Problem of slip definition in driving systems

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Abstract. The pneumatic tire slip phenomena in vehicle driving system have been investigated for 100 years. Many models describe the tire –road force generation base on it, but in literature we can find various definition of slip. In this paper authors present the most known model of force generation and slip definitions and discuss consequences of choice of them. The model of force generation which is the combination of tire to road friction models and force carrying by deformable running elements which are getting and losing contact with road seems to be very simple and intuitive for process explanation. The implementation of the tire deformation model is the base for many models but is not easy to implement it into dynamic computer modelling of process. So for many years the most common models base on slip function. In paper the concept model is based on deformation introduced and its carry out balance is presented. This model has been adapted to other friction gears like belt gear or friction wheels modelling. The deformation model appears to be quite universal and developable to the energy efficiency analyses or acoustic wave generation.

1 Introduction

Friction joints and gears have been widely used for thousands of years. They can be found in Roman war machines as well as in tools reconstructions based on paintings or exponents found in the Egyptian pyramids. The friction that makes difficult the pulling of large stone blocks became an ally when wooden beams with a circular cross-section were inserted between the ground and the stone block.

Cable-driven pulleys or lever blocks are still used in simple machines. It is possible to dwell for a long time on how this friction allows movement, and the lack of it causes loss of control over movement. Friction forces were described relatively early, and work on their modelling is still ongoing and new aspects of this phenomenon are still discovered and discussed. New materials combined with modern mechatronic devices and advanced control algorithms offer new very interesting possibilities for constructing sophisticated mechanisms. The transfer of forces through friction can be modelled in many ways, which are general in nature or dedicated to specific processes.

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In the domain of construction and dynamics of mobile construction machines, the interaction between the wheel and the ground is the crucial problem of the driving and traction system. So many researchers have been modelling this process since the 1930s, [1] and many physical and mathematical models have been developed.
We can distinguish two main methodology of models building.
The first called physical model consist of schema of process which displays the relation between input and output values with variable state and physical laws which describes the relations. Basing on it we get the mathematical equations which can be used in computer modelling. To build such a model we need to understand the process.

The second methodology of achieving mathematical equation is carried out in "shortcuts" by experiments, to collect a sufficiently large amount of data, in order to use statistical methods to find relationships between the quantities we take as input and the output quantities. This is how the so-called mathematical modelling is giving so called engineering formulas that allows us to make design calculations using simplified methods, without delving into the details of the process.
The same methodology of data collection and competition with simulation result is used to verification of model. The difference between calculation and experiment result inspire to physical model improving.

2 Slip models

In the 1950s, it was determined that the driving force generated between the wheel and the ground depends on the so-called “slip”. The concept of slippage is used in almost all models describing this phenomenon: Beccker [2], Grecenko [3], Kutzbach [4] and Pacejko [5].
Some models were developed in the form of charts of slip to force relation. Following the friction force relation to load and friction coefficient (eq.1) the dependence of the coefficient of adhesion μ on the slip s, which were used to determine the driving force is presented in graphical forms of chart or tables.

$$F(s) = N\mu(s) \qquad (1)$$

The example of graphical representation – model of μ(s) developed by Kutzbach is presented in Fig.1 [4].

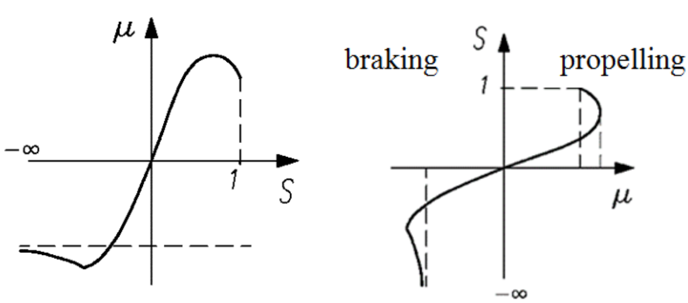

Fig. 1. Diagram of the wheel’s slip as a function of the propelling force coefficient [5].

