An accurate lumped convective plus radiative heat exchanger model for performance-based modelling

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Abstract. There are several thermo-fluid process modelling tools available on the market which can be used to analyze the off-design performance of thermal plants. These tools all offer the user with a simple convective heat exchanger component that requires the design-base process conditions as inputs. The tools would then calculate an effective overall heat transfer factor (UA) and make use of gas flow mass ratios to scale the UA value for off-design conditions. The models employed in these tools assume that the contribution of gas radiation is insignificant, hence only applies convection scaling laws. This paper presents an improved model which considers the contribution of the gas and particle radiation, as is often encountered in the first few heaters in coal fired boilers and heat recovery steam generators. A more fundamental scaling law is applied for the convection scaling and incorporates a cleanliness factor which allows for the consideration of fouling of the heater surfaces. The model's performance was validated against a discretized tube-level heater model that solves the fundamental convection and radiation terms. The model is accurate within 1% for the cases considered, as compared to more than 20% error if radiation contribution is not considered.

1 Introduction

The ability to accurately predict the heat transfer occurring in a thermal application is key to a successful design. It is therefore not unusual to find a vast volume of literature dealing with heat exchanger performance modelling. The methods range from lumped heuristic and empirical models, to detailed discretised analysis, and more recently high resolution CFD analyses. The choice of method employed by the designer or analyst is a function of the expected outcome as well as the available information about the specific heat exchanger.

There are various commercial software tools available which implements one or multiple methods. The tools generally offer three approaches: a) determine the physical design requirements of a heat exchanger, given certain performance needs; b) determine the performance of a heat exchanger given detailed design information; c) determine the performance of a heat exchanger given a known operating point. This paper deals with the last approach where the analyst is usually interested in the off-design performance of an existing heater without detailed knowledge of the physical design or physical process

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occurring during the heat transfer. More specifically, the focus is on gas-to-fluid heat exchangers such as superheaters found in typical steam generating plants.

2 Literature

The following is a brief discussion of the methods employed by some power plant modelling software tools and researchers. It is by no means comprehensive to cover all tools and methods available, but a common trend is observable. Also, the method described here is not necessarily the only option available to the user of the mentioned software.

In the case where the user only specifies the design point performance of a heater, the method used by the software is sometimes called a "black box" model, or "performance" model and is essentially a lumped model. The inputs for these are typically the inlet thermal properties and mass flow of both streams, as well as one stream's exit condition. The software will then use these, along with a user specified heat exchanger type (cross-flow, parallel flow or counter flow), and determine the design base overall heat transfer factor (UA_D) such that:

$$\begin{aligned}&\dot{Q}_D = \varepsilon_D C_{min}\left(T_{H.in} - T_{C.in}\right)\\ &\text{with } \varepsilon_D = f(Ntu,...)\\ &\text{and } Ntu = \frac{UA_D}{C_{min}}\end{aligned} \tag{1}$$

This is the well-known ε-NTU method which defines the heat exchanger's effectiveness (ε) in terms of the Number of Transfer Units (*NTU*) which in turn is a function of the UA value and the minimum stream's thermal capacitance, with $C_{\min} = \min(C_H \dot{m}_H, C_C \dot{m}_C)$. The subscript *D* denotes the Design conditions, and H and C points to the Hot and Cold stream respectively. In some cases, the equivalent log mean temperature difference (LMTD) method is used, which requires a flow correction factor *F* in the case of cross flow.

In the off-design condition, the design UA value is scaled based on the change of process conditions to arrive at an off-design UA value, which can then be used to calculate the heat transfer using the same ε–NTU method. The method of scaling varies slightly between various tools, but the most significant factor is the ratio of the off-design mass flow vs. the design flow, raised to some exponent.

