Optimal Designs of Tilting-Pad Thrust Bearing operation with the combination of numerical and machine learning techniques

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Abstract. Hydrodynamic thrust bearings are machine elements used in many rotating machinery in order to support axial loads. The investigation of the lubrication in such mechanisms using numerical analysis methods has been the major subject of many studies over the years. Furthermore, the evolution of technology in the last decade brought the concept of industry 4.0 and machine learning techniques have started to play important role in the operational optimization of such assemblies. The aim of this study is to examine optimal designs of tilting pad thrust bearings by combining numerical analysis and machine learning techniques.

1 Introduction

Hydrodynamic thrust bearings are machine elements designed to carry axial loads in rotating machinery. Studies of thrust bearing's hydrodynamic lubrication with numerical analysis methods have been the subject of many works over the last years. Nowadays, the revolutionary ideas of Industry 4.0 have been established new opportunities for more complex computational analysis. In terms of the traditional numerical analysis and the contribution of machine learning techniques the target has been set to minimize friction losses and build hydrodynamic thrust bearing designs with a "greener" environmental effect [1-2].

It is well known that, hydrodynamic thrust bearings have a variety of commercial and industrial applications. The last few years, machine learning techniques are widely used from researchers of the field in fault diagnosis, prognosis and life estimation of bearings [3-5].

The aim of this study is to combine numerical and machine learning techniques in order to predict the design hydrodynamic parameters of tilting pad thrust bearings operating

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in a variety of rotating velocities and loads. Until now, literature review shows that there is no similar application on this field.

The Reynolds equations are solved numerically using the finite difference, central differences, technique. An iterative algorithm is built in order to calculate the load carrying capacity and the friction force developed over the pad's surface. A multi-grade SAE 10W40 and a bio-lubricant ASW 100 are used in the current investigation in a variety of loads and rotational speeds. A data lake is then built and used as an input to a machine learning algorithm in order to investigate optimal combinations for the bearing's operation. The simple and multi variable, linear and polynomial regression models are used and compered for their accuracy to predict friction energy losses for the examined operation conditions. AWS100 is found to be the optimum lubrication solution for the examined cases.

2 Theory

2.1 Basic Assumptions

To begin with, the lubricant is assumed to be Newtonian, isoviscous and incompressible. The flow inside the examined domain is considered to be laminar and isothermal. Cavitation effects are not taken into consideration and the minimum pressure value is assumed to be above the lubricants vapour pressure. No- slip condition is considered between the lubricant and the wall of the bearing and the film thickness is assumed to be small in comparison to the radius of curvature of the bearing surfaces.

2.2 Governing Equations

The fundamental hydrodynamic characteristics of the bearing are calculated by solving the Reynolds equation. Applying the basic assumptions in the Mass conservation and Momentum equations the generalised Reynolds equation can be deduced into its 2-D form (3) that is used in the current study:

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 \mathbf{V} \tag{2}$$

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\mu U \frac{\partial h}{\partial x} \tag{3}$$

The film thickness h along the pad surface is calculated as a function of x:

$$h = f(x) = h_1 + \frac{x}{B}(h_1 - h_0) \tag{4}$$

The average vertical force exerted to a pad area is calculated by integrating the pressure, p, above the pad's surface:

$$F_p = \frac{1}{N_p} \int_A p dA = W \tag{5}$$

The stress tensor is calculated by the following equation:

$$\bar{\tau} = \mu \left[\left(\nabla \vec{\upsilon} + \left(\nabla \vec{\upsilon} \right)^T \right) - \frac{2}{3} \nabla \cdot \vec{\upsilon} \bar{\bar{I}} \right] \tag{6}$$

The friction force on the pad is then calculated by integrating the shear stress τ over the pad's surface:

$$F_{fr_i} = \iint_{A_i} \tau_i \, dA_i \quad ,i = j \text{ or } b \tag{7}$$

2.3 Viscosity Model

For the current study a viscosity model based on Sutherland's law is used in order to approach as realistically as possible the temperature and viscosity conditions inside the bearing pad during operation. In order to build this model experimental temperature values were obtained by [6] Specific coefficients have been calculated in order to adapt the model to current study needs.

