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The causal relationship k

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Abstract. Aim: A detailed and sophisticated analysis of causal relationships and chains of causation in medicine, life and other sciences by logically consistent statistical methods in the light of empirical data is still not a matter of daily routine for us.
Methods: In this publication, the relationship between cause and effect is characterized while using the tools of classical logic and probability theory.
Results: Methods how to determine conditions are developed in detail. The causal relationship k has been derived mathematically from the axiom +1 = +1.
Conclusion: Non-experimental and experimental data can be analysed by the methods presented for causal relationships.

1 Introduction

Before we try to describe the relationship between a cause and an effect mathematically in a logically consistent way, it is vital to consider whether it is possible to achieve such a goal in principle. In other words, why should we care about the nature of causation at all? Causation seemed painfully important to some[1–11], but not to others. The trial to establish a generally accepted mathematical concept of causation is aggravated especially by the countless attacks [12] on **the principle of causality**[1, 7, 13–16] by many authors which even tried to get rid of this concept altogether and by the very long and rich history of the denialism of causality in Philosophy, Mathematics, Statistics, Physics and a number of other disciplines too. However, it is by no means a hopeless case to mathematise the relationship between a cause and an effect in accordance with the basic laws of classical logic, statistics and probability theory. In point of fact, George Boole (1815 - 1864) has been one of the first who successfully mathematised classical logic[17, 18]. Meanwhile, Boolean algebra is widely used and of highest value. However, logical connectives (also called logical operators) like conjunction (denoted as $\wedge$), disjunction (denoted as $\vee$) or negation (denoted as $\neg$) et cetera which are used to conjoin two statements P_t and Q_t to form another statement can be used under conditions of probability theory too. Especially conditions like necessary and sufficient **conditions** et cetera can be expressed mathematically while using the tools of probability theory. In this context, notable proponents of *conditionalism*[19] like the German anatomist and pathologist David Paul von Hansemann [20] (1858 - 1920) and the biologist and physiologist Max Richard Constantin Verworn [21] (1863 - 1921) and of course other authors too favoured conditions one-sidedly with no objective reason. In his essay “Kausale und konditionale Weltanschauung”Verworn himself ignores cause and effect relationships completely. Verworn demands: “**Das Ding ist also identisch mit der Gesamtheit seiner**

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Bedingungen."[21] In this publication, we will develop new mathematical methods in order to recognise necessary conditions, sufficient conditions, cause effect relationships et cetera with the tools of statistics and probability theory.

2 Material and methods

2.2 Definitions

Classical logic is a branch of philosophy but equally a branch of mathematics too. A Boolean variable, named after George Boole, represents mathematically (either +0 or +1) the two truth values of classical logic and Boolean algebra [17]. However, it is very remarkable that Gottfried Wilhelm Leibniz (1646 - 1716) [22] published 1703 the first self-consistent binary number system [23, 24] representing all numeric values while using typically +0 (zero, false) and +1 (one, true).

2.2.1 The number +0

Definition 1 (The number +0). *The number +0 is defined [7–11] as the expression*

$$+0 \equiv (+1) \times (+0) \equiv (+0) \times (+1) \equiv +1 - 1 \tag{1}$$

2.2.2 The number +1

Definition 2 (The number +1). *The number +1 is defined [7–11] as the expression*

$$+1 \equiv +1 + 0 \equiv +1 - 0 \tag{2}$$

2.2.3 The probability of a single event

Definition 3 (The probability of a single event). *In consideration of the definitions before, let $p({}_{R}X_{t})$ represent the probability of a single event ${}_{R}X_{t}$ at Bernoulli trial t. Let $\Psi\left({}_{R}X_{t}\right)$ represent the wavefunction, a probability amplitude [25] of an event or of finding an event inside a set at a given (period of) time / Bernoulli trial [26] t. Let $\Psi^{*}\left({}_{R}X_{t}\right)$ denote the complex conjugate of the wave-function. In general, it is*

$$\begin{aligned}
p\left({}_{R}X_{t}\right) &\equiv p\left({}_{R}X_{t}\right) \times \frac{({}_{R}X_{t})}{({}_{R}X_{t})} \equiv \frac{p\left({}_{R}X_{t}\right) \times ({}_{R}X_{t})}{({}_{R}X_{t})} \equiv \frac{E({}_{R}X_{t})}{{}_{R}X_{t}} \\
&\equiv p\left({}_{R}X_{t}\right) \times \frac{p\left({}_{R}X_{t}\right) \times ({}_{R}X_{t} \times {}_{R}X_{t})}{p\left({}_{R}X_{t}\right) \times ({}_{R}X_{t} \times {}_{R}X_{t})} \equiv \frac{p\left({}_{R}X_{t}\right) \times p\left({}_{R}X_{t}\right) \times ({}_{R}X_{t} \times {}_{R}X_{t})}{p\left({}_{R}X_{t}\right) \times ({}_{R}X_{t} \times {}_{R}X_{t})} \equiv \frac{E\left({}_{R}X_{t}\right)^{2}}{E\left({}_{R}X_{t}{}^{2}\right)} \\
&\equiv \Psi({}_{R}X_{t}) \times \Psi^{*}({}_{R}U_{t})
\end{aligned} \tag{3}$$

2.2.4 The n-th moment expectation value of X

Definition 4 (The n-th moment expectation value of X). *Let ${}_{R}X_{t}$ denote an event (at a certain (period of) time or **Bernoulli trial t** [26]. Let $p({}_{R}X_{t})$ represent the probability of an event at a given Bernoulli trial t. Let $E({}_{R}X_{t}{}^{n})$ denote the n-th moment **expectation value** [27, 28] of ${}_{R}X_{t}$. Let $E({}_{R}X_{t}{}^{1})$ denote the first moment expectation value of ${}_{R}X_{t}$. Let $E({}_{R}X_{t}{}^{2})$ denote the second moment expectation value of ${}_{R}X_{t}$. In general, **the n-th moment expectation value of ${}_{R}X_{t}$** is defined as*

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}\right) \equiv \left(\underbrace{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{1} \times \ldots}_{(n-times)}\right) \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)$$
$$\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}\right) \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{4}$$

Furthermore, it is

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}\right)^{\mathrm{m}} \equiv \left(\underbrace{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{1} \times \ldots}_{(n-times)}\right)^{\mathrm{m}} \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{m}}$$
$$\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{\mathrm{n}}\right)^{\mathrm{m}} \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{\mathrm{m}} \tag{5}$$

The first moment expectation value of ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$ follows as

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{1}\right) \equiv \left(\underbrace{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{1}}_{(one-times)}\right) \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)$$
$$\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{1}\right) \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)$$
$$\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{6}$$

The second moment expectation value of ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$ follows as

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) \equiv \left(\underbrace{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{1}}_{(two-times)}\right) \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)$$
$$\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) \times p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \tag{7}$$

2.2.5 The n-th moment expectation value of anti X

Definition 5 (The n-th moment expectation value of anti X). *Let $p({}_{R}X_{t})$ represent the probability of a single event ${}_{R}X_{t}$ at a given Bernoulli trial t. Let $(1-p({}_{R}X_{t}))$ represent the probability that a single event ${}_{R}X_{t}$ will not occur, will not exist at a given Bernoulli trial t. Let $E({}_{R}\underline{X}_{t}{}^{n})$ denote the n-th moment* ***expectation value*** *[27, 28] of anti ${}_{R}X_{t}$. Let $E({}_{R}\underline{X}_{t}{}^{1})$ denote the first moment expectation value of anti ${}_{R}X_{t}$. Let $E({}_{R}\underline{X}_{t}{}^{2})$ denote the second moment expectation value of anti ${}_{R}X_{t}$. In general,* ***the n-th moment expectation value of anti ${}_{R}X_{t}$*** *is defined as*

$$E\left({}_{R}\underline{X}_{t}{}^{n}\right) \equiv \left(\underbrace{{}_{R}X_{t}{}^{1} \times {}_{R}X_{t}{}^{1} \times {}_{R}X_{t}{}^{1} \times \ldots}_{(n-times)}\right) \times \left(1 - p\left({}_{R}X_{t}\right)\right)$$
$$\equiv \left({}_{R}X_{t}{}^{n}\right) \times \left(1 - p\left({}_{R}X_{t}\right)\right) \tag{8}$$