$$UA \approx UA_D \left(\frac{\dot{m}}{\dot{m}_D}\right)^a \tag{2}$$

This ratio originates from the common shape of the Nusselt number correlation when considering forced convection of fluids, which is:

$$Nu = C \cdot Re^a Pr^b \tag{3}$$

If one assumes that the gas stream is the dominant thermal resistance, equation (2) an (3) can be combined and rewritten using the known definitions for Nusselt, Reynolds and Prandtl number. The coefficient *C* cancels out when considering only convection ratios. The exponent *a* varies based on the tube arrangement for tubular heat exchangers, but is typically in the range of 0.5 to 0.8 [1], while the exponent *b* is generally taken as $\frac{1}{3}$. The off-design UA value is thus:

$$UA = UA_D \left(\frac{\dot{m}_g}{\dot{m}_{g.D}}\right)^a \left(\frac{k_g}{k_{g.D}}\right)^{2/3} \left(\frac{\mu_g}{\mu_{g.D}}\right)^{1/3-a} \left(\frac{cp_g}{cp_{g.D}}\right)^{1/3} \tag{4}$$

The commercial software Ebsilon offers a Component 61 which splits the UA into an internal (liquid side) and external (gas side) value, and then applies the following scaling laws [2]:

$$UA_l = UA_{l.D}\left(\frac{\dot{m}_l}{\dot{m}_{l.D}}\right)^{a_l}$$
$$UA_g = UA_{g.D}\left(\frac{\dot{m}_g}{\dot{m}_{g.D}}\right)^{a_g}\left(1-\frac{\left(T_{g.av}-T_{g.av.D}\right)}{2000K}\right) \tag{5}$$

The user is required to enter values for $UA_{l.D}$ and $UA_{g.D}$. The help file does not suggest a suitable value for the exponents. The temperature difference term added to the gas side scaling is probably trying to compensate for fluid property changes.

The commercial tool PEPSE has a performance mode for all its heat exchanger components, however the manual does not state how the calculation is done [3]. Based on the required inputs, it is likely that it also uses a mass flow ratio scaling.

The software Thermoflex uses a multi-parameter thermal resistance scaling, and translated into UA values results in [4]:

$$UA_l = UA_{l.D}\left(\frac{\dot{m}_l}{\dot{m}_{l.D}}\right)^{a_l}\left(\frac{T_l}{T_{l.D}}\right)^{c_l}\left(\frac{p_l}{p_{l.D}}\right)^{d_l}$$
$$UA_g = UA_{g.D}\left(\frac{\dot{m}_g}{\dot{m}_{g.D}}\right)^{a_g}\left(\frac{T_g}{T_{g.D}}\right)^{c_g}\left(\frac{M_g}{M_{g.D}}\right)^{d_g} \tag{6}$$

The user enters the ratios for liquid, gas and wall resistance vs overall resistance, and this is used to determine the individual stream's design UA. No guidelines are given for the value of the exponents. It is interesting to note that for steam (liquid side)† the fluid property scaling is related to temperature and pressure, while the gas side is scaled using temperature and the fluid's average molecular weight M. The only reason why the molecular weight would change is if the gas composition changes, which probably has the greatest effect on the specific heat c_P.

The VirtualPlant software is part of the EtaPro performance and condition monitoring platform [5]. For its boiler and steam generator heat exchangers, it uses the following scaling law:

$$UA = UA_D\left(\frac{\dot{m}_g}{\dot{m}_{g.D}}\right)^{a}\left(\frac{k_g}{k_{g.D}}\right)^{2/3}\left(\frac{\mu_g}{\mu_{g.D}}\right)^{2/3}\left(\frac{c_{P_g}}{c_{P_{g.D}}}\right)^{1/3} \tag{7}$$

The default suggestion is a = 0.8. It closely resembles the fundamental equation (4), except for a small difference in the viscosity ratio exponent.

Nord used Aspen HYSYS to model the steam generator of a combined cycle, and applied the following scaling [6]:

$$UA = UA_D\left(\frac{\dot{m}_g}{\dot{m}_{g.D}}\right)^{0.57} \tag{8}$$

† Although the steam in power plant superheaters are in the gaseous phase, its pressure and flowrate are typically so high that the heat transfer coefficients resembles that of a liquid.