$$\mu = \mu_V e^A \tag{8}$$

$$A = C_2^\mu \left(\frac{1}{\mathrm{T}} + \frac{1}{C_1^\mu}\right) + C_3^\mu \left(\frac{1}{\mathrm{T}} + \frac{1}{C_1^\mu}\right) \tag{9}$$

3 Numerical Analysis

Typical 2D meshes consisting of 2500 finite cells per bearing pad are used in this study. The corresponding finite cell number in the flow direction is 50, along the x direction. Similarly, the corresponding finite cell number in the radial direction –y direction- is 50. The grid is modified with load variation keeping the total number of finite cells constant. Spatial resolution tests were performed with differences under 1% between typical and fine meshes.

The bearing walls are assumed to be impermeable. The bottom wall is assumed to be stationary. The upper wall is assumed to be rotating at a constant speed ω, where $U=\omega R_{mean}$ at the pad's mid sector. The inlet and outlet surfaces are assumed as opening with $p=p_{atm}$. An outflow condition is taken into consideration for the bearing sides R_{in} and R_{out}. No inflow condition is applied on the lubrication domain. Finally, the constant pressure $p=p_{atm}$ is applied on the bearing's ambient area.

The pressure P_{ij} is calculated on its node using an iterative algorithm based on the finite difference methodology (central differences). For this purpose, the 2D Reynolds equation is modified accordingly:

$$P_{i,j} = C_n P_n + C_s P_s + C_e P_e + C_w P_w + G \tag{10}$$

The balance of the pad is calculated by applying the Newton- Raphson method on the tilting pad parameter <<k>>. The algorithm runs repeatedly for <<k>> until both Pressure and Balance convergence criteria are met.

$$Err_{press} = \frac{\sum_1^N \left| p_i^j - p_{i-1}^j \right|}{\sum_1^N \left| p_i^j \right|} \leq 1 \times 10^{-6} \tag{11}$$

$$Err_{balance} = \frac{\sum_1^N \left| k_i^j - k_{i-1}^j \right|}{\sum_1^N \left| k_i^j \right|} \leq 1 \times 10^{-4} \tag{12}$$

4 Machine Learning Techniques

Regression techniques are widely used in machine learning applications for two main reasons: simplicity and accuracy. For the case studies presented here, simple and multi variable, linear and polynomial regression models are built in order to correlate the response with the prediction values. Equation 13 represents the Simple Linear Regression Model. For i=n observations the matrix format of the system is generated:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon \tag{13}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & x_{21} \\ 1 & x_{12} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{1n} & x_{2n} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} \text{ or} \tag{14}$$

$$Y = XB$$

For the current study <<y>>, is the dependant variable representing Frictional Energy Losses in Joules and x_1, x_2 are independent variable representing the load [N] and the rotating velocity [rpm] of the bearing. β_1, β_2 are the slopes of the regression line while β_0 represents the y-intercept of the regression line.

For the 2nd Order Polynomial Regression model the X matrix is formatted accordingly:

$$X = \begin{bmatrix} 1 & x_{11} & x_{21} & {x_{11}}^2 & x_{11}x_{21} & {x_{21}}^2 \\ 1 & x_{12} & x_{22} & {x_{12}}^2 & x_{12}x_{22} & {x_{22}}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1n} & x_{2n} & {x_{1n}}^2 & x_{1nx_{2n}} & {x_{2n}}^2 \end{bmatrix} \tag{15}$$

In order to evaluate the goodness of fit for the generated curves the coefficient of determination R^2 is calculated in all cases.

$$R^2 = 1 - \frac{\sum_1^n (y - \hat{y})^2}{\sum_1^n (y - \bar{y})^2} \tag{16}$$

Where, $\hat{y}$ are the values calculated from the regression model and $\overline{y}$ represent the mean of the y values.

5 Results

Simulations were performed for rotating speeds from 2000 to 12000 rpm and applied external loads from 650 to 2300 N. Two different lubricants were studied and compered: the multi grade SAE 10W40 and the bio-lubricant AWS 100. The minimum film thickness was 4μm in all studied cases. The numerical analysis model was validated with the experimental data from the paper of Bielec and Leopard [6].