The first moment expectation value of anti ${}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$ follows as

$$E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}{}^{1}\right) \equiv \left(\underbrace{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{1}}_{(one-times)}\right) \times \left(1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)$$
$$\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{1}\right) \times \left(1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)$$
$$\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \times \left(1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right) \tag{9}$$

The second moment expectation value of anti $_{\mathrm{R}}\mathrm{X}_{\mathrm{t}}$ follows as

$$
\begin{aligned}
E\left({}_{\mathrm{R}}\underline{X}_{\mathrm{t}}{}^{2}\right) &\equiv \left(\underbrace{{}_{\mathrm{R}}X_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}X_{\mathrm{t}}{}^{1}}_{(two-times)}\right) \times \left(1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right) \\
&\equiv \left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) \times \left(1 - p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)
\end{aligned}
\tag{10}
$$

2.2.6 The n-th moment expectation value of U and W

Definition 6 (The n-th moment expectation value of U and W). *Let $p({}_{R}U_{t}, {}_{R}W_{t})$ represent the joint probability of an occurring of the events ${}_{R}U_{t}$ and ${}_{R}W_{t}$ at the same (period of time or) Bernoulli trial t. Let $E({}_{R}U_{t}{}^{n}, {}_{R}W_{t}{}^{n})$ denote the n-th moment expectation value of ${}_{R}U_{t}$ and ${}_{R}W_{t}$. Let $E({}_{R}U_{t}{}^{1})$ denote the first moment expectation value of ${}_{R}U_{t}$. In general,* ***the n-th moment expectation value of $_{R}U_{t}$ and $_{R}W_{t}$*** *is defined as*

$$
\begin{aligned}
E\left({}_{R}U_{t}{}^{n}, {}_{R}W_{t}{}^{n}\right) &\equiv \left(\underbrace{\left({}_{R}U_{t}{}^{1} \times {}_{R}W_{t}{}^{1}\right) \times \left({}_{R}U_{t}{}^{1} \times {}_{R}W_{t}{}^{1}\right) \times \ldots}_{(n-times)}\right) \times p\left({}_{R}U_{t}, {}_{R}W_{t}\right) \\
&\equiv \left({}_{R}U_{t}{}^{n} \times {}_{R}W_{t}{}^{n}\right) \times p\left({}_{R}U_{t}, {}_{R}W_{t}\right)
\end{aligned}
\tag{11}
$$

The first moment expectation value of $_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ and $_{\mathrm{R}}\mathrm{W}_{\mathrm{t}}$ follows as

$$
\begin{aligned}
E\left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{1}, {}_{\mathrm{R}}W_{\mathrm{t}}{}^{1}\right) &\equiv \left(\underbrace{{}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}W_{\mathrm{t}}{}^{1}}_{(one-times)}\right) \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right) \\
&\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}}{}^{1} \times {}_{\mathrm{R}}W_{\mathrm{t}}{}^{1}\right) \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right) \\
&\equiv \left({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}}\right) \times p\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right)
\end{aligned}
\tag{12}
$$

2.2.7 The variance

Definition 7 (The variance). *Sir Ronald Aylmer Fisher (1890 – 1962), an English statistician, "the single most important figure in 20th century statistics"[29] coined the term variance as follows: "It is therefore desirable* ***in analysing the causes*** *of variability to deal with the square of the standard deviation as the measure of variability. We shall term this quantity the Variance ... "[see 30, p. 399] Again, let $p({}_{R}X_{t})$ represent the probability of a single event ${}_{R}X_{t}$ at a given point in space-time or Bernoulli trial t. Let $E({}_{R}X_{t})$ denote again the expectation value of ${}_{R}X_{t}$. The expectation value of ${}_{R}X_{t}$ is defined as*

$$
E\left({}_{R}X_{t}\right) \equiv p\left({}_{R}X_{t}\right) \times \left({}_{R}X_{t}\right) \equiv \Psi\left({}_{R}X_{t}\right) \times {}_{R}X_{t} \times \Psi^{*}\left({}_{R}X_{t}\right) \tag{13}
$$

The expectation value of the other of ${}_{R}X_{t}$, of the complementary [7, 10] of ${}_{R}X_{t}$, of the opposite of ${}_{R}X_{t}$, of ***the anti $_{R}X_{t}$****, denoted by ${}_{R}\underline{X}_{t}$, is defined as*

$$
E\left({}_{R}\underline{X}_{t}\right) \equiv \left(1 - p\left({}_{R}X_{t}\right)\right) \times \left({}_{R}X_{t}\right) \tag{14}
$$

In this context, $E({}_{R}X_{t}{}^{2})$ is the expectation value of the second moment of ${}_{R}X_{t}$. The expectation value of ${}_{R}X_{t}{}^{2}$ is defined as

$$
E\left({}_{R}X_{t}{}^{2}\right) \equiv p\left({}_{R}X_{t}\right) \times \left({}_{R}X_{t}{}^{2}\right) \equiv p\left({}_{R}X_{t}\right) \times \left({}_{R}X_{t} \times {}_{R}X_{t}\right) \tag{15}
$$

Let $\sigma({}_RX_t)$ denote the standard deviation of ${}_RX_t$. Let $\sigma({}_RX_t)^2$ denote the variance of ${}_RX_t$. In general, the variance [see 31, p. 42] is defined[7, 10] as

$$\begin{aligned}\sigma({}_RX_t)^2 &\equiv \sigma\left({}_RX_t\right) \times \sigma\left({}_RX_t\right)\\ &\equiv E\left({}_RX_t - E\left({}_RX_t\right)\right)^2\\ &\equiv E\left({}_RX_t{}^2\right) - \left(E\left({}_RX_t\right)\right)^2\\ &\equiv \left({}_RX_t{}^2 \times p\left({}_RX_t\right)\right) - \left(p\left({}_RX_t\right) \times {}_RX_t\right)^2\\ &\equiv \left({}_RX_t{}^2\right) \times \left(p\left({}_RX_t\right) - p\left({}_RX_t\right)^2\right)\\ &\equiv \left({}_RX_t{}^2\right) \times \left(p\left({}_RX_t\right) \times \left(1 - p\left({}_RX_t\right)\right)\right)\\ &\equiv {}_RX_t \times \left(p\left({}_RX_t\right) \times {}_RX_t \times \left(1 - p\left({}_RX_t\right)\right)\right)\\ &\equiv E\left({}_RX_t\right) \times {}_RX_t \times \left(1 - p\left({}_RX_t\right)\right)\\ &\equiv E\left({}_RX_t\right) \times E\left({}_R\underline{X}_t\right)\end{aligned} \tag{16}$$

From equation 16 follows that

$${}_RU_t \equiv \frac{\sigma({}_RU_t)}{\sqrt[2]{p\left({}_RU_t\right) \times \left(1 - p\left({}_RU_t\right)\right)}} \tag{17}$$

and that

$${}_RW_t \equiv \frac{\sigma({}_RW_t)}{\sqrt[2]{p\left({}_RW_t\right) \times \left(1 - p\left({}_RW_t\right)\right)}} \tag{18}$$

2.2.8 The n-th moment co-variance

Definition 8 (**The n-th moment co-variance**). *Let $p({}_RU_t,\ {}_RW_t)$ represent the joint probability of ${}_RU_t$ and ${}_RW_t$ at the same (period of time) Bernoulli trial t. Let $E({}_RU_t{}^n,\ {}_RW_t{}^n)$ denote the n-th moment expectation value of ${}_RU_t$ and ${}_RW_t$. Let $E({}_RU_t{}^n)$ denote the n-th moment expectation value of ${}_RU_t$. Let $E({}_RW_t{}^n)$ denote the n-th moment expectation value of ${}_RW_t$. Let $\sigma({}_RU_t,\ {}_RW_t)$ denote the co-variance between ${}_RU_t$ and ${}_RW_t$. In general,* ***the n-th moment co-variance between ${}_RU_t$ and ${}_RW_t$*** *is defined[7] as*

$$\begin{aligned}\sigma\left({}_RU_t{}^n, {}_RW_t{}^n\right) &\equiv \left({}_RU_t{}^n \times {}_RW_t{}^n\right) \times \left(p\left({}_RU_t, {}_RW_t\right) - \left(p\left({}_RU_t\right) \times p\left({}_RW_t\right)\right)\right)\\ &\equiv \left(\left({}_RU_t{}^n \times {}_RW_t{}^n \times p\left({}_RU_t, {}_RW_t\right)\right) - \left(\left({}_RU_t{}^n \times {}_RW_t{}^n\right) \times p\left({}_RU_t\right) \times p\left({}_RW_t\right)\right)\right)\\ &\equiv E\left({}_RU_t{}^n, {}_RW_t{}^n\right) - \left({}_RU_t{}^n \times p\left({}_RU_t\right)\right) \times \left({}_RW_t{}^n \times p\left({}_RW_t\right)\right)\\ &\equiv E\left({}_RU_t{}^n, {}_RW_t{}^n\right) - \left(E\left({}_RU_t{}^n\right) \times E\left({}_RW_t{}^n\right)\right)\end{aligned} \tag{19}$$