In his book, Ganapathy suggests the following scaling for modelling heat recovery steam generators [7]:

$$UA = UA_D \left(\frac{\dot{m}_g}{\dot{m}_{g.D}} \right)^{0.65} \left(\frac{F_g}{F_{g.D}} \right) \left(\frac{\dot{m}_l}{\dot{m}_{l.D}} \right)^{0.15}$$
$$with \;\; F = \frac{c_P^{\,0.33} k^{0.67}}{\mu^{0.32}} \tag{9}$$

It is unsure why the liquid mass flow ratio is included in this fashion, as the combined effect of inner and outer heat transfer is not the product of the two, but rather the reciprocal sum.

3 Model definition

The primary drawback with all the methods employed in the presented literature is that it ignores the contribution of thermal radiation to the overall heat transfer and how it changes with plant load. Although this may be suitable for low temperature heat exchangers, it is insufficient for those that are close to the fuel combustion, such as superheaters in a coal fired boiler. The ratio of thermal radiation compared to the total heat transfer can be more than 50% [8]. If the overall heat transfer is scaled only on mass flow, one may make a substantial error for off-design conditions, especially in the case where the temperature remains relatively high while the mass flow reduces.

Let us define a ratio of the heat transfer due to thermal radiation relative to the total heat transfer at design conditions as R_r. Using the general equation for thermal radiation, one can write the thermal radiation component as:

$$\dot{Q}_{rD} = R_r \cdot \dot{Q}_D = \sigma \varepsilon A F \left(T_{g.D}{}^4 - T_{l.D}{}^4 \right) \tag{10}$$

$T_{g.D}$ and $T_{l.D}$ is the average temperature of the gas and liquid stream between inlets and outlets. This is a simplification of the actual process occurring, most significantly because one assumes radiation occurs between the gas and fluid, instead of the gas and the outer wall surface. This assumption implies that the thermal resistance on the fluid-side, as well as the wall resistance is substantially lower than the resistance on the gas side, making the wall temperature on the gas side to be approximately the same as the fluid temperature. This is generally an adequate assumption for gas to liquid heat exchangers.

The term $\sigma \varepsilon A F$ can be called the radiation factor of the heat exchanger. It contains the collective effect of thermal emissivity, radiation surface area and radiation view factor. Apart from minor changes to the effective gas emissivity, this radiation factor can be assumed to remain constant irrespective of the process conditions.

One can also write the radiation heat transfer as an effective UA value such that:

$$\dot{Q}_{r.D} = UA_{r.D} \left(T_{g.D} - T_{l.D} \right) \tag{11}$$

The overall design UA_D is calculated with the design process conditions and the appropriate ε-NTU correlation. This enables one to determine the convective $UA_{c.D}$ value at design conditions, still assuming the wall resistance is insignificant:

$$UA_{c.D} = UA_D - UA_{r.D} \tag{12}$$

For off-design conditions, the convection portion UA_c can be calculated using equation (4). The radiation-convection is determined using the constant $\sigma \varepsilon A F$ factor and the off-design temperatures by combining equation (10) and (11):

$$UA_r = \sigma\varepsilon AF \cdot \frac{\left(T_g^{\ 4} - T_l^4\right)}{\left(T_g - T_l\right)} \tag{13}$$

This results in the effective off-design UA value:

$$UA = UA_c + UA_r \tag{14}$$

The design conditions are typically for a clean heat exchanger. Many software tools provide the user with a correction factor option to adjust the overall heat transfer to account for fouling, hence:

$$UA_{fouled} = CF \cdot UA_{clean} \tag{15}$$

The *CF* factor can also be termed a Cleanliness Factor. The effective thermal resistance due to the added fouling R_{foul} can be obtained from:

$$UA_{fouled} = \left(UA_{clean}^{\ -1} + R_{foul}\right)^{-1} \tag{16}$$

Thus, for a given cleanliness factor, the resulting fouling resistance is:

$$R_{foul} = \frac{1/CF - 1}{UA_D} \tag{17}$$

The overall UA value for a fouled heat exchanger at off-design conditions is thus given as:

$$UA = \left(\frac{1}{UA_r + UA_c} + R_{foul}\right)^{-1} \tag{18}$$

This method is different to what most software tools use, as they would typically scale the off-design UA factor with the cleanliness factor (equation (15)). This is not entirely correct, as it implies that the convection and radiation is proportionally affected by fouling. Fouling rather results in a constant thermal resistance in the heat transfer path. As can be seen from equation (18).