For this model, the slip is defined as:

$$s = \frac{V_d - V_r}{V_d} \qquad (2)$$

where V_d – is the peripheral speed of wheel (the result of wheel rotation speed multiplied by dynamic radius of wheel), V_r- the speed of vehicle.
We can see, that the slip can take value in the range of :$\langle -\infty; 1\rangle$.
The characteristic is asymmetric, and for s=0 the force is equal to zero. As we know, it is not correct, because vehicle which is standing on the slope is kept without movement by friction force with slip equal zero [10]. For such a model, vehicle would never start to move, because at the start condition, the slip and the propelling are equal zero.
Nevertheless this model is very popular for modelling the agriculture machinery, when working process starts with some movement speed [4].
The uniform definition of slip for driving/propelling and pulled/braking wheel make the computer modelling very easy, especially when we focus only on linear (market in grey in Fig 2) in range of slip $\langle -s_D; s_D\rangle$. Similar approach for linearization is implemented in Dahl model [6].

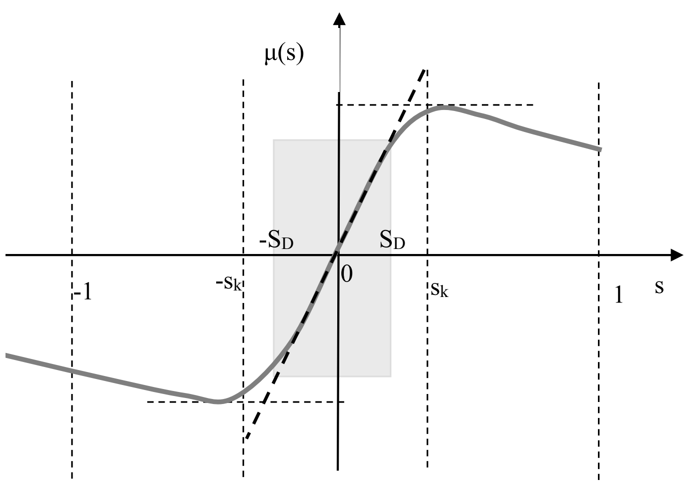

Fig.2. Classical relation curves of driving force coefficient (μ) to slip (s) μ=f(s) with linear part of relation

In literature we can find another analogous definition of slip κ:

$$\kappa = \frac{V_d - V_r}{V_r} \qquad (3)$$

where V_d – is the peripheral speed of wheel (the result of wheel rotation speed multiplied by dynamic radius of wheel), V_r- the speed of vehicle.
We can see, that the slip κ can take value in the range of :$\langle -1; \infty\rangle$.
When we compare influence of definition of the model we can notice, that for starting point with propelling wheel (the wheel is rotating $V_d \neq 0$ and $V_r = 0$) $s=1$ and $\kappa = \infty$. But then we brake with locked wheel ($V_r \neq 0$ and $V_d = 0$) s=-∞ and $\kappa = -1$. So for implementing these models to computing we have to remember about these infinities conditions.

One the most important work on tire modelling has been done by Hans Pacejko [5-8] [11] who discovered the magic formula and developed the brush model for explanation the slip phenomena. He created his "magic formula" analyzing the experimental data and adjusting the mathematical function (eq. 4) which the best suits to the chart for longitudinal slip σ to propelling force and drift angle α to lateral force relations. The shape of this diagram (Fig.3.) has been obtained experimentally, and the explanation of the process of the formation of forces was treated as a secondary issue [5,8].

$$F(\sigma) = d \cdot sin\{c \cdot arctan[b \cdot (1-x) \cdot k + x \cdot arctan(b \cdot k)]\} \cdot \qquad (4)$$