From equation 19 follows equally that

$$\begin{aligned}\sigma\left({}_RU_t, {}_RW_t\right) &\equiv \left({}_RU_t \times {}_RW_t\right) \times \left(p\left({}_RU_t, {}_RW_t\right) - \left(p\left({}_RU_t\right) \times p\left({}_RW_t\right)\right)\right)\\ &\equiv \left(\left({}_RU_t \times {}_RW_t \times p\left({}_RU_t, {}_RW_t\right)\right) - \left(\left({}_RU_t \times {}_RW_t\right) \times p\left({}_RU_t\right) \times p\left({}_RW_t\right)\right)\right)\\ &\equiv E\left({}_RU_t, {}_RW_t\right) - \left({}_RU_t \times p\left({}_RU_t\right)\right) \times \left({}_RW_t \times p\left({}_RW_t\right)\right)\\ &\equiv E\left({}_RU_t, {}_RW_t\right) - \left(E\left({}_RU_t\right) \times E\left({}_RW_t\right)\right)\end{aligned} \tag{20}$$

Equation 20 demands too that

$${}_RU_t \times {}_RW_t \equiv \frac{\sigma\left({}_RU_t, {}_RW_t\right)}{\left(p\left({}_RU_t, {}_RW_t\right) - \left(p\left({}_RU_t\right) \times p\left({}_RW_t\right)\right)\right)} \tag{21}$$

2.2.9 Two by two table of Bernoulli random variables

Definition 9 (Two by two table of Bernoulli random variables).

The two by two or contingency table as introduced by Karl Pearson[32] in 1904 harbours a large variety of topics and debates. Central to these is the problem to apply the laws of classical logic on data sets, which concerns the justification of inferences that extrapolate from sample data to general facts. However, a contingency table is still an appropriate theoretical model too for studying the relationships between two *Bernoulli[33] (i. e. +0/+1) distributed random variables* existing or occurring at the same *Bernoulli trial* [26] (period of time) t. In this context, let a Bernoulli distributed random variable A_t denote a risk factor, a condition or a cause et cetera and occur or exist with the probability $p(A_t)$ at the *Bernoulli trial* [26] (period of time) t. Let $E(A_t)$ denote the expectation value of A_t. In the case of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E(A_t) &\equiv A_t \times p(A_t) \\ &\equiv p(a_t) + p(b_t) \\ &\equiv (+0+1) \times p(A_t) \\ &\equiv p(A_t) \end{aligned} \tag{22}$$

Let a Bernoulli distributed random variable B_t denote an outcome, a conditioned event or an effect and occur or exist et cetera with the probability $p(B_t)$ at the Bernoulli trial (period of time) t. Let $E(B_t)$ denote the expectation value of B_t. It is

$$\begin{aligned} E(B_t) &\equiv B_t \times p(B_t) \\ &\equiv p(a_t) + p(c_t) \\ &\equiv (+0+1) \times p(B_t) \\ &\equiv p(B_t) \end{aligned} \tag{23}$$

Let $p(a_t)= p(A_t \wedge B_t)$ denote the joint probability distribution of A_t and B_t at the same Bernoulli trial (period of time) t. In general it is

$$\begin{aligned} E(a_t) &\equiv E(A_t \wedge B_t) \\ &\equiv (A_t \times B_t) \times p(A_t \wedge B_t) \\ &\equiv p(A_t \wedge B_t) \\ &\equiv p(a_t) \end{aligned} \tag{24}$$

Let $p(b_t)= p(A_t \wedge \neg B_t)$ denote the joint probability distribution of A_t and not B_t at the same Bernoulli trial (period of time) t. In general it is

$$\begin{aligned} E(b_t) &\equiv E(A_t \wedge \neg B_t) \\ &\equiv (A_t \times \neg B_t) \times p(A_t \wedge \neg B_t) \\ &\equiv p(A_t \wedge \neg B_t) \\ &\equiv p(b_t) \end{aligned} \tag{25}$$

Let $p(c_t)= p(\neg A_t \wedge B_t)$ denote the joint probability distribution of not A_t and B_t at the same Bernoulli trial (period of time) t. In general it is

$$\begin{aligned} E(c_t) &\equiv E(\neg A_t \wedge B_t) \\ &\equiv (\neg A_t \times B_t) \times p(\neg A_t \wedge B_t) \\ &\equiv p(\neg A_t \wedge B_t) \\ &\equiv p(c_t) \end{aligned} \tag{26}$$

Let $p(d_t) = p(\neg A_t \wedge \neg B_t)$ denote the joint probability distribution of not A_t and not B_t at the same Bernoulli trial (period of time) t. In general it is

$$\begin{aligned} E(d_t) &\equiv E(\neg A_t \wedge \neg B_t) \\ &\equiv (\neg A_t \times \neg B_t) \times p(\neg A_t \wedge \neg B_t) \\ &\equiv p(\neg A_t \wedge \neg B_t) \\ &\equiv p(d_t) \end{aligned} \tag{27}$$

In general, it is

$$p(a_t) + p(b_t) + p(c_t) + p(d_t) \equiv +1 \tag{28}$$

Table 1 provide us with an overview of the definitions above.

Table 1: The two by two table of Bernoulli random variables

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	$p(a_t)$	$p(b_t)$	$p(A_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

2.2.10 Two by two table of Binomial random variables

Definition 10 (Two by two table of Binomial random variables).

Under conditions where *the probability of an event, an outcome, a success et cetera is* ***constant*** *from Bernoulli trial to Bernoulli trial t*, it is

$$\begin{aligned} A &= N \times E(A_t) \\ &\equiv N \times (A_t \times p(A_t)) \\ &\equiv N \times (p(A_t) + p(B_t)) \\ &\equiv N \times p(A_t) \end{aligned} \tag{29}$$

and

$$\begin{aligned} B &= N \times E(B_t) \\ &\equiv N \times (B_t \times p(B_t)) \\ &\equiv N \times (p(A_t) + p(c_t)) \\ &\equiv N \times p(B_t) \end{aligned} \tag{30}$$

where N denotes the population size. Furthermore, it is

$$a \equiv N \times (E(A_t)) \equiv N \times (p(A_t)) \tag{31}$$

and

$$b \equiv N \times (E(B_t)) \equiv N \times (p(B_t)) \tag{32}$$

and

$$c \equiv N \times (E(c_t)) \equiv N \times (p(c_t)) \tag{33}$$

and

$$d \equiv N \times (E(d_t)) \equiv N \times (p(d_t)) \tag{34}$$

and

$$a + b + c + d \equiv A + \underline{A} \equiv B + \underline{B} \equiv N \tag{35}$$

Table 2 provide us again an overview of a two by two table of Binomial random variables.

Table 2: The two by two table of Binomial random variables

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	a	b	A
A_t	FALSE	c	d	$\underline{A}$
		B	$\underline{B}$	N

2.2.11 Independence

Definition 11 (Independence).

In general, an event A_t at the Bernoulli trial t need not but can be independent of the existence or of the occurrence of another event B_t *at the same* Bernoulli trial t. Mathematically, independence [34, 35] in terms of probability theory is defined at the same (period of) time t (i. e. Bernoulli trial t) as

$$p(A_t \land B_t) \equiv p(A_t) \times p(B_t) \tag{36}$$

2.2.12 Dependence

Definition 12 (Dependence).