Fig. 1 represents the variation of Friction Energy Losses according to velocity and load carrying capacity of the pad. Raise in rotational velocity with constant load application leads to more energy losses due to friction. Moreover, raise in load carrying capacity of the pad with constant speed leads to reduction of friction energy losses. Finally, AWS 100 shows lower energy losses compared to SAE 10W40 in all studied cases.

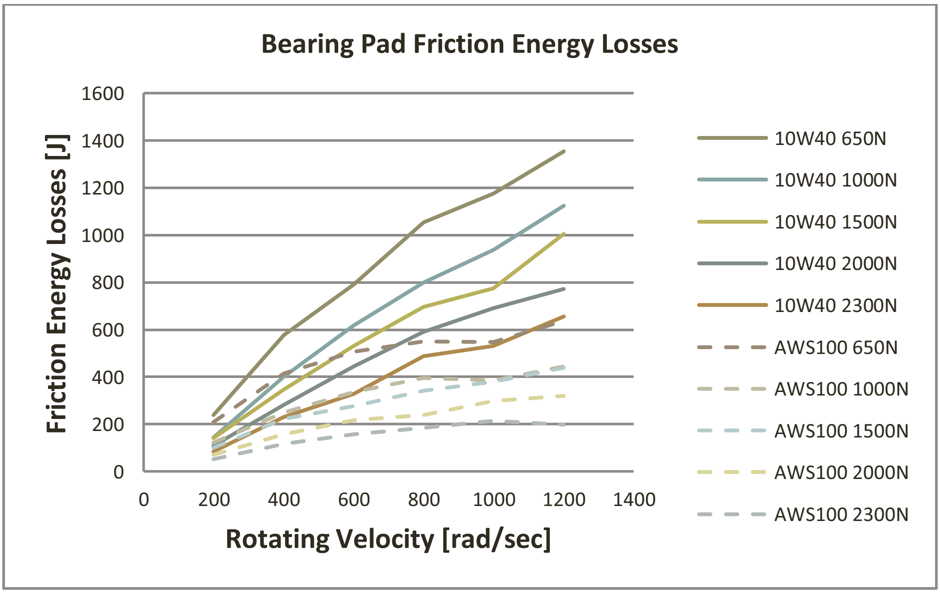

Fig. 1: Friction Energy Losses Numerical Analysis results according to Rotating Velocity and Pad's Load Carrying Capacity.

Fig. 2 bellow shows the graphical representation of the simple linear regression model built for AWS 100 for a load of 2300N. In this case R_{sq}=0.86 which means that the friction energy losses can be predicted with 86% accuracy from the current model.

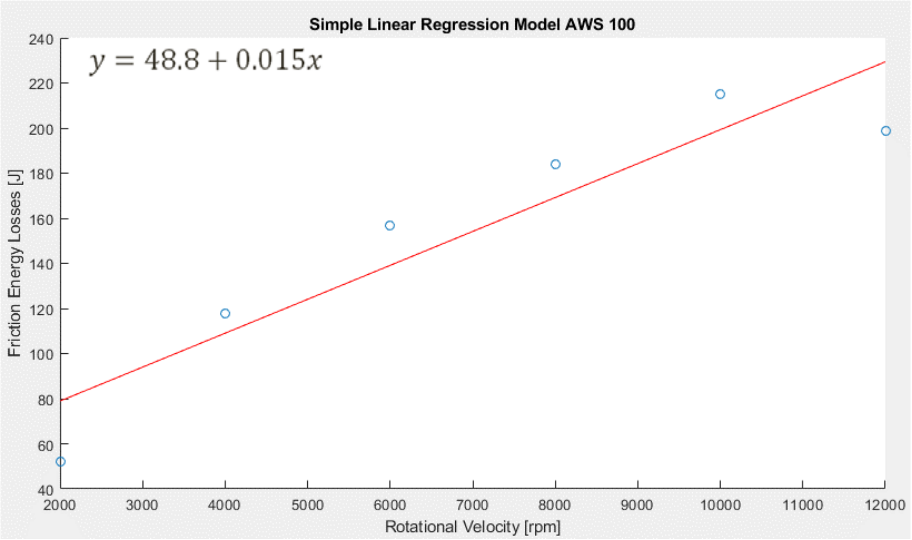

Fig. 2 : Simple Linear Regression Model AWS 100, Load= 2300N.