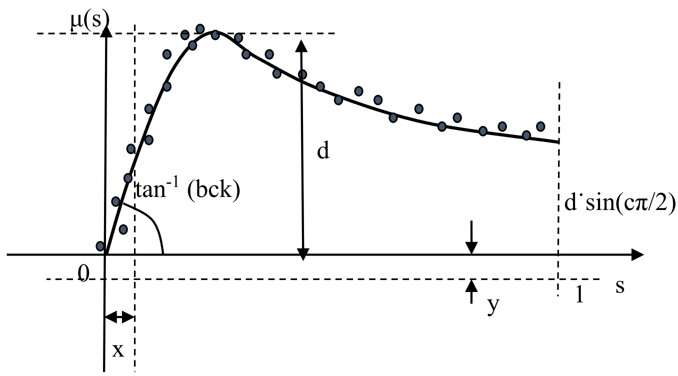

Fig.3. The "Pacejka magic formula" explanation [5] data from experiments. The force for "zero slip" is flowing – describe by one of adjustable parameters.

For this model the slip is defined as follow:

$$\sigma = \begin{cases} \frac{V_d - V_r}{V_d}, \ V_r < V_d \\ \frac{V_r - V_d}{V_r}, \ V_r \geq V_d \end{cases} \qquad (5)$$

where V_d – is the peripheral speed of wheel (the result of wheel rotation speed multiplied by dynamic radius of wheel), V_r- the speed of vehicle.

In this case we have two different definition for driving wheel ($V_d > V_r$) and for braking wheel ($V_d < V_r$). When we implement this model into computer program we have decide which definition we use and when. This disadvantage is compensated by symmetry of diagram and limitation of slip, because we avoid the "zero value" in denominator. The slip σ is in the range of :$\langle -1; 1 \rangle$. In this model for slip zero, the force is produced. But another problem appears, when the vehicle is standing on the horizontal flat surface, the propelling force should be zero [7-11].

Because of that, again we refer to the linearized model presented in Fig. 4.

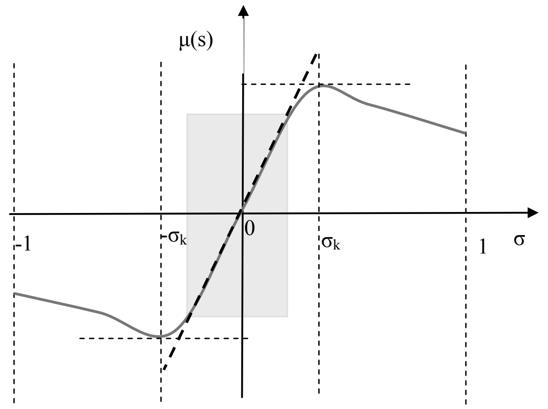

Fig.4. Classical relation curves of driving force coefficient (μ) to slip (s) μ=f(s)

Again we have problem for slip equal to zero, so we have to use some trick for computer modelling [9-11].

3 The friction force transmission by tire deformation model

Hans Pacejka presented the brush model which explains the phenomena of slip. The modified brush model with elastic element between brushes is presented in Fig. 5. In Fig 5a the wheel is standing without horizontal forces. For simplification the pressure on the road under wheel is marked as plane. When horizontal force **R** appears, the brushes are deformed but in the same extend. The friction force $\sum F_N$ is transmitted to the wheel axle by flexible/deformable tire elements.