The off-design UA factor can now be used with the appropriate ε-NTU or LMTD method to calculate the accurate off-design performance.

4 Model validation

Gwebu and Rousseau published results from modelling a superheater in a discretized pipe-per-pipe network approach [9]. The significant contribution was a special pipe component developed in the Flownex Simulation Environment [10] which includes accurate correlations for internal and external convection, pipe and fouling layer conductivity, as well as thermal radiation from the surrounding gas. The method is similar to the work done by Trojan and Taler [11].

This special pipe component was used in this study to model a hypothetical multi-pass heat exchanger which resembles a typical final superheater of a subcritical coal fired boiler. **Figure 1** shows an image of the heater model in Flownex SE. The model considers one tube bank only, and consists of 8 longitudinal passes, each discretized into 6 vertical pipe elements. The geometrical parameters of each pipe element as well as the design base process conditions and performance are given in **Table 1**. It is worth-while to note that this specific heater transfers 51.9% of the heat via thermal radiation.

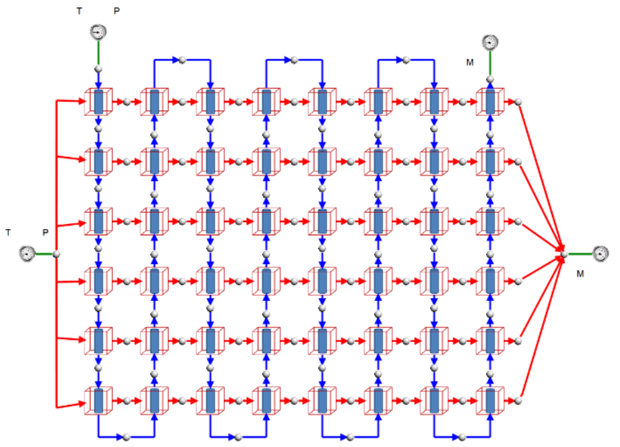

Figure 1. Flownex Model of the validation heat exchanger.

The IF97 formulation of water and steam as per IAPWS [12] was used for the liquid side, and ideal gas air for the gas side. It is assumed that there is no maldistribution of temperature or mass flow per pipe element. The discretized model was used to predict the "true" result of the heater at off-design conditions and was then compared to the newly proposed scaling method implemented in a lumped model.

Table 1. Inputs and Process Results for the heat exchanger.

Geometry	Properties
Pipe increment length : 1.0 m Pipe outer dia. : 45 mm Pipe inner dia. : 30 mm Gas volume length : 200 mm Gas volume width : 500 mm	Wall emissivity : 0.7 Gas emissivity : 0.1
Process inputs	**Process results**
Steam inlet :17 MPa, 450 °C Steam mass flow : 0.3 kg/s Gas inlet : 100 kPa, 850 °C Gas mass flow : 1.2 kg/s	Steam outlet : 530.08 °C Gas outlet : 795.58 °C Total heat : 75.631 kW Radiation heat : 39.274 kW Design UA : 0.230 kW/K

The lumped model used the ε-NTU formulation of a pure parallel flow heat exchanger. This is not entirely correct, as there are better models to use for multi-pass cross flow heat exchangers, but this would require more detailed information from the user. Recalling that the main goal of the lumped model is to predict off-design performance with little to no physical inputs, these models typically only provide for pure parallel flow, counter flow, or cross flow with both streams un-mixed.

The first set of validation calculations focussed on the ability of the model to properly account for the thermal radiation contribution. Two scenarios were studied: a) change in gas mass flow; and b) change in gas temperature. For both cases the lumped model was solved with the actual design base radiation ratio, and a radiation ratio of zero. The zero condition would resemble the results produced by traditional methods. The total heat transfer and the steam outlet temperature were used as primary indicators. The curves in **Figure 2** show the percentage error for the heat transfer, as well as the absolute steam temperature error for the various cases.