Fig. 3 shows the 2nd order polynomial regression model of AWS 100 at a load of 2300N. Since R_{sq}=0.99 the model show 99% accuracy in predicting the friction energy losses according to rotational velocity for the current case studied.

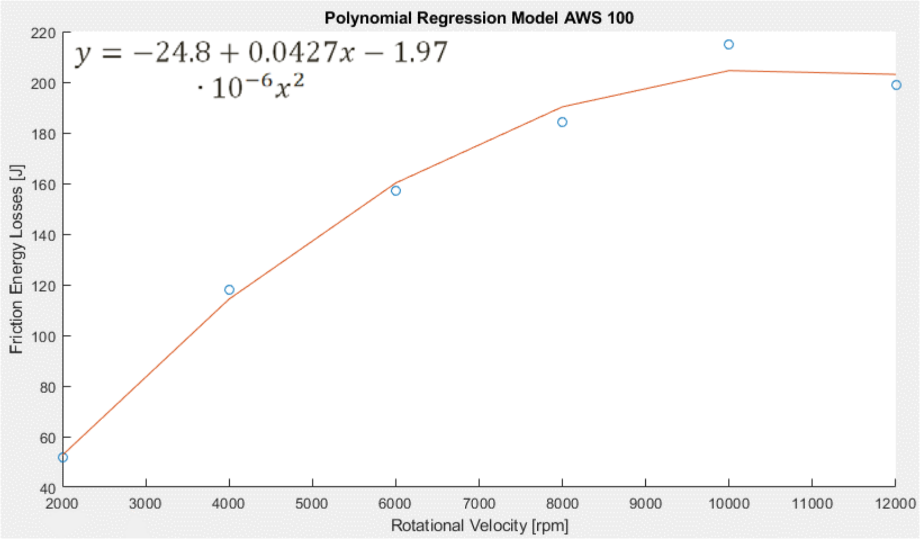

Fig. 3 : Polynomial Regression Model AWS 100, Load= 2300N.

Fig. 4 is the graphical representation of the multi variable linear regression model of AWS 100. R_{sq}=0.88 which means that this linear model has 88% accuracy in predicting the friction losses of the pad according to load carrying capacity and rotational velocity.

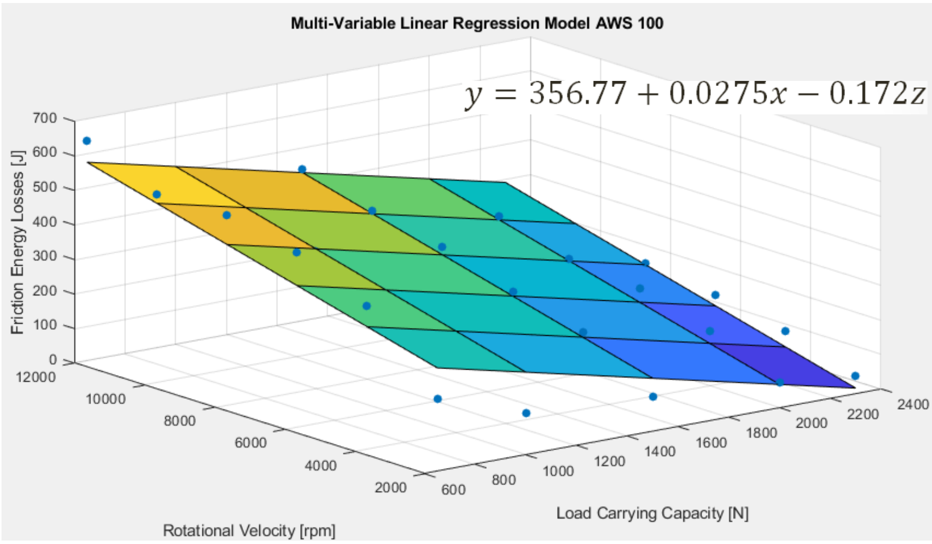

Fig. 4 : Multi Variable Linear Regression model AWS 100.

