The solving of optimizing the structure of a transport node problem by the fuzzy set method

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Abstract. One of the main tasks in the road traffic organization is to create fast, convenient and safe traffic on the road network. For this purpose, the main factors influencing the technical and economic indicators for the construction of intersections of various complexity are determined [1, 2]. These include: capital costs, taking into account reconstruction and installation; the area of the territory for the intersection device; the degree of danger of the intersection; maintenance costs; accident rate of the site; road capacity; impact on the environmental situation of the environment.

Choosing the ratio of intersections with the use of technical means of traffic management in the overall structure of a transport hub is a typical optimization problem [3,4]. However, it is impossible to construct an objective function using classical methods of mathematics due to inaccurate information about the listed factors. To cope with this problem will help the field of mathematics, which is called fuzzy sets [5, 6].

To select the optimal ratio of regulated and unregulated intersections with the use of technical means of traffic management in the overall structure of the transport hub, it is proposed to use seven fuzzy criteria corresponding to the above factors:

T_1 – capital expenditures including reconstruction and installation technical means of traffic management, K, o.e.

T_2 – the area of the alienated territory for the organization of the event, S, o.e.

T_3 – degree of danger of crossing, m, o.e.

T_4 – maintenance costs, C, o.e.

T_5 – accident rate of the site, TA, o.e.

T_6 – road capacity, P, o.e.

T_7 – impact on the environmental situation of the environment, Ec, o.e.

The quantitative assessment of each criterion is made on the basis of expert assessments and statistical data on the range of their changes.

The solution to this problem is made by one option-criteria of varying degrees of importance.

In fact, the criteria of the T_j, have different degrees of importance, since the degree of their significance in the overall structure of the criteria is different. For example, the criteria T_2, T_3, T_5 and T_6 depend not only on the location of the transport node, but also on the configuration of the intersection, the location of the centers of attraction, the availability of

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technical means of traffic management and the functional features of the transport node. When the criteria T_j, have different importance, then each of them is assigned a degree of importance $\alpha_j \geq 0$, (the more important the criterion, the greater a), and the selection rule takes the form:

$$M = T_1^{\alpha_1} \cap T_2^{\alpha_2} \cap \ldots \cap T_j^{\alpha_n}, \tag{1}$$

where $\alpha_j \geq 0$;
$j = 1..n$;

$$\frac{1}{n}\sum_{j=1}^{n}\alpha_j = 1,$$

n – number of criteria.

The coefficients of relative importance are determined based on the procedure of paired comparison of criteria. First, a matrix of paired comparisons K is formed, the elements of which are found in table 1 and satisfy the following conditions: $m_{ii} = 1$; $m_{ij} = 1/m_{ij}$:

$$K = \left(m_{ij}\right) = \begin{pmatrix} m_{11} & m_{12} & .. & m_{1n} \\ m_{21} & m_{22} & .. & m_{2n} \\ .. & .. & .. & .. \\ m_{n1} & m_{n2} & .. & m_{nn} \end{pmatrix}. \tag{2}$$

For the purposes of expert evaluation, a 9-point scale of correspondences was adopted based on the Saati hierarchy analysis method [9].

Table 1 The scale of relative importance of criteria

The intensity of the relative importance of the criteria points	The degree of significance of the criteria	Explanation of the experts ' assessment of the importance of the criteria
1	Equal importance	The importance of the criterion (factors) $T_i=T_j$
3	Moderate superiority of one over the other	Experience and judgment give an easy superiority to one criterion over another
5	Substantial superiority	The available data shows a marked superiority of T_i over T_j
7	Very strong superiority	The superiority of the criterion T_i over T_j is obvious
9	Absolute superiority	Очевидность превосходства over T_j подтверждается всеми имеющимися признаками The evidence of the superiority of the T_i over the T_j is confirmed by all the available signs
2, 4, 6, 8	Intermediate decisions between two adjacent judgments	They are used in compromise cases

Next, the eigenvector of the K–w matrix corresponding to the maximum eigenvalue of $z_{\max}$ is determined.

The desired coefficients α_j are obtained by multiplying the elements w_i (priority vector T_j) by n (the number of criteria under consideration $n = 6$) to fulfill the condition [10]:

$$\alpha_j = n \cdot w_i. \tag{3}$$

As a result of statistical and expert evaluation, the topics for linguistic variables and the range of their possible values were selected [7, 8]. To describe the criteria T_1 – T_7, four functional input linguistic variables (terms) are introduced. The ranges of linguistic variables are given in relative units for each T_j criterion and are presented in Table 2.