The dependence of events [see 1, p. 57-61] is defined as

$$p\left(\underbrace{A_t \land B_t \land C_t \land \ldots}_{n}\right) \equiv \sqrt[n]{\underbrace{p(A_t) \times p(B_t) \times p(C_t) \times \ldots}_{n}} \tag{37}$$

2.2.13 Exclusion relationship

Definition 13 (Exclusion relationship [*EXCL*]).

Mathematically, the exclusion (EXCL) relationship, denoted by $p(A_t \mid B_t)$ in terms of statistics and probability theory, is defined[see 1, p. 68-70] as

$$\begin{aligned} p(A_t \mid B_t) &\equiv p(b_t) + p(c_t) + p(d_t) \\ &\equiv \frac{N \times (p(b_t) + p(c_t) + p(d_t))}{N} \\ &\equiv \frac{\sum_{t=1}^{N} (\underline{A}_t \vee \underline{B}_t)}{N} \equiv \frac{b + c + d}{N} \\ &\equiv +1 \end{aligned} \tag{38}$$

Based on the Henry M. Sheffer, 1913 relationship, it is $p(A_t \wedge B_t) \equiv 1 - p(A_t \mid B_t)$ (see table 3).

Table 3: A_t excludes B_t and vice versa.

		Conditioned (COVID-19) B_t		
		TRUE	FALSE	
Condition (Vaccine)	TRUE	**+0**	$p(b_t)$	$p(A_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

Remark 1. *Pfizer Inc. and BioNTech SE announced on Monday, November 09, 2020 - 06:45am results from a Phase 3 COVID-19 vaccine trial with 43.538 participants which provides evidence that their vaccine (BNT162b2) is preventing COVID-19 in participants without evidence of prior SARS-CoV-2 infection. In toto, 170 confirmed cases of COVID-19 were evaluated, with 8 in the vaccine group versus 162 in the placebo group. The exclusion relationship can be calculated as follows.*

$$\begin{aligned} p(Vaccine : BNT162b2 \mid COVID-19(infection)) &\equiv p(b_t) + p(c_t) + p(d_t) \\ &\equiv 1 - p(a_t) \\ &\equiv 1 - \left(\frac{8}{43538}\right) \\ &\equiv +0,99981625 \end{aligned} \tag{39}$$

with a P Value = 0,000184.

2.2.14 The goodness of fit test of an exclusion relationship

Definition 14 (The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship).

Under some well known circumstances, testing hypothesis about an exclusion relationship $p(A_t \mid B_t)$ is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\tilde{\chi}^2{}_{\text{Calculated}}\left(\left(A_{\text{t}} \mid B_{\text{t}}\right) \mid A\right) \equiv \frac{(b-(a+b))^2}{A} + \frac{((c+d)-\underline{A})^2}{\underline{A}} \equiv \frac{a^2}{A} + 0 \equiv \frac{a^2}{A} \quad (40)$$

or equally as

$$\tilde{\chi}^2{}_{\text{Calculated}}\left(\left(A_{\text{t}} \mid B_{\text{t}}\right) \mid B\right) \equiv \frac{(c-(a+c))^2}{B} + \frac{((b+d)-\underline{B})^2}{\underline{B}} \equiv \frac{a^2}{B} + 0 \equiv \frac{a^2}{B} \quad (41)$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of an exclusion relationship/distribution $p(A_t \mid B_t)$, in which case the null hypothesis to be accepted. Yate's [36] continuity correction has not been used under these circumstances.

2.2.15 The left-tailed p Value of an exclusion relationship

Definition 15 (The left-tailed p Value of an exclusion relationship).

It is known that as a sample size, N, increases, a sampling distribution of a special test statistic approaches the normal distribution (central limit theorem). Under these circumstances, the left-tailed (lt) p Value [37] of an exclusion relationship can be calculated as follows.

$$pValue_{\text{lt}}\left(A_{\text{t}} \mid B_{\text{t}}\right) \equiv 1 - e^{-\left(1-p\left(A_{\text{t}} \mid B_{\text{t}}\right)\right)} \equiv 1 - e^{-(a/N)} \quad (42)$$

A low p-value may provide some evidence of statistical significance.

2.2.16 Neither nor conditions

Definition 16 (Neither A_t nor B_t conditions [*NOR*]).

Mathematically, a neither A_t nor B_t condition (rejection, Jean Nicod's statement 1924) relationship (NOR), denoted by $p(A_t \downarrow B_t)$ in terms of statistics and probability theory, is defined [see 1, p. 68-70] as

$$
\begin{aligned}
p\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right) &\equiv p\left(d_{\mathrm{t}}\right) \\
&\equiv \frac{N-\sum_{t=1}^{N}\left(A_{\mathrm{t}} \vee B_{\mathrm{t}}\right)}{N} \equiv \frac{\sum_{t=1}^{N}\left(\underline{A}_{\mathrm{t}} \wedge \underline{B}_{\mathrm{t}}\right)}{N} \equiv \frac{N \times\left(p\left(d_{\mathrm{t}}\right)\right)}{N} \\
&\equiv \frac{d}{N} \\
&\equiv +1
\end{aligned} \tag{43}
$$

2.2.17 The Chi square goodness of fit test of a neither nor condition relationship

Definition 17 (The $\tilde{\chi}^2$ goodness of fit test of a neither A_t nor B_t condition relationship).

A neither A_t nor B_t condition relationship $p(A_t \downarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution). The $\tilde{\chi}^2$ goodness of fit test of a neither A_t nor B_t condition relationship with degree of freedom (d. f.) of d. f. = 1 may be calculated as

$$
\begin{aligned}
\tilde{\chi}^2_{\text{Calculated}}\left(\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right) \mid A\right) &\equiv \frac{(d-(c+d))^2}{\underline{A}} + \\
&\quad \frac{((a+b)-A)^2}{A} \\
&\equiv \frac{c^2}{\underline{A}} + 0
\end{aligned} \tag{44}
$$

or equally as

$$
\begin{aligned}
\tilde{\chi}^2_{\text{Calculated}}\left(\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right) \mid B\right) &\equiv \frac{(d-(b+d))^2}{\underline{B}} + \\
&\quad \frac{((a+c)-B)^2}{B} \\
&\equiv \frac{b^2}{\underline{B}} + 0
\end{aligned} \tag{45}
$$

Yate's [36] continuity correction has not been used in this context.

2.2.18 The left-tailed p Value of a neither nor B condition relationship

Definition 18 (The left-tailed p Value of a neither A_t nor B_t condition relationship).

The left-tailed (lt) p Value [37] of a neither A_t nor B_t condition relationship can be calculated as follows.

$$
\begin{aligned}
pValue_{\mathrm{lt}}\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right) &\equiv 1-e^{-\left(1-p\left(A_{\mathrm{t}} \downarrow B_{\mathrm{t}}\right)\right)} \\
&\equiv 1-e^{-p\left(A_{\mathrm{t}} \vee B_{\mathrm{t}}\right)} \\
&\equiv 1-e^{-((a+b+c)/N)}
\end{aligned} \tag{46}
$$

where ∨ may denote disjunction or logical inclusive or. In this context, a low p-value indicates again a statistical significance. In general, it is $p(A_t \vee B_t) \equiv 1 - p(A_t \downarrow B_t)$ (see table 4).

Table 4: Neither A_t nor B_t relationship.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	0	0	0
	NO	0	1	1
		0	1	1

2.2.19 Necessary condition

Definition 19 (Necessary condition [*Conditio sine qua non*]).

Mathematically, the necessary condition (SINE) relationship, denoted by $p(A_t \leftarrow B_t)$ in terms of statistics and probability theory, is defined [see 1, p. 15-28] as

$$\begin{aligned} p(A_t \leftarrow B_t) \equiv p(A_t \vee \underline{B}_t) &\equiv \frac{\sum_{t=1}^{N} (A_t \vee \underline{B}_t)}{N} \\ &\equiv p(a_t) + p(b_t) + p(d_t) \\ &\equiv \frac{N \times (p(a_t) + p(b_t) + p(d_t))}{N} \\ &\equiv \frac{a+b+d}{N} \\ &\equiv +1 \end{aligned} \tag{47}$$

It is $p(A_t - < B_t) \equiv 1 - p(A_t \leftarrow B_t)$ (see Table 5).

