Using Full Field Data to Produce a Single Indentation Test for Fully Characterising the Mooney Rivlin Material Model

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Abstract. A theoretical testing method for fully characterising the Mooney-Rivlin three-parameter hyper-elastic material model is proposed by capturing full-field digital image correlation (DIC) data, namely displacement field and indentation force data. A finite element model with known parameters will act as the experimental model against which all data will be referenced (a preliminary test case). Going forward this stand-in model will be replaced with physical test data. This paper also introduces a new concept, the concept of hyperplanes. These hyperplanes represent regions in the force and displacement field data where all the objective function values are equal. The paper concludes that the Mooney-Rivlin material model can theoretically be fully characterised in a single indentation test. By applying the methods discussed in the paper when using full-field data operating under the assumption of hyper-planes.

1 Introduction

Hyper-elastic materials have found themselves in numerous applications ranging from simple watertight seals to primary components in soft-robotics [11]. Simulating the behaviour of these materials for improved design requires the use of hyper-elastic material models. Performing Inverse finite element analysis (FE analysis) on an indentation test may improve the process of isolating the correct material parameters. Since this loading method induces complex load cases in the deformed material. Inverse FE analysis also has the advantage in that it may reduce the engineering time for obtaining a suitable material model for FE analysis [10].

General testing standards usually require samples of specific geometries, for example, a "dog bone" sample in tensile tests. Having specific geometries is usually not a limiting factor when the materials are readily available. However, in situations such as when working with small biomedical tissue, there may not be enough material to produce the required geometry. Due to their limited size, it may also not be feasible to clamp the samples and perform uniaxial or biaxial tensile tests. These problems guided the research behind this paper to develop a testing method focused on small test samples with simple geometries.

This paper introduces a new concept, the concept being hyperplanes. It was found that when performing inverse FE analysis for a single dataset (displacement field data or force data) that the optimiser would never converge to the same single point in the design space for different starting positions. The design variables in this analysis being the parameters of the

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Mooney-Rivlin three-parameter model as seen in equation 1. Even though these converged points were in different locations of the design space, they all had similar or even identical objective function values. This meant that all of these points would result in a material behaviour that would fit that optimised dataset with minimal error. However, then evaluating the material behaviour for the other dataset would reveal significant error scores. This indicates those sets of parameters are sub-optimal and do not represent the correct material behaviour. Further investigation on the response surface showed these points were not caught in local minimums as they had the same objective function values as the known solution's set of parameters. It was also observed that all of these points appear to lie on a linear plane "hyperplane" as seen in Figure 3. Any set of material parameters that lie on this plane would result in identical material behaviour for that particular dataset. The exact equation for the hyperplane region is still not yet determined. However, in this investigation, it will be assumed that a linear equation will be sufficient to represent this region.

2 Methods and Materials

2.1 Analysis Method

Understanding how well the inverse FE analysis method performs under ideal circumstances, as in the case of FE model data, provides valuable insight on which factors are the most relevant before attempting to do the same analysis on physical sample data. The evaluated material model in this paper will be the Mooney-Rivlin three-parameter model as seen in equation 1. This model applies strain energy density to assess material behaviour [2]. Where W , I_1 , I_2 , and I_3 respectively represent strain energy density, the first, second, and third stretch invariants. Parameters C_{10} , C_{01} , and C_{20} are the material constants. These materials constants represent the design variables in this analysis. When the set of constants fully represent the material behaviour, the model is then classified as characterised.

$$W = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) + C_{20} (I_1 - 3)^2 \quad (1)$$

The analysis is broken up into three distinct phases operating under the core assumption that there exists regions or hyperplanes in the design space. Evidence supporting this assumption can be shown from Jensen [7]. He identified that simulated indentation forces had non-unique sets of material parameters. These different parameter sets would result in identical indentation forces being witnessed. Bresler [4] and Murdock, et al [1] both witnessed that optimising the objective function for a single dataset would not always provide a good fit for another dataset or loading case. With [1] showing that the absolute values of the material parameters being poorly correlated with the objective function value, and their product and intern their ratio having a strong correlation.

Phase one would be locating the low objective function regions where the hyperplane can be found by using an optimisation algorithm to find a point on the hyperplane. If the optimiser converges it will be assumed that point lies on the hyperplane. This must be done for both the displacement field and indentation force data under a single variate optimisation scheme using DOT [8] (Design Optimization Tools) as the optimiser.