Table 2. The range of changes in the linguistic variables of each criterion

№	Linguistic variables	Thermae			
		ZP	PS	PM	PB
	T_1, K	[0; 0,1; 0,3]	[0,25; 0,4; 0,55]	[0,47; 0,6; 0,73]	[0,7; 0,9; 1]
	T_2, S	[0; 0,05; 0,2]	[0,15; 0,3; 0,45]	[0,4; 0,55; 0,7]	[0,65; 0,9; 1]
	T_3,m	–	[0; 0,15; 0,4]	[0,3; 0,5; 0,7]	[0,6; 0,85; 1]
	T_4, C	–	[0; 0,2; 0,4]	[0,2; 0,5; 0,8]	[0,6; 0,8; 1]
	T_5, TA	[0; 0,05; 0,3]	[0,25; 0,4; 0,55]	[0,45; 0,6; 0,75]	[0,7; 0,95; 1]
	T_6, P	–	[0; 0,15; 0,35]	[0,2; 0,5; 0,8]	[0,65; 0,85; 1]
	T_7, Ec	–	[0; 0,1; 0,35]	[0,3; 0,5; 0,7]	[0,65; 0,9; 1]

ZP – «positive close to zero; PS– «positive small»; PM – «positive average»; PB – « positive large»

The construction of the accessory functions for each criterion begins with an estimated calculation of the possible range of changes in quantitative estimates at comparable prices, and the results are tabulated. Then statistical processing is performed and the number of hits of the calculated values in one of the intervals ((v_{ij}), i =1÷6, j =1÷9) is determined. The values of the membership functions are calculated using the formula [11]:

$$\mu_{ij} = \frac{v_{ij}}{v_{i\max}}, \tag{4}$$

where $v_{i\max}$ – maximum value of the element in the i -th row.

We will select the optimal structure of the transport hub with the use of technical means of organizing traffic in this way, taking into account the different importance of the criteria.

In accordance with the scale of relative importance, a matrix K of the above requirements for the transport node is formed:

$$K = (m_{ij}) = \begin{pmatrix} 1 & 7 & 5 & 1 & 5 & 1 & 1 \\ 0,143 & 1 & 3 & 0,2 & 0,2 & 0,333 & 7 \\ 0,2 & 0,333 & 1 & 0,2 & 0,333 & 1 & 0,2 \\ 1 & 5 & 5 & 1 & 1 & 1 & 0,333 \\ 0,2 & 5 & 3 & 1 & 1 & 3 & 0,333 \\ 1 & 3 & 1 & 1 & 0,333 & 1 & 0,333 \\ 1 & 0,143 & 5 & 3 & 3 & 3 & 1 \end{pmatrix}$$

– moderate superiority of criteria is accepted T_1 and T_7 above the criteria T_4 and T_6;

– significant superiority of the criteria T_1 and T_7 above the criterion T_3 and T_4, T_5 над T_2;

– very strong criteria superiority T_1 and T_7 above the criterion T_2;

– moderate superiority of criteria T_4 and T_5 above the criterion T_6;

– moderate superiority of the criterion T_3 above the criterion T_2;

– equal importance of criteria T_1 and T_7, T_4 and T_6;

– moderate superiority T_3, T_6 above the criterion T_2;

– moderate superiority to criteria T_5 above the criterion T_3;

– equal importance of criteria T_6 and T_3, T_4 and T_5;

– moderate superiority of the criterion T_7 above the criterion T_5;

– significant superiority of the criterion T_4 above the criterion T_3.

Based on the obtained matrix *K*, the eigenvector of pairwise comparisons of this matrix is determined:

$$w = \begin{pmatrix} 0,601 \\ 0,362 \\ 0,08 \\ 0,371 \\ 0,358 \\ 0,25 \\ 0,416 \end{pmatrix};$$

Hence the relative importance coefficients are:

$$\alpha_1 = n \cdot w_1 = 7 \cdot 0,601 = 4,207;$$
$$\alpha_2 = n \cdot w_2 = 7 \cdot 0,362 = 2,534;$$
$$\alpha_3 = n \cdot w_3 = 7 \cdot 0,08 = 0,56;$$
$$\alpha_4 = n \cdot w_4 = 7 \cdot 0,371 = 2,597;$$
$$\alpha_5 = n \cdot w_5 = 7 \cdot 0,358 = 2,506;$$
$$\alpha_6 = n \cdot w_6 = 7 \cdot 0,25 = 1,75;$$
$$\alpha_7 = n \cdot w_7 = 7 \cdot 0,416 = 2,912.$$