Table 5: Necessary condition.

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	$p(a_t)$	$p(b_t)$	$p(A_t)$
A_t	FALSE	**+0**	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

Remark 2. *A necessary condition A_t is characterized itself by the property that another event B_t will not occur if A_t is not given, if A_t did not occur [1–11].* ***Example****. A human being cannot live without water. A human being cannot live without gaseous oxygen et cetera. Water itself is a necessary*

condition of human life. However, gaseous oxygen is a necessary condition of human life too. Thus far, even if water is given and even if water is a necessary condition of human life, without gaseous oxygen there will be no human life. In general, if a conditioned or an outcome B_t depends on the necessary condition A_t and equally on numerous other necessary conditions, an event B_t will not occur if A_t itself is not given independently of the occurrence of other necessary conditions.

2.2.20 The Chi-square goodness of fit test of a necessary condition relationship

Definition 20 (The $\tilde{\chi}^2$ goodness of fit test of a necessary condition relationship).

Under some well known circumstances, hypothesis about the conditio sine qua non relationship $p(A_t \leftarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or χ^2-distribution), first described by the German statistician Friedrich Robert Helmert [38] and later rediscovered by Karl Pearson [39] in the context of a goodness of fit test. The $\tilde{\chi}^2$ goodness of fit test of a conditio sine qua non relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}(A_t \leftarrow B_t \mid B) &\equiv \frac{(a-(a+c))^2}{B} + \\ &\quad \frac{((b+d)-\underline{B})^2}{\underline{B}} \\ &\equiv \frac{c^2}{B} + 0 \\ &\equiv \frac{c^2}{B} \end{aligned} \tag{48}$$

or equally as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}(A_t \leftarrow B_t \mid \underline{A}) &\equiv \frac{(d-(c+d))^2}{\underline{A}} + \\ &\quad \frac{((a+b)-A)^2}{A} \\ &\equiv \frac{c^2}{\underline{A}} + 0 \\ &\equiv \frac{c^2}{\underline{A}} \end{aligned} \tag{49}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. It has not yet been finally clarified whether the use of Yate's [36] continuity correction is necessary at all.

2.2.21 The left-tailed p Value of the conditio sine qua non relationship

Definition 21 (The left-tailed p Value of the conditio sine qua non relationship).

The left-tailed (lt) p Value [37] of the conditio sine qua non relationship can be calculated as follows.

$$\begin{aligned} pValue_{\text{lt}}(A_t \leftarrow B_t) &\equiv 1 - e^{-(1-p(A_t \leftarrow B_t))} \\ &\equiv 1 - e^{-(c/N)} \end{aligned} \tag{50}$$

2.2.22 Sufficient condition

Definition 22 (Sufficient condition [*Conditio per quam*]).

Mathematically, the sufficient condition (IMP) relationship, denoted by $p(A_t \rightarrow B_t)$ in terms of statistics and probability theory, is defined[see 1, p. 68-70] as

$$\begin{aligned} p(A_t \rightarrow B_t) \equiv p(\underline{A}_t \vee B_t) &\equiv \frac{\sum_{t=1}^{N} (\underline{A}_t \vee B_t)}{N} \\ &\equiv p(a_t) + p(c_t) + p(d_t) \\ &\frac{N \times (p(a_t) + p(c_t) + p(d_t))}{N} \\ &\equiv \frac{a + c + d}{N} \\ &\equiv +1 \end{aligned} \tag{51}$$

It is $p(A_t > -B_t) \equiv 1 - p(A_t \rightarrow B_t)$ (see Table 6).

Table 6: Sufficient condition.

		Conditioned B_t TRUE	Conditioned B_t FALSE	
Condition	TRUE	$p(a_t)$	**+0**	$p(A_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

Remark 3. *A sufficient condition A_t is characterized by the property that another event B_t will occur if A_t is given, if A_t itself occured [1–11].* ***Example****. The ground, the streets, the trees, human beings and many other objects too will become wet during a heavy rain. Especially,* ***if*** *it is raining (event A_t),* ***then*** *human beings will be wet (event B_t). However, even if this is a common human wisdom, a human being equipped with an appropriate umbrella (denoted by R_t) need not to become wet even during a heavy rain. An appropriate umbrella (R_t) is similar to an event which can counteract the occurrence of another event (B_t) and can be understood something as an anti-dot of another event. In other words, an appropriate umbrella is an antidote of the effect of rain on human body, an appropriate umbrella has the potential to protect humans from the effect of rain on their body. It is a good rule of thumb that the following relationship*

$$p(A_t \rightarrow B_t) + p(R_t \wedge B_t) \equiv +1 \tag{52}$$

indicates that R_t is an antidote of A_t. However, taking a shower, swimming in a lake et cetera may make human hair wet too. More than anything else, however, these events does not affect the final outcome, the effect of raining on human body.

2.2.23 The Chi square goodness of fit test of a sufficient condition relationship

Definition 23 (The $\tilde{\chi}^2$ goodness of fit test of a sufficient condition relationship).

Under some well known circumstances, testing hypothesis about the conditio per quam relationship $p(A_t \rightarrow B_t)$ is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a conditio per quam relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left(A_t \rightarrow B_t \mid A\right) &\equiv \frac{(a-(a+b))^2}{A} + \\ &\quad \frac{((c+d)-\underline{A})^2}{\underline{A}} \\ &\equiv \frac{b^2}{A} + 0 \\ &\equiv \frac{b^2}{A}\end{aligned} \tag{53}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left(A_t \rightarrow B_t \mid \underline{B}\right) &\equiv \frac{(d-(b+d))^2}{\underline{B}} + \\ &\quad \frac{((a+c)-B)^2}{B} \\ &\equiv \frac{b^2}{\underline{B}} + 0 \\ &\equiv \frac{b^2}{\underline{B}}\end{aligned} \tag{54}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of the conditio per quam relationship/distribution $p(A_t \rightarrow B_t)$, in which case the null hypothesis accepted. Yate's [36] continuity correction has not been used in this context.

2.2.24 The left-tailed p Value of the conditio per quam relationship

Definition 24 (The left-tailed p Value of the conditio per quam relationship).

The left-tailed (lt) p Value [37] of the conditio per quam relationship can be calculated as follows.

$$\begin{aligned}pValue_{\text{lt}}\left(A_t \rightarrow B_t\right) &\equiv 1 - e^{-\left(1-p\left(A_t \rightarrow B_t\right)\right)} \\ &\equiv 1 - e^{-(b/N)}\end{aligned} \tag{55}$$

Again, a low p-value indicates a statistical significance.

2.2.25 Necessary and sufficient conditions

Definition 25 (Necessary and sufficient conditions [*EQV*]).

The necessary and sufficient condition (EQV) relationship, denoted by $p(A_t \leftrightarrow B_t)$ in terms of statistics and probability theory, is defined[see 1, p. 68-70] as

$$\begin{aligned} p(A_t \leftrightarrow B_t) &\equiv \frac{\sum_{t=1}^{N} ((A_t \vee \underline{B}_t) \wedge (\underline{A}_t \vee B_t))}{N} \\ &\equiv p(a_t) + p(d_t) \\ &\equiv \frac{N \times (p(a_t) + p(d_t))}{N} \\ &\equiv \frac{a+d}{N} \\ &\equiv +1 \end{aligned} \tag{56}$$

2.2.26 The Chi square goodness of fit test of a necessary and sufficient condition relationship

Definition 26 (The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship).

Even the necessary and sufficient condition relationship $p(A_t \leftrightarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}(A_t \leftrightarrow B_t \mid A) &\equiv \frac{(a-(a+b))^2}{A} + \\ &\quad \frac{d-((c+d))^2}{\underline{A}} \\ &\equiv \frac{b^2}{A} + \frac{c^2}{\underline{A}} \end{aligned} \tag{57}$$

or equally as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}(A_t \leftrightarrow B_t \mid B) &\equiv \frac{(a-(a+c))^2}{B} + \\ &\quad \frac{d-((b+d))^2}{\underline{B}} \\ &\equiv \frac{c^2}{B} + \frac{b^2}{\underline{B}} \end{aligned} \tag{58}$$

The calculated $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship can be compared with a theoretical chi-square value at a certain level of significance α. Under conditions where the observed values are equal to the expected/theoretical values of a necessary and sufficient condition relationship/distribution $p(A_t \leftrightarrow B_t)$, the $\tilde{\chi}^2$-distribution equals zero. It is to be cleared whether Yate's [36] continuity correction should be used at all.