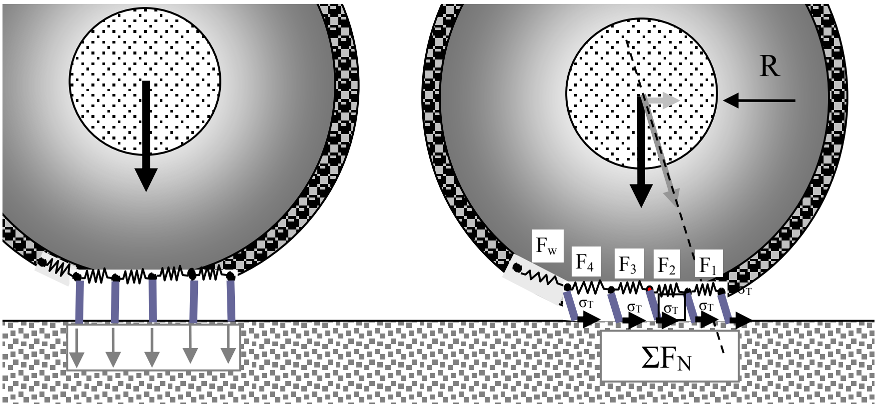

Fig.5. Force acting on no rotating wheel

When wheel stars rotating the new undistorted part of tire (point A in fig, 6a is entering in contact with the road and it is stopped and kept sticking to the road by friction force between tire and road. Because wheel is rotating the part of tire is deformed and transmits the force to the wheel rim. But meantime the deformed part of tire (marked as the B point in fig 6a) carrying out some part of transmitted force. To keep the propelling force of the constant value the balance of new and carried out deformation have to be ensured, so the wheel is rotating faster than moves the axle of wheel.

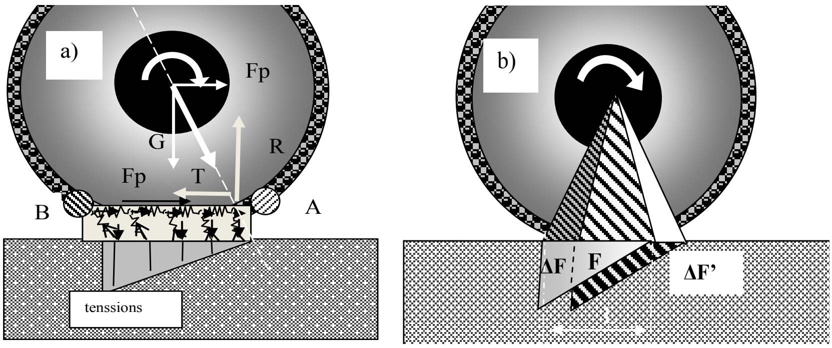

Fig.6. Force layout under rotating wheel [13]

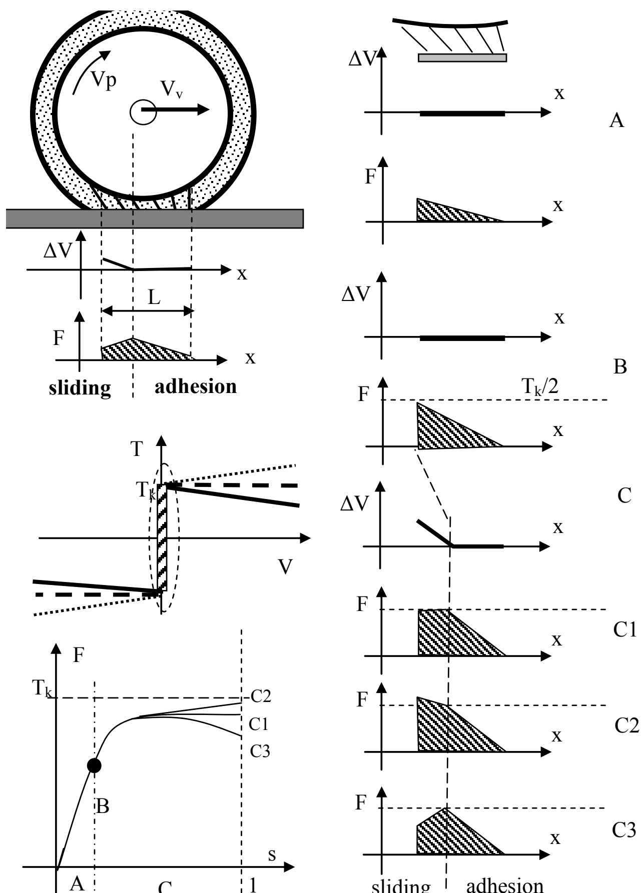

Fig.7. Model of longitudinal force generation process [12]. Vp – peripheral wheel velocity, V_V= vehicle speed, ΔV – relative movement speed (sliding speed) of tire finite element on the road, T- friction force, T_k – maximum value of static friction, A – zone of tire to road adhesion, B- the point of maximum friction without sliding, C- zone of combined friction: static and kinetic (adhesion and slide). E – zone of rolling resistance overcoming (the ordinate axis is movable).