The results clearly show how the new model performs exceptionally well in capturing the actual off-design performance, even in the very extreme conditions as was tested. A maximum heat error of 1.6% and 0.9 K is reported when radiation is included, as compared to the very large errors from traditional models which do not include the effect of thermal radiation.

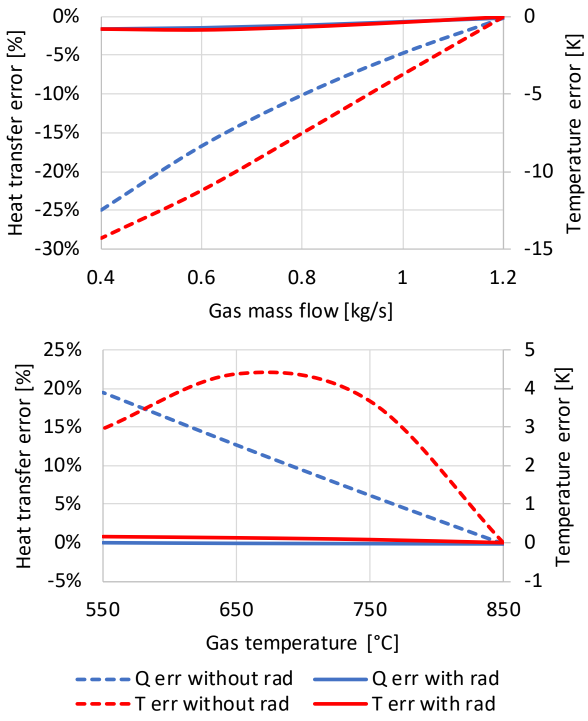

Figure 2. Difference between models with and without the Radiation Effect for off-design gas mass flow (top) and inlet temperature (bottom).

The next case considers the inclusion of a fouling layer. For this, a 1mm thick layer of fouling with arbitrary thermal conductivity of 0.07 W/mK was added to the outside of all the pipes. When solved for the design base input conditions, an exit steam temperature of 510.6 °C and 58.3 kW of heat was reported. This translates into a fouled overall $UA = 0.1686$ kW/K, and thus a cleanliness factor $CF = 0.733$.

Two sets of results are reported in **Figure 3** where the inlet temperature and mass flow of the gas was varied together. It is clear how the traditional method of scaling the overall UA for fouling results in substantial errors as compared to the new method which rather includes a fouling resistance. Using the fouling resistance, the maximum heat error is about 3% at the lowest flow rate considered, with a temperature error of only 0.4 K. Any change in wall thermal emissivity caused by the fouling is not considered in the model.

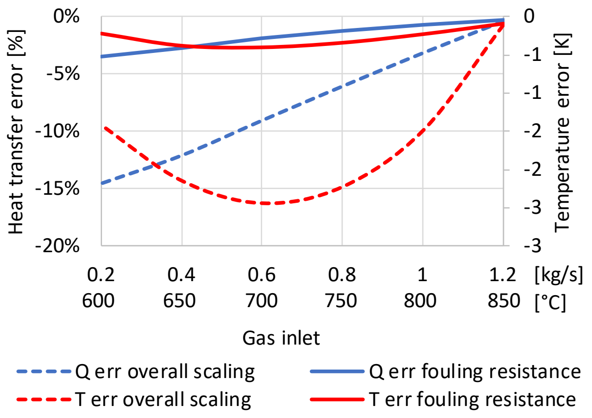

Figure 3. Difference between models using Overall Scaling or a Fouling Resistance for off-design gas inlet conditions.

The primary reason for the new model producing relatively large errors as compared to the previous case, is the fact that the 1mm fouling thickness added to the pipe resulted in a noticeable change in the gas volume around the pipe. This changed the mean beam length used for radiation as well as the stream length used for convection around the pipe. The actual thermal radiation ratio changed from 0.519 to 0.565, but this change was not included into the lumped model, as one would typically ignore the geometric effect that fouling has on a heat exchanger.