2.2.27 The left-tailed p Value of a necessary and sufficient condition relationship

Definition 27 (The left-tailed p Value of a necessary and sufficient condition relationship).

The left-tailed (lt) p Value [37] of a necessary and sufficient condition relationship can be calculated as follows.

$$\begin{aligned} pValue_{\mathrm{lt}}(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}}) &\equiv 1 - e^{-\left(1-p\left(A_{\mathrm{t}} \leftrightarrow B_{\mathrm{t}}\right)\right)} \\ &\equiv 1 - e^{-((b+c)/N)} \end{aligned} \tag{59}$$

In this context, a low p-value indicates again a statistical significance. Table 7 may provide an overview of the theoretical distribution of a necessary and sufficient condition.

Table 7: Necessary and sufficient condition.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	1	0	1
	NO	0	1	1
		1	1	2

2.2.28 Either or conditions

Definition 28 (Either A_t or B_t conditions [*NEQV*]).

Mathematically, an either A_t or B_t condition relationship (NEQV), denoted by p(A_t >-< B_t) in terms of statistics and probability theory, is defined[see 1, p. 68-70] as

$$\begin{aligned} p(A_{\mathrm{t}} > - < B_{\mathrm{t}}) &\equiv \frac{\sum_{t=1}^{N} \left((A_{\mathrm{t}} \wedge \underline{B}_{\mathrm{t}}) \vee (\underline{A}_{\mathrm{t}} \wedge B_{\mathrm{t}}) \right)}{N} \\ &\equiv p(b_{\mathrm{t}}) + p(c_{\mathrm{t}}) \\ &\equiv \frac{N \times (p(b_{\mathrm{t}}) + p(c_{\mathrm{t}}))}{N} \\ &\equiv \frac{b+c}{N} \\ &\equiv +1 \end{aligned} \tag{60}$$

It is $p(A_{\mathrm{t}} > - < B_{\mathrm{t}}) \equiv 1 - p(A_{\mathrm{t}} < - > B_{\mathrm{t}})$ (see Table 8).

Table 8: Either A_t or B_t relationship.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	0	1	1
	NO	1	0	1
		1	1	2

2.2.29 The Chi-square goodness of fit test of an either or condition relationship

Definition 29 (The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship).

An either or condition relationship p(A_t >-< B_t) can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left((A_t > - < B_t) \mid A\right) &\equiv \frac{(b-(a+b))^2}{A} + \\ &\quad \frac{c-((c+d))^2}{\underline{A}} \\ &\equiv \frac{a^2}{A} + \frac{d^2}{\underline{A}}\end{aligned} \tag{61}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}\left((A_t > - < B_t) \mid B\right) &\equiv \frac{(c-(a+c))^2}{B} + \\ &\quad \frac{b-((b+d))^2}{\underline{B}} \\ &\equiv \frac{a^2}{B} + \frac{d^2}{\underline{B}}\end{aligned} \tag{62}$$

Yate's [36] continuity correction has not been used in this context.

2.2.30 The left-tailed p Value of an either or condition relationship

Definition 30 (The left-tailed p Value of an either or condition relationship).

The left-tailed (lt) p Value [37] of an either or condition relationship can be calculated as follows.

$$\begin{aligned}pValue_{\text{lt}}\left(A_t > - < B_t\right) &\equiv 1 - e^{-\left(1-p\left(A_t > - < B_t\right)\right)} \\ &\equiv 1 - e^{-((a+d)/N)}\end{aligned} \tag{63}$$

In this context, a low p-value indicates again a statistical significance.

2.2.31 Causal relationship k

Definition 31 (Causal relationship k).

Nonetheless, mathematically, the causal relationship [1–5] between a cause U_t (German: Ursache) and an effect W_t (German: Wirkung), denoted by $k(U_t, W_t)$, is defined *at each single Bernoulli trial t* in terms of statistics and probability theory as

$$k(U_t, W_t) \equiv \frac{\sigma(U_t, W_t)}{\sigma(U_t) \times \sigma(W_t)} \equiv \frac{p(U_t \wedge W_t) - p(U_t) \times p(W_t)}{\sqrt[2]{(p(U_t) \times (1 - p(U_t))) \times (p(W_t) \times (1 - p(W_t)))}} \tag{64}$$

where $\sigma(U_t, W_t)$ denotes the co-variance between a cause U_t and an effect W_t *at every single Bernoulli trial t*, $\sigma(U_t)$ denotes the standard deviation of a cause U_t at the same single Bernoulli trial t, $\sigma(W_t)$ denotes the standard deviation of an effect W_t at same single Bernoulli trial t. Table 9 illustrates the theoretically possible relationships between a cause and an effect.

Table 9: Sample space and the causal relationship k

		Effect B_t		
		TRUE	FALSE	
Cause	TRUE	$p(a_t)$	$p(b_t)$	$p(U_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{U}_t)$
		$p(W_t)$	$p(\underline{W}_t)$	+1

2.3 Axioms

2.2.1 Axiom I. Lex identitatis

In this context, we define[7] axiom I as

$$+1 = +1 \tag{65}$$

2.2.2 Axiom II. Lex contradictionis

In this context[7], axiom II or **lex contradictionis**, the negative of lex identitatis, or

$$+0 = +1 \tag{66}$$

is of no minor importance too.

2.2.3 Axiom III. Lex negationis

$$\neg(0) \times 0 = 1 \tag{67}$$

where $\neg$ denotes (logical [18] or natural) negation[7]. In this context, there is some evidence that $\neg(1) \times 1 = 0$. In other words, it is $(\neg(1) \times 1) \times (\neg(0) \times 0) = 1$.

3 Results

3.1 Anti Chebyshev - The exact probability of a single event

Theorem 1 (Anti Chebyshev - The exact probability of a single event)**.** *The Pafnuty Lvovich Chebyshev's (1821 – 1894) inequality (also called the Irénée-Jules Bienaymé [40] (1796 – 1878) – Chebyshev inequality) enables us to obtain bounds on the probability of an event when both the mean and variance of a random variable are known. According to Kolmogoroff[see 31, p. 42], Chebyshev's inequality can be changed to*

$$p\left(\mid {}_{R}X_{t} - E\left({}_{R}X_{t}\right) \mid \geq \sqrt[2]{E\left({}_{R}X_{t}{}^{2}\right)}\right) \leq \frac{\sigma({}_{R}X_{t})^{2}}{E\left({}_{R}X_{t}{}^{2}\right)} \tag{68}$$

However, it is necessary to emphasize that Chebyshev's inequality as proved by Chebyshev [41] himself in 1867 and later by his student Andrey Markov (1856–1922) in his 1884 Ph.D. thesis provides in this form only an approximate value of ***the exact probability of a single event****. Thus far and in contrast to Chebyshev's inequality [see 31, p. 42], the exact value of the probability of a single event (****Chebyshev's equality****) is given by the relationship*

$$p\left({}_{R}X_{t}\right) \equiv 1 - \frac{\sigma({}_{R}X_{t})^{2}}{E\left({}_{R}X_{t}{}^{2}\right)} \tag{69}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{70}$$

is true, **then** the conclusion

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv 1 - \frac{\sigma({}_{\mathrm{R}}X_{\mathrm{t}})^{2}}{E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right)} \tag{71}$$

is also true, the absence of any technical errors presupposed. The premise

$$+\,1 \equiv +1 \tag{72}$$

is true. Multiplying this premise (i. e. axiom) by the variance $\sigma({}_{\mathrm{R}}\mathrm{X}_{\mathrm{t}})^2$

$$\sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2} \equiv \sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2} \tag{73}$$

Equation 73 can be rearranged (see definition 7, equation 16) as

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) - \left(E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)^{2} \equiv \sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2} \tag{74}$$

or as

$$E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right) \equiv \left(E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)^{2} + \sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2} \tag{75}$$

The normalised form of the variance follows as

$$\frac{\left(E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)^{2}}{E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right)} + \frac{\sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2}}{E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right)} \equiv +1 \tag{76}$$

Rearranging equation 76, it is

$$\frac{\left(E\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)\right)^{2}}{E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right)} \equiv 1 - \frac{\sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2}}{E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right)} \tag{77}$$

Equation 77 simplifies (see definition 3, equation 3) as

$$p\left({}_{\mathrm{R}}X_{\mathrm{t}}\right) \equiv 1-\frac{\sigma\left({}_{\mathrm{R}}X_{\mathrm{t}}\right)^{2}}{E\left({}_{\mathrm{R}}X_{\mathrm{t}}{}^{2}\right)} \tag{78}$$

Quod erat demonstrandum.