For vehicle wheel the contact tire road patch usually consist of two parts, one adhesion and the second sliding part as it is presented in Fig. 7. When the carried by flexible elements forces overcome the critical value, the tension under some part of tire overcomes the static friction tension and this part of tire starts to slide. The force generated under sliding part of tire depends on the kinetic friction model. The basic friction models are presented in Fig.8. Following the friction model we have first order Columbus model where friction keeps constant value for sliding surfaces, or the second order when is increase or decrease with sliding speed. Following the Lu-Gre model [6] we can take into account the Stribeck effect. Taken friction model influences on the diagram of *F(s)*.

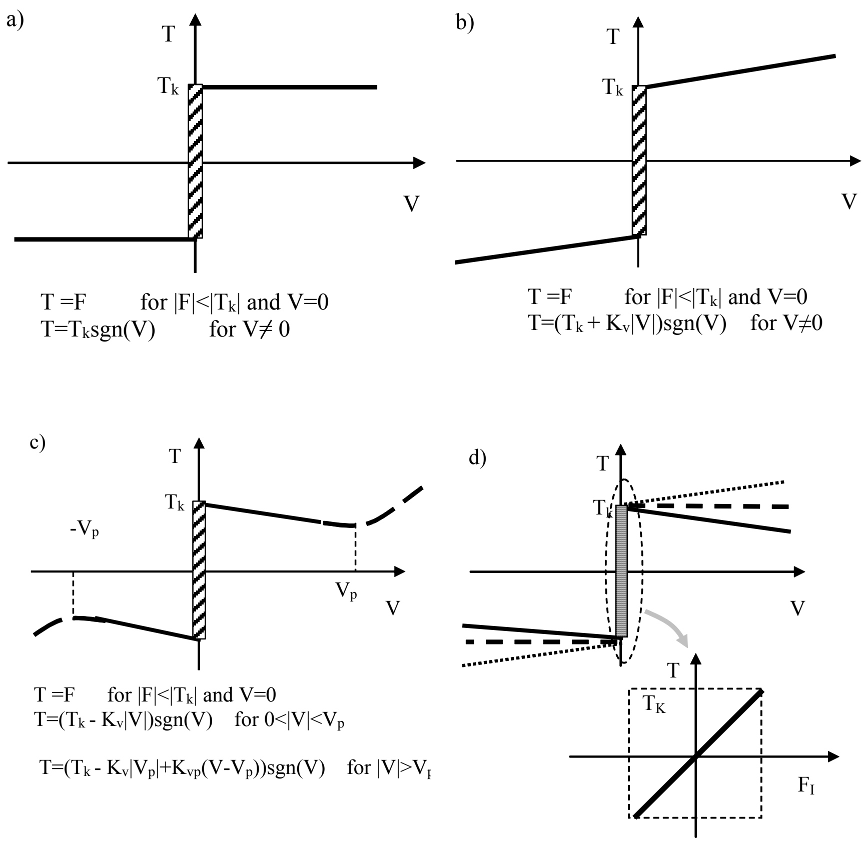

Fig.8. The Columbus friction model a) constant friction value during movement b) friction force increases with the speed c) the Stribeck effect –friction force decreases with the speed growing (the part for low sliding speed), d) the friction value as the answer for driving force in static state [12].

Analysing the deformation and flexibility of carried up elements of rotating tire we can evaluate efficiency (losses of energy) caused by slip.

Analysing the model [19] of slip origin, we can evaluate its value depends mainly on the deformation at the adhesion part of contact patch. If we assume the critical deformation of tire, the slip is increasing because the length of adhesion decreases with sliding.
The value of presented model, which focuses mainly on force carrying deformation than on slip value, would be more valuable, if it would be used also for other system with friction force transmission.