The assumption that the thermal resistance on the liquid side is insignificant was tested by varying the steam mass flow from 0.3 kg/s down to 0.05 kg/s. The maximum heat error was only 0.42%, and temperature error of 0.8 K. At the lowest mass flow, the convection coefficient on the steam side was calculated with the detail model as 550 W/m^{2}K, compared to only 16 W/m^{2}K on the gas side. This therefore confirms that one may ignore the effect of the convection on the liquid side.

5 Practical example

Taler and colleagues have modelled a complex superheater using a discretised method, as well as using CFD analysis [13]. The exact geometry and inputs that was published were used to model the heater using the special pipe component in Flownex as is shown in **Figure 4**. The inputs and process parameters are given in **Table 2**. The paper does not report the wall or gas emissivity used, hence typical values were used.

For the design case process conditions with a fouling layer of 1.8 mm at 0.18 W/mK conductivity, the discretized model reported results which are very close to those from Taler. This can be seen in **Table 3**. The heat error is 2.4%. The good accuracy of the results adds further confidence to the validation done previously, as it demonstrates that the special pipe component does capture the fundamental physical processes correctly.

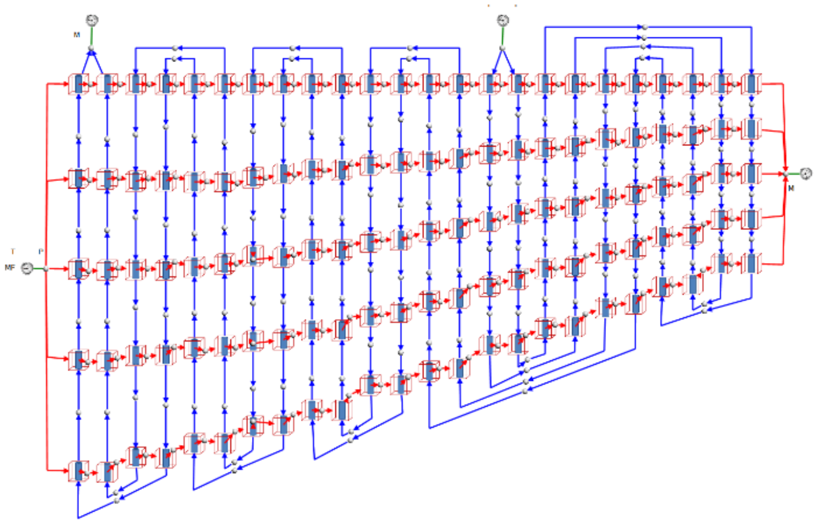

Figure 4. Discretized Flownex model of the heater exchanger analysed by Taler et al. [13]

Table 2. Inputs and Process results for the clean heat exchanger used by Taler.

Geometry	Properties
Pipe increment length : 1.068 m – 0.892 m Pipe outer dia. : 42 mm Pipe inner dia. : 32 mm Gas volume length : 180 mm Gas volume width : 104 mm	Wall emissivity : 0.7 Gas emissivity : 0.1 Ideal gas Air
Process inputs	**Process results**
Steam inlet : 9.61 MPa, 317.2 °C Steam mass flow : 0.668 kg/s Gas inlet : 100 kPa, 862 °C Gas mass flow : 0.815 kg/s	Steam outlet : 420.6 °C Gas outlet : 589.1 °C Total heat : 253.56 kW Radiation heat : 67.12 kW Design UA : 0.7209 kW/K

Table 3. Result comparison for the fouled heat exchanger at Design conditions.

Property	Taler's	This study
Steam outlet Temp. [°C]	399.1	402.2
Gas outlet Temp. [°C]	651.9	648.5
Total heat transfer [kW]	212.5	217.5

Because of the strange flow arrangement of the superheater, it is not obvious which ε-NTU correlation should be used in the lumped model. The model was solved using the three different options and compared with the discretized model results. To capture all changing effects, an artificial "start-up" scenario was implemented where the steam and gas mass flow increases with the gas inlet temperature, but the steam inlet temperature was kept constant. The detail model steam exit temperature and heat transfer results are shown in **Figure 5**. Also shown is the heat transfer error from the three types of flow options using the lumped model.