The below figure (Fig. 5) summarizes the prediction results for the two lubricants AWS100 and SAE 10W40 in all examined cases of rotational velocity and load carrying capacity, using the Multi-Variable 2nd Order Polynomial Regression Model. The red dots correspond to the numerical values of SAE 10W40 while the blue ones to the AWS 100. The upper surface belongs to the Prediction Model for the Friction Energy Losses of SAE 10W40 with an accuracy of 99% (R_{sq}=0.99) while the lower surface corresponds to the Prediction Model of AWS 100 with 94% accuracy (R_{sq}=0.94).

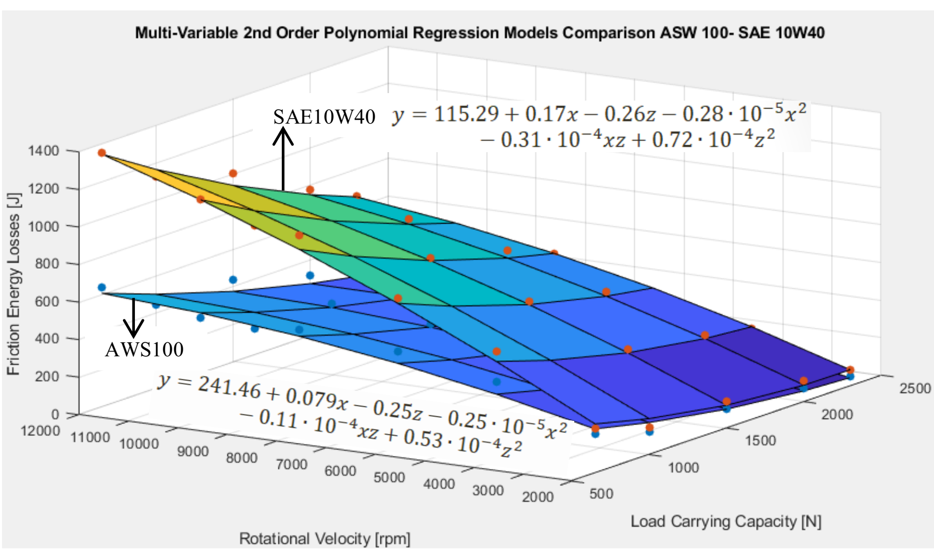

Fig. 5 : Multi Variable 2nd Order Polynomial Regression Models Comparison between AWS 100 and SAE 10W40

In all studied cases there is a good agreement between the numerical and the prediction model's results. Furthermore, multi variable regression models are preferred from the simple ones since the prediction becomes more accurate by taking more aspects into consideration. The multi variable 2nd order polynomial regression model has a better

Goodness of Fit from the multi-variable linear regression model. Higher power polynomials were examined but no sufficient improvement in the goodness of fit was noticed compared to the increase in computational complexity. As a result, the Multi-Variable 2nd Order Polynomial Regression Model was chosen as the best fit for the current study. In all studied cases the thrust pad lubricated with AWS 100 shows the lowest Friction Energy Losses.

6 Conclusions

Summarizing the conclusions of this investigation:

- Raise in rotational velocity with constant load application leads to more energy losses due to friction.
- Raise in load carrying capacity of the pad with constant speed leads to reduction of friction energy losses.
- The Simple Linear Regression Model has a prediction accuracy of 86%.
- The 2nd Order Polynomial Regression Model has a prediction accuracy of 99%.
- The Multi-Variable Linear Regression Model has a prediction accuracy of 88%.
- The Multi-Variable 2nd Order Polynomial Regression model shows prediction accuracy values 94 and 99% for the two examined lubricants.
- In general Multi-Variable prediction models are preferred to the single predictor models because they provide a wider range of examination for the same results.
- The Multi-Variable 2nd order Polynomial Regression Model was the most accurate for the ones examined.
- There was a good agreement between the numerical and the prediction models.
- AWS 100 shows significant reduction in friction energy losses compared to SAE 10W40 that reach up to 70% in the highest rotational velocity with the maximum applied load.

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