Remark 4. ***Example.*** *Let* ${}_{R}X_{t} = (A_{t} \vee \underline{B}_{t})$. *The exact probability of this relationship follows according to theorem 1, equation 78 as* $1-\frac{\sigma\left(A_{t} \vee \underline{B}_{t}\right)^{2}}{E\left(\left(A_{t} \vee \underline{B}_{t}\right)^{2}\right)}$ *The exact probability[7] of other events, including the numerous relationships developed in this publication can be calculated by the same method too.*

3.2 The causal relationship k

Theorem 2 (Causal relationship k)**.** *Thus far, let* $p({}_{R}U_{t})$ *represent the probability from the point of view of a stationary observer R of a certain cause* ${}_{R}U_{t}$ *(in German: U like Ursache), i. e. a random variable or a quantum mechanical observable or a cluster inside a set, at a certain Bernoulli trial t. Let* $E({}_{R}U_{t}{}^{2})$ *denote the expectation value of the cause* ${}_{R}U_{t}{}^{2}$. *Let* $E({}_{R}U_{t})$ *denote the expectation value of the cause* ${}_{R}U_{t}$. *Let* $\sigma({}_{R}U_{t})$ *denote the standard deviation of the cause* ${}_{R}U_{t}$. *Let* $\sigma({}_{R}U_{t})^{2}$ *denote the variance of the cause* ${}_{R}C_{t}$. *Let* $p({}_{R}W_{t})$ *represent the probability from the point of view of a stationary observer R of its own effect* ${}_{R}W_{t}$ *(in German: W like Wirkung), i. e. a random variable or a quantum mechanical observable or a cluster inside a set, at a certain Bernoulli trial t. Let* $E({}_{R}W_{t}{}^{2})$ *denote the expectation value of the effect* ${}_{R}W_{t}{}^{2}$. *Let* $E({}_{R}W_{t})$ *denote the expectation value of the effect* ${}_{R}W_{t}$. *Let* $\sigma({}_{R}W_{t})$ *denote the standard deviation of the effect* ${}_{R}W_{t}$. *Let* $\sigma({}_{R}W_{t})^{2}$ *denote the variance of the effect* ${}_{R}W_{t}$. *Let* $\sigma({}_{R}U_{t}, {}_{R}W_{t})$ *denote the co-variance of cause* ${}_{R}U_{t}$ *and effect* ${}_{R}W_{t}$. *The causal relationship, denoted as* $k\left({}_{R}U_{t}, {}_{R}W_{t}\right)$, *inside a sets can be calculated as*

$$\begin{aligned}
k\left({}_{R}U_{t}, {}_{R}W_{t}\right) &\equiv \frac{\sigma\left({}_{R}U_{t}, {}_{R}W_{t}\right)}{\sqrt[2]{\sigma\left({}_{R}U_{t}\right)^{2} \times \sigma\left({}_{R}W_{t}\right)^{2}}} \\
&\equiv \frac{\sigma\left({}_{R}U_{t}, {}_{R}W_{t}\right)}{\sigma\left({}_{R}U_{t}\right) \times \sigma\left({}_{R}W_{t}\right)} \\
&\equiv \frac{\left({}_{R}U_{t} \times {}_{R}W_{t}\right) \times \left(p\left({}_{R}U_{t}, {}_{R}W_{t}\right)-\left(p\left({}_{R}U_{t}\right) \times p\left({}_{R}W_{t}\right)\right)\right)}{\sqrt[2]{\left(\left({}_{R}U_{t}{}^{2}\right) \times \left(p\left({}_{R}U_{t}\right) \times \left(1-p\left({}_{R}U_{t}\right)\right)\right) \times \left({}_{R}W_{t}{}^{2}\right) \times \left(p\left({}_{R}W_{t}\right) \times \left(1-p\left({}_{R}W_{t}\right)\right)\right)\right)}} \\
&\equiv \frac{\left({}_{R}U_{t} \times {}_{R}W_{t}\right) \times \left(p\left({}_{R}U_{t}, {}_{R}W_{t}\right)-\left(p\left({}_{R}U_{t}\right) \times p\left({}_{R}W_{t}\right)\right)\right)}{\left({}_{R}U_{t} \times {}_{R}W_{t}\right) \times \sqrt[2]{\left(\left(p\left({}_{R}U_{t}\right) \times \left(1-p\left({}_{R}U_{t}\right)\right)\right) \times \left(p\left({}_{R}W_{t}\right) \times \left(1-p\left({}_{R}W_{t}\right)\right)\right)\right)}} \\
&\equiv \frac{\left(p\left({}_{R}U_{t} \wedge {}_{R}W_{t}\right)-\left(p\left({}_{R}U_{t}\right) \times p\left({}_{R}W_{t}\right)\right)\right)}{\sqrt[2]{\left(\left(p\left({}_{R}U_{t}\right) \times \left(1-p\left({}_{R}U_{t}\right)\right)\right) \times \left(p\left({}_{R}W_{t}\right) \times \left(1-p\left({}_{R}W_{t}\right)\right)\right)\right)}}
\end{aligned} \tag{79}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1=+1}_{(Premise)} \tag{80}$$

is true, **then** the conclusion

$$k\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right) \equiv \frac{\sigma\left({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}\right)}{\sigma({}_{\mathrm{R}}U_{\mathrm{t}}) \times \sigma({}_{\mathrm{R}}W_{\mathrm{t}})} \tag{81}$$

is also true, the absence of any technical errors presupposed. The premise or respectively axiom I

$$+1 \equiv +1 \tag{82}$$

is true. Multiplying this premise (i. e. axiom I) by the a single cause $({}_{\mathrm{R}}U_{\mathrm{t}})$ at a certain Bernoulli trial t, it is

$$({}_{\mathrm{R}}U_{\mathrm{t}}) \equiv ({}_{\mathrm{R}}U_{\mathrm{t}}) \tag{83}$$

Multiplying the cause (equation 83) by its own effect $({}_{\mathrm{R}}W_{\mathrm{t}})$ it is

$$({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}}) \equiv ({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}}) \tag{84}$$

This basic relationship between a cause and an effect as described by equation 84 may be of use especially under conditions of classical logic (either +0 or +1 values) and is illustrated in detail by table 10.

Table 10: Causal relationship under conditions of classical logic

Bernoulli trial t	${}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$	${}_{\mathrm{R}}\mathrm{W}_{\mathrm{t}}$	$({}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}} \land {}_{\mathrm{R}}\mathrm{W}_{\mathrm{t}}) \equiv ({}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}} \times {}_{\mathrm{R}}\mathrm{W}_{\mathrm{t}})$
1	+1	+1	+1
2	+1	+0	+0
3	+0	+1	+0
4	+0	+0	+0
...	...	...	...