Phase two is where the equations of the hyperplanes are identified. This method requires at least three converged points to fit a linear equation by least squares regression. This means a total of six converged points from phase one are needed to extract the force and displacement hyperplanes. The hyperplane's equation is represented by equation 2. Parameters a_1 through to a_4 are the fitted constants obtained from the least-squares regression.

$$f(C_{10}, C_{01}, C_{20}) = a_1 \cdot C_{10} + a_2 \cdot C_{01} + a_3 \cdot C_{20} + a_4 \quad (2)$$

Phase three is where the intersection between the hyperplanes from the displacement field and force dataset is evaluated. Setting these two linear equations equal to each other results in a one-dimension line in hyperspace. This line can be seen as one dimensional since all parameters are coupled to a single variable α . The hyperplanes can be seen in Figures 3a and 3b. A one-dimensional search algorithm was then used to complete inverse analysis.

2.2 Experimental Model

The geometry under investigation was a 20x20x5 mm cuboid with a cylindrical indenter placed lengthwise along the top surface's centreline with a length of 30 mm with a radius of 3 mm. Because this geometry is symmetric about its centre planes it can be taken advantage of as only a quarter of the geometry needs to be simulated. This significantly reduces the computational cost. The indenter, however, does not need symmetry applied to it in the simulation. This is because the indenter was generated using a geometric surface and not with deformable elements. Applying symmetry conditions results in a simulated sample with dimensions 10x10x5 mm and the indenter being placed along the top surface's edge as seen in Figure 1.

As in the physical test, the indentation depth will be physically controlled. Therefore to replicate this in the FE model, displacement control for an overall indentation depth of 3 mm over 1 second to induce material deformation was used. The model also simulates the situation in which digital image correlation (DIC) would be used for extracting the displacement field dataset. By evaluating only the surface nodes and elements on a single face of the sample as seen in Figure 1, the simulation represents the physical setup.

To generate the theoretical experimental data, the same model used in the inverse FE analysis was run once with significantly denser mesh and only surface data on a single face was extracted mimicking the type of data that would result from the use of DIC.

Figure 1: Simulated sample

2.3 Numerical Pipeline

The program used to perform the FE analysis in this investigation was MSC Marc Mentat 2021. This software was favourable since it allows for control in either batch mode or direct control through python during operation using the PyMentat modules [5]. Operating the program in batch mode allows for the generating of a procedure file. This file would contain

all the required steps for updating the finite element model. Allowing for the program to be fully controlled in code, giving the optimiser control of the program.

The initial optimisation phase was performed using sequential quadratic programming (SQP) in the DOT software package. Figure 2 summarises the three analysis phases. With phase one being the primary loop identifying points on the hyperplanes for both datasets. Phase two where the two linear hyperplanes are fitted through least squares regression. Setting these two linear equations equal to each other results in a one-dimension line. This is where phase three comes in using the method of golden sections on the Force dataset at a different indentation depth in the sample. The indentation force for a different depth was found to have its unique hyperplane as well. Which allows for a single point on the intersection line to be found.

Figure 2: Numerical pipeline flowchart

2.4 Objective Functions

Inverse FE analysis at its core uses some method of optimisation to determine the material parameters of an evaluated sample. This is achieved by minimising an objective function based on error data generated from the experimental and simulation models. Since displacement field and indentation force are very different in terms of their data structure, two objective functions were needed.

2.4.1 Displacement Field Objective Function

Root mean square (RMS) was used to determine the error between the simulated and measured displacement fields. This final RMS error is represented by a single value for each FE simulation. It comprises of three sub RMS scores, one for each coordinate direction as represented by equations 3, 4, and 5. Directly summing up these equations will result in a single error value that describes how similar the two displacement fields are. However, this now introduces bias into the objective function favouring the larger displacement errors. The simulation has an indenter that moves in the Y-direction and therefore it is expected that Y-displacements will always be larger than displacements in the X and Z-directions. An attempt to remove this bias was to normalize the displacements with their respective maximum displacements in that direction, this is represented by equation 6. [10] used the same set of

equations when characterising a non-linear orthotropic through bubble inflation tests. Where n represents the time step in the simulation to start the summation, m is the time in the simulation were to end the summation, N represents the number of surface nodes, and M being the total number of time steps being evaluated.

