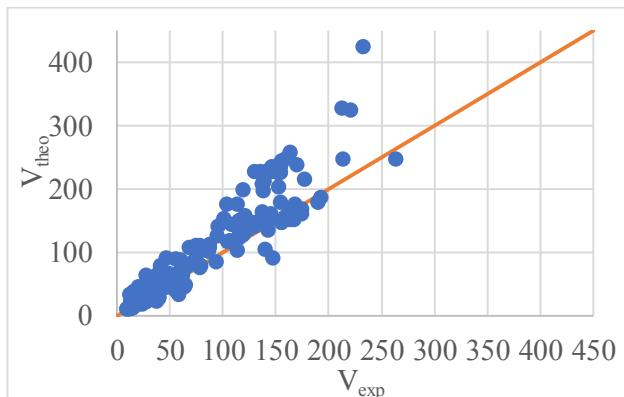


Fig. 7. Comparison of theoretical values of shear resistance according to the proposed procedure (at $x_A=a-d$) with experimental data (296 elements reinforced with FRP with a shear span $a/d \geq 2,5$)

6 Conclusion

This paper presents a methodology for calculating the shear resistance of concrete elements reinforced with FRP bars, based on the provisions of the Critical Shear Crack Theory (CSCT) and the modified “*bond-slip*” law. This procedure has been verified on the results of our own experiments, which shows good convergence. For design purposes, a failure criterion was obtained against the background of an experimental database (296 beam elements with a shear span $a/d \geq 2,5$).

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