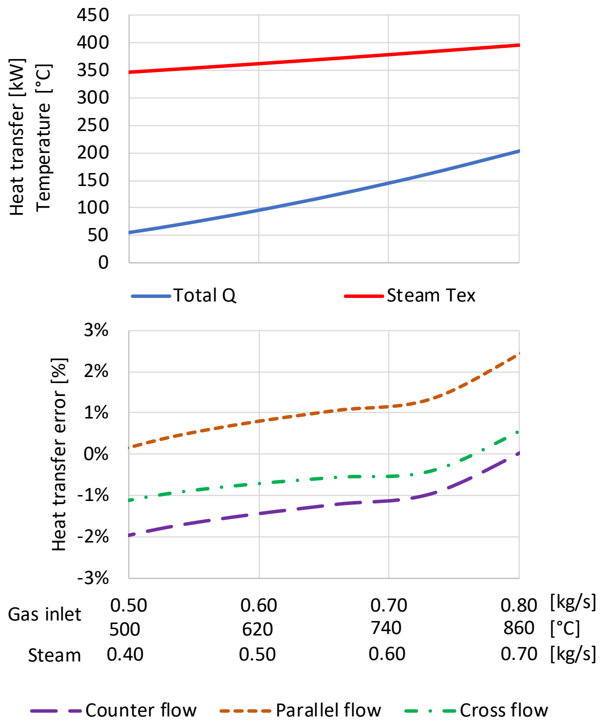

Figure 5. Results for off-design detail analysis of the chosen heat exchanger, along with the heat transfer error for the three flow options using the lumped model.

The errors are quite small for all the flow configuration options, but it does appear as-if the cross-flow with both streams un-mixed option would be the best type to use for this specific heater, producing errors within 1% of the accurate result.

6 Conclusion

It has been shown that many commercial software tools make use of only a mass flow ratio to scale the heat transfer coefficient for gas-to-liquid heat exchangers in the off-design performance calculations. A new approach was presented where the overall UA term is split into a convection and a radiation term and treated separately. Furthermore, a fouling factor is correctly introduced into the overall UA calculation based on a user-supplied cleanliness factor.

The new model has been tested against a discretized pipe-per-pipe heat exchanger model which demonstrated the improved accuracy for off-design performance prediction. The model remains very accurate, even for substantially large deviations from design conditions.

The only additional parameter required from the user is the radiation ratio. This would require some informed estimate of the radiation contribution, either from analogy of similar heaters, a detailed CFD result, or a hand calculation of the radiation and convection terms for the first tube in the heat exchanger using the method described by Trojan and Taler [11]. It is therefore recommended that this model be incorporated in commercial software to produce a more accurate result, without the need for detailed geometry.

The method is applicable for any gas-to-liquid heat exchanger where the gas side's heat transfer dominates the overall heat transfer such as in a water heater / economiser, a convective evaporator, or a superheater at typical power plant pressures. It is believed that this model would even produce reasonable results for a platen superheater of a fired boiler, even though this type of heater also experiences direct radiation from the flame ball. Because the inlet gas temperature to the platen will also change if the flame temperature changes, the model should be able to capture the effect of a change in flame temperature. This is the subject for a future study.

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Nomenclature

a	Mass flow ratio exponent []
b, c, d	Fluid property ratio exponents []
ε	Effectiveness, Emissivity []
CF	Cleanliness factor []
M	Molecular weight [mol/g]
Q	Rate of heat transfer [W]
R_r	Radiation ratio []
R_{foul}	Fouling factor [K/W]
T	Temperature [°C]
UA	Heat transfer factor [W/K]

Subscripts

av	Average
c	Convection
D	Design
r	Radiation
H	Hot
C	Cold

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