It should be noted at this early stage that we will not further discuss at this point the relationship $1^1 \times 1^1 \equiv 1^2$ or the relationship $0^1 \times 0^1 \equiv 0^2$ [9]. As a side effect, table 10 provide an evidence of the identity of the logical operation, denoted by $\land$, and the algebraic operation multiplication. However, as soon as ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ or ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{t}}$ take on other values than either +0 or +1, the previous relationship (equation 84) is not appropriate enough to describe the causal relationship completely and may lead to a **cum hoc ergo propter hoc** logical fallacy [42]. In general, it is necessary to deal with such circumstances too. According to equation 17 it is

$${}_{\mathrm{R}}U_{\mathrm{t}} \equiv \frac{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})}{\sqrt[2]{p\,({}_{\mathrm{R}}U_{\mathrm{t}}) \times (1 - p\,({}_{\mathrm{R}}U_{\mathrm{t}}))}} \tag{85}$$

with the consequence that equation 84 changes to

$$({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}}) \equiv \left(\frac{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})}{\sqrt[2]{p\,({}_{\mathrm{R}}U_{\mathrm{t}}) \times (1 - p\,({}_{\mathrm{R}}U_{\mathrm{t}}))}}\right) \times {}_{\mathrm{R}}W_{\mathrm{t}} \tag{86}$$

According to equation 18 it is

$${}_{\mathrm{R}}W_{\mathrm{t}} \equiv \frac{\sigma({}_{\mathrm{R}}W_{\mathrm{t}})}{\sqrt[2]{p\,({}_{\mathrm{R}}W_{\mathrm{t}}) \times (1 - p\,({}_{\mathrm{R}}W_{\mathrm{t}}))}} \tag{87}$$

Therefore, equation 86 changes to

$$({}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}}) \equiv \left(\frac{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})}{\sqrt[2]{p\,({}_{\mathrm{R}}U_{\mathrm{t}}) \times (1 - p\,({}_{\mathrm{R}}U_{\mathrm{t}}))}}\right) \times \left(\frac{\sigma({}_{\mathrm{R}}W_{\mathrm{t}})}{\sqrt[2]{p\,({}_{\mathrm{R}}W_{\mathrm{t}}) \times (1 - p\,({}_{\mathrm{R}}W_{\mathrm{t}}))}}\right) \tag{88}$$

According to definition 8, equation 21, it is

$${}_{\mathrm{R}}U_{\mathrm{t}} \times {}_{\mathrm{R}}W_{\mathrm{t}} \equiv \frac{\sigma\,({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}})}{(p\,({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}) - (p\,({}_{\mathrm{R}}U_{\mathrm{t}}) \times p\,({}_{\mathrm{R}}W_{\mathrm{t}})))} \tag{89}$$

Simplifying equation 88, it is

$$\left(\frac{\sigma\,({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}})}{(p\,({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}) - (p\,({}_{\mathrm{R}}U_{\mathrm{t}}) \times p\,({}_{\mathrm{R}}W_{\mathrm{t}})))}\right) \equiv \left(\frac{\sigma({}_{\mathrm{R}}U_{\mathrm{t}})}{\sqrt[2]{p\,({}_{\mathrm{R}}U_{\mathrm{t}}) \times (1 - p\,({}_{\mathrm{R}}U_{\mathrm{t}}))}}\right) \times \left(\frac{\sigma({}_{\mathrm{R}}W_{\mathrm{t}})}{\sqrt[2]{p\,({}_{\mathrm{R}}W_{\mathrm{t}}) \times (1 - p\,({}_{\mathrm{R}}W_{\mathrm{t}}))}}\right) \tag{90}$$

Further rearrangement of equation 90 yields the causal relationship between the cause ${}_{\mathrm{R}}\mathrm{U}_{\mathrm{t}}$ and the effect ${}_{\mathrm{R}}\mathrm{W}_{\mathrm{t}}$, denoted as $k\,({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}})$, as

$$\begin{aligned} k\,({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}) &\equiv \frac{\sigma\,({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}})}{\sigma({}_{\mathrm{R}}U_{\mathrm{t}}) \times \sigma({}_{\mathrm{R}}W_{\mathrm{t}})} \\ &\equiv \frac{(p\,({}_{\mathrm{R}}U_{\mathrm{t}}, {}_{\mathrm{R}}W_{\mathrm{t}}) - (p\,({}_{\mathrm{R}}U_{\mathrm{t}}) \times p\,({}_{\mathrm{R}}W_{\mathrm{t}})))}{\sqrt[2]{p\,({}_{\mathrm{R}}U_{\mathrm{t}}) \times (1 - p\,({}_{\mathrm{R}}U_{\mathrm{t}})) \times p\,({}_{\mathrm{R}}W_{\mathrm{t}}) \times (1 - p\,({}_{\mathrm{R}}W_{\mathrm{t}}))}} \end{aligned} \tag{91}$$

Quod erat demonstrandum.

3.3 The law of nature relationship g

Theorem 3 (The law of nature relationship g)**.** *In this theorem, we are specifying the probability measure on the sample space of an experiment as being equal to p = 1. In other words, it is for sure that an event occurred. Thus far, let the sample space Y_t denote the set of all possible outcomes of an experiment at a certain Bernoulli trial t. Let y_t denote a random variable, a real-valued function defined on a single element of the sample space Y_t at a Bernoulli trial t. In general, it is $Y_t \equiv \{y_{1\,t}, y_{2\,t}, \ldots, y_{n\,t},\}$. Let $E(y_t)$ denote the expectation value of y_t. Let the sample space X_t denote the set of all possible outcomes of X at a certain Bernoulli trial t. Let x_t denote a random variable, a real-valued function defined on a single element of the sample space X_t at a Bernoulli trial t. In general, it is $X_t \equiv \{x_{1\,t}, x_{2\,t}, \ldots, x_{n\,t},\}$. Let $E(y_t)$ denote the expectation value of y_t. Let $f(x_t)$ denote a mathematical function which describes the behaviour of each element of a set X_t, let $E(f(\ x_t))$ denote the expectation value of $f(x_t)$. The law of nature relationship is based on a quantity dominated, mechanical understanding of the relationship between two factors like y_t and $f(\ x_t)$. Let $g(y_t, f(x\ x_t))$ denote the law of nature relationship, 'der gesetzmäßige Zusammenhang'. The law of nature relationship is defined as*

$$g\,(y_t, f\,(x_t)) \equiv \frac{\sigma\,(y_t, f\,(x_t))}{\sigma\,(y_t) \times \sigma\,(f\,(x_t))} \tag{92}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{93}$$

is true, **then** the conclusion

$$g(y_t, f(x_t)) \equiv \frac{E((y_t - E(y_t)) \times (f(x_t) - E(f(x_t))))}{E(y_t - E(y_t)) \times E(f(x_t) - E(f(x_t)))} \equiv \frac{\sigma(y_t, f(x_t))}{\sigma(y_t) \times \sigma(f(x_t))} \tag{94}$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{95}$$

is true. Multiplying this premise (i. e. axiom) by y_t, it is

$$y_t \equiv y_t \tag{96}$$

The law of nature relationship $\mathbf{g(y_t, f(x_t))}$ is based on the demand that an outcome, denoted as $\mathbf{y_t}$ is determined exactly by $f(x_t)$ at every run of an experiment, at every Bernoulli trial t. In other words, it is $\mathbf{y_t = f(x_t)}$. Based on this fundamental assumption, equation 94 can be rearranged as

$$y_t \equiv f(x_t) \tag{97}$$

Equation 97 leads to

$$E(y_t) \equiv E(f(x_t)) \tag{98}$$

Equation 97 demands too that

$$y_t^2 \equiv f(x_t)^2 \tag{99}$$

Equation 99 demands that

$$E\left(y_t^2\right) \equiv E\left(f(x_t)^2\right) \tag{100}$$

Equation 97 can be rearranged as

$$y_t - E(y_t) \equiv f(x_t) - E(y_t) \tag{101}$$

According to equation 98, equation 101 changes to

$$y_t - E(y_t) \equiv f(x_t) - E(f(x_t)) \tag{102}$$

In other words, we must accept the equality of

$$E(y_t - E(y_t)) \equiv E(f(x_t) - E(f(x_t))) \tag{103}$$

By squaring equation 103, it is

$$E(y_t - E(y_t))^2 \equiv E(f(x_t) - E(f(x_t)))^2 \tag{104}$$

or

$$E(y_t - E(y_t))^2 \equiv E(f(x_t) - E(f(x_t))) \times E(f(x_t) - E(f(x_t))) \tag{105}$$

or

$$E(y_t - E(y_t)) \times E(y_t - E(y_t)) \equiv E(f(x_t) - E(f(x_t))) \times E(f(x_t) - E(f(x_t))) \quad (106)$$

Based on equation 103, equation 106 can be rearranged as

$$E(y_t - E(y_t)) \times E(f(x_t) - E(f(x_t))) \equiv E((f(x_t) - E(f(x_t))) \times (f(x_t) - E(f(x_t)))) \quad (107)$$