4 V belt transmission

The belt transmission consists of a propelling wheel and a driven wheel or wheels. The forces generating the moments of forces are transmitted via a V-belt with a structure as shown in Fig.9

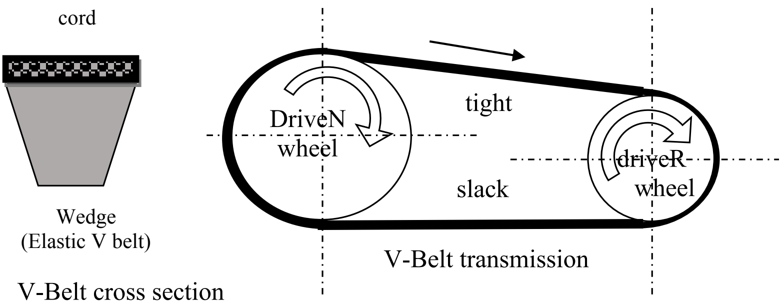

Fig.9. Structure of V-belt transmission

The strip of V-belt consists of a cord reinforced with a metal wire - almost not stretchable, and a wedge-shaped deformable part welded to it. The deformation of the wedge part depends on the speed of the cord and the wheel on which the wedge tip is adjacent.
In order to check the possibility of generalizing the method presented for the co-operation of the wheel with the ground, a similar model of the V-belt deformation process and the generation of forces was adopted.
In the case of the driven wheel, we can again assume that (with neglecting the mass of the deformable element) the part of the wedge whose vertex comes into contact with the driven wheel is not deformed. As the wheel turns slower than the cord moves, the deformation of the wedge part occurs, increasing with the angle of wrap. The total moment is the total moment derived from the stresses in contact with the wheel of the deformed parts of the wedge. These stresses are transferred to the cord and the taut belt is pulled by the drive pulley. The deformable part of the wedge, outside the circle, returns to its undeformed state.

As the belt comes into contact with the drive pulley, the wedge portion accelerates and is gradually deformed, transferring the force between the drive pulley and the cord. As the tension is transferred to the cord above the portion of the belt contacting the drive pulley, the tension is reduced with the angle of wrap. At the end of contact, the belt loses contact with the drive pulley, loses tension.
The model of the cord as a deformable element may help to explain this model. The belt is then stretched between the driven and driving pulley, but is compressed again over the driving pulley to a relaxed state.

The deformation model correctly describes the behaviour of a belt transmission, and can be a tool for analysing a transmission with many pulleys with different radii and angles of contact.
In principle, in belt transmissions, belt slippage is an undesirable effect. However, the occurrence of a kinetic belt slip reduces the transmitted forces and speeds. These

dependencies are not linear and, similarly to the tire model, they depend on the type of kinetic friction model. The Fig.10 presents the tension and deformation of belt wedge.

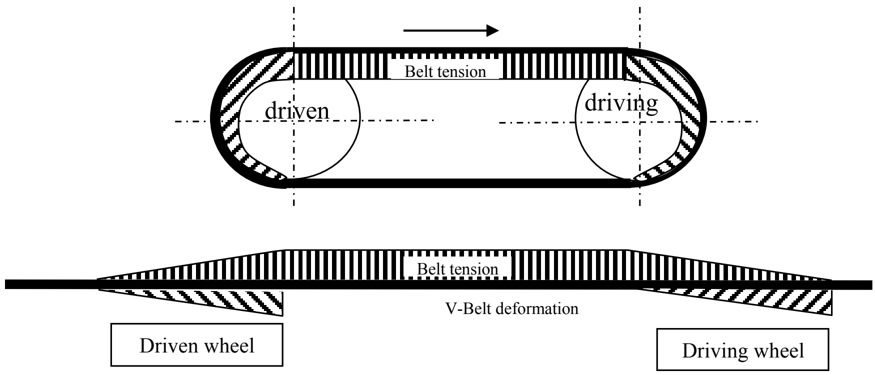

Fig.10. The tension and deformation of the belt on the wheel

5 Friction wheels gear

Another model of the friction gear is the gear in which the rubbing elements are the wheels. To facilitate the analysis, the same dynamic radii of the wheels were assumed. Wheels transmitting moments of force to each other are deformed. This results in the speed of the contacting parts of the tire being different from both the peripheral speed of the drive wheel and the driven wheel. The third speed of the "V" contact layer appears (Fig.11). Recognizing that the deformation of the wheels depends on this speed and the peripheral speed, we can determine the forces, again resulting from the deformation of the elastic part of the wheels.