$$e_x(m) = \sqrt{\frac{\sum_{i=1}^N (d_{\text{exp}_x}(m) - d_{\text{sim}_x}(m))^2}{N}} \quad (3)$$

$$e_y(m) = \sqrt{\frac{\sum_{i=1}^N (d_{\text{exp}_y}(m) - d_{\text{sim}_y}(m))^2}{N}} \quad (4)$$

$$e_z(m) = \sqrt{\frac{\sum_{i=1}^N (d_{\text{exp}_z}(m) - d_{\text{sim}_z}(m))^2}{N}} \quad (5)$$

$$e_{\text{disp}} = \frac{1}{M} \sum_{i=n}^N \frac{e_x(i)}{\max(|d_{\text{exp}_x}(i)|)} + \frac{e_y(i)}{\max(|d_{\text{exp}_y}(i)|)} + \frac{e_z(i)}{\max(|d_{\text{exp}_z}(i)|)} \quad (6)$$

2.4.2 Indentation Force Objective Function

The objective function for the indentation force was simpler than the displacement field in that there is only one point of measurement. This point would be measured in one direction for each indentation depth and not a "field". The chosen objective function represents the difference in measured force between the experimental and simulated results would increase linearly with the absolute difference between the two values. This objective function is represented by equation 7. Where n represents the time step in the simulation to start the summation and m is the time in the simulation were to end the summation.

$$e_{\text{force}} = f_{\text{exp}}(d) - f_{\text{sim}}(d) \quad (7)$$

3 Results

3.1 Phase One and Two Convergence Points Results

After the SQP algorithm in DOT identified many convergence points for each respective dataset it was found, as expected, that none of the starting points converged to the same point in design space across different datasets. This was also true within each dataset itself that it would be extremely unlikely that two starting points in the displacement field or indentation force data would converge to the same point as seen in Figures 3a and 3b. However, these points did display the core behaviour behind this paper that the assumption of a hyperplane. That being, any point on the plane would result in identical material behaviour for the given data type and that the region in which these points lie can be represented by a linear equation. Its validity is demonstrated since it was possible to fit the first-order surface through the converged points with almost all of these points being on that plane. The correlation coefficients for the linear fits on the two hyperplanes were $R_{\text{force}} = 0.999$ and $R_{\text{disp}} = 1.000$, a near-perfect correlation score providing further evidence supporting the assumption of hyperplanes being present in the data sets. It is also worth noting that none of the starting points did converge to correct material parameters as prescribed in the high density mesh finite element model which was used to generate the synthetic experimental data in this research paper. These findings agree with what was witnessed in the literature.

(a) Front view of the force hyperplane showing converged design points

(b) Front view of the displacement hyperplane showing converged design points

Figure 3: Force convergence with linear fit

3.2 Phase Three Golden Section Results

The results for phase three were that the method of golden sections was successfully able to find the correct set of material parameters that fit both the displacement field and indentation force data simultaneously when following the one-dimensional line that is generated at the intersection point between the two linear surfaces, also referred to as hyperplanes. This final convergent point from the golden section phase also represents the correct material parameters for the Mooney-Rivlin model with a total magnitude of 2.31% error from the prescribed values and was convergent at a single point along the intersection line as seen in Figures 4a through to 4d. Keep in mind the reason for the golden sections algorithm being able to optimise the parameters along the two hyperplanes. The reason being phase three used the indentation force at a different depth to perform that section of the analysis. This different depth was found to have its hyperplane which allowed for a single optimal point to exist on the one-dimensional line. Figure 4 shows a history plot of the golden section algorithm converging to single set of parameters along the one-dimensional line.

Figure 4: Golden section optimisation on intersection line between the two hyperplanes

4 Conclusion

In this paper, a method of performing indentation tests for fully characterising the three-parameter Mooney-Rivlin model. This method is based on the assumption that within the data sets there are hyper-planes with fixed relationships between the material parameters wherein the objective function is equally optimal. It was demonstrated for both the force and displacement field data that this assumption is applicable since the linear plane was able to be fit through the converged points for random starting positions in the design space with near-perfect correlation coefficients of $R_{force} = 0.999$ and $R_{disp} = 1.000$.

When optimising parameters along the intersection line of the two hyper-planes it was demonstrated in section 3.2 that evaluating the indentation force for a different depth results in the objective function being convergent to a single point. The results, therefore, demonstrate that operating under the assumption of hyperplanes in the data sets the Mooney-Rivlin Material model can be fully characterised with a low level of error of only 2.31%.

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