Hence, membership functions with a modified set of requirements are obtained:

$$\mathrm{T}_1^{\alpha 1} = \mathrm{T}_1^{4,207} = \begin{Bmatrix} 0,054/A_1; 0,197/A_2; 0,117/A_3; 0,0011/A_4; \\ 0,117/A_5; 0,012/A_6; 0,298/A_7; 0,0642/A_8; \end{Bmatrix}$$
$$\mathrm{T}_2^{\alpha 2} = \mathrm{T}_2^{2,534} = \begin{Bmatrix} 1/A_1; 0,336/A_2; 0,098/A_3; 0,0046/A_4; \\ 0,0057/A_5; 0,173/A_6; 0,274/A_7; 0,568/A_8; \end{Bmatrix};$$
$$\mathrm{T}_3^{\alpha 3} = \mathrm{T}_3^{0,56} = \begin{Bmatrix} 1/A_1; 0,883/A_2; 0,751/A_3; 0,333/A_4; \\ 0,858/A_5; 0,333/A_6; 0,883/A_7; 1/A_8; \end{Bmatrix};$$
$$\mathrm{T}_4^{\alpha 4} = \mathrm{T}_4^{2,597} = \begin{Bmatrix} 1/A_1; 0,01/A_2; 0,025/A_3; 0,616/A_4; \\ 0,579/A_5; 0,033/A_6; 1/A_7; 1/A_8; \end{Bmatrix};$$
$$\mathrm{T}_5^{\alpha 5} = \mathrm{T}_5^{2,506} = \begin{Bmatrix} 0,96/A_1; 0,572/A_2; 0,302/A_3; 0,101/A_4; \\ 0,034/A_5; 0,101/A_6; 0,025/A_7; 0,572/A_8; \end{Bmatrix};$$
$$\mathrm{T}_6^{\alpha 6} = \mathrm{T}_6^{1,75} = \begin{Bmatrix} 0,05/A_1; 0,496/A_2; 0,677/A_3; 0,483/A_4; \\ 0,076/A_5; 0,05/A_6; 0,604/A_7; 0,722/A_8; \end{Bmatrix};$$
$$\mathrm{T}_7^{\alpha 7} = \mathrm{T}_7^{2,912} = \begin{Bmatrix} 1/A_1; 0,055/A_2; 0,133/A_3; 0,4/A_4; \\ 0,433/A_5; 0,009/A_6; 0,025/A_7; 0,069/A_8; \end{Bmatrix}.$$

A visual representation of the modified set of requirements with different importance of criteria, from which the degree of influence of each criterion on the choice of the optimal structure of the transport node with the use of technical means of traffic management is clearly visible, is shown in Figure 1.

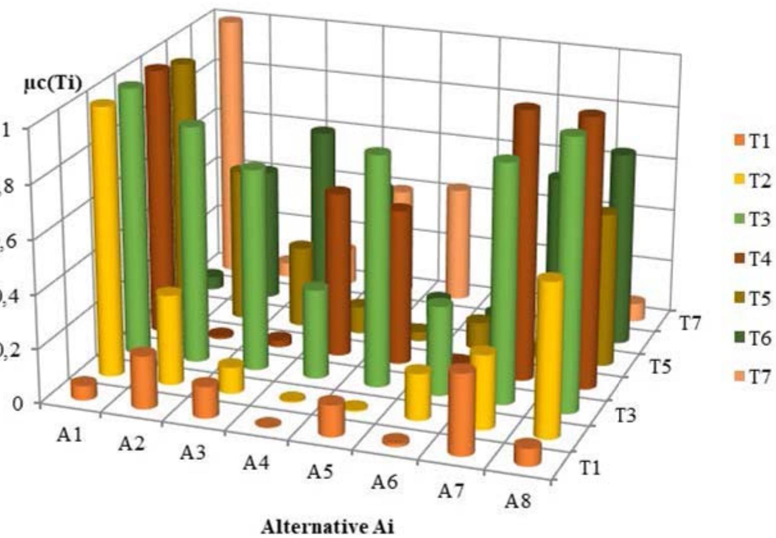

Fig. 1. Evaluation of the functions of belonging to the types of transport node under different criteria with different degrees of importance

Using the proposed rule, we first determine the minimum values of the membership function, from which we choose the maximum. Based on the decision according to the algorithm proposed above, the best option, taking into account the different importance of the criteria, is the eighth choice.

$$F_8^* = \{0,0642;\ 0,568;\ 1;\ 1;\ 0,572;\ 0,722;\ 0,069\}.$$

Given that this solution allows to more accurately take into account the impact of the criteria, we accept the structure of the transport node (two-level interchange + traffic light object).

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