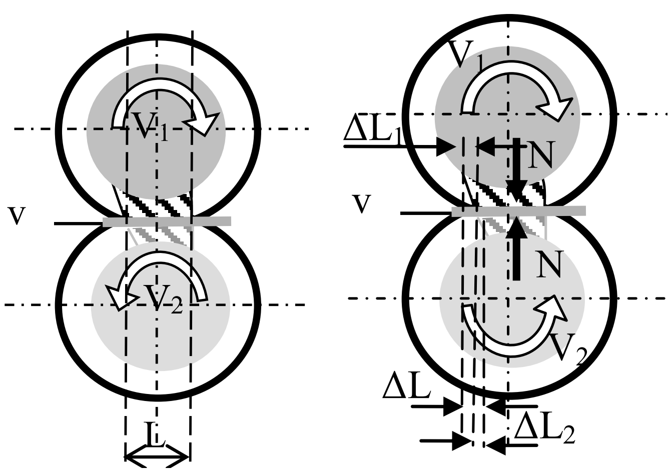

Fig.11. The wheel gear deformation

In order to maintain the balance of forces, i.e. the equality of the moments transferred between the wheels, we are able to determine the speed V so that the slippages of both wheels are the same for the same parameters of the driving and driven wheels.
If we ignore the presence of an intermediate layer, a paradox arises. The decision to choose the definition of a slip based only on the velocities V_1 and V_2 leads to the paradox of

incompatibility of the transmitted forces, or introduces asymmetries to the relationship between the deformation and the transmitted force.
Some discrepancies can be explained by the difference in active length ΔL of both wheels and particular deformations ΔL_1 and ΔL_2 of them. Taking into account the impact of this difference, quite interesting relationships are obtained, but it opens a new discussion, which goes beyond the assumed area of this paper.

6 Conclusions

The adoption of the slip-based force transfer model does not fully describe the process. Due to the complexity of the skid process from deformation and kinetic skid, it is better to use the deformation of elastic elements through which the force flows.

The use of the model of the development of forces based only on slip, in the case of computer modelling, may lead to ambiguity and complicate the model recording for the vicinity of the speed of movement close to zero.

All models, which are limited to determining the forces only on the basis of slip, are valid for certain ranges of velocity and thrust forces.

Modelling the deformation method is closely related to the modelling of tangential and kinetic friction between two surfaces. This gives a very large field for research, especially when considering the structures of the materials from which the contact surfaces are made.
Controlling the ratio of the friction gears (made of low-deforming elements) by changing the pressure allows the slippage of the transmitted force and speed to be changed. This changes the ratio of the speed of the driving wheel to the driven wheel, while maintaining the equality of the moments. There are power losses related to the control of its transmission.

References

1. B. Goriaczkin, Theory of Wheel, (1937).
2. M. Bekker,: Introduction to terrain-vehicle systems. (1969).
3. A. Grečenko, Agrartechnische Kolloquium, **69,** 4 (1989)
4. D. Kutzbach, Grundladen Landtechnik **32,** 2 (1982).
5. H.Pacejka, Tire and Vehicle Dynamics, (2006).
6. G. Erdogan, Vehicle system dynamics IJVMM **51**, 5 (2013)
7. M. Schreiber, D. Kutzbach, Terramechanics, **44**, 6 (2007)
8. L. Jakliński, Models of pneumatic wheel effecting on the soil (1999)
9. T. Mirosław, Z. Żebrowski, Theoretical and Experimental Analysis, Springer Proceedings in Mathematics & Statistics, pp. 261-278 (2016)
10. T. Mirosław, Z. Żebrowski, Przegląd Mechaniczny **6'13,** pp. 37-41 (2013).
11. Z. Zebrowski, The Methodology of wheeled tractor work automation. (2013)
12. T. Miroslaw, ZAMM **101,** 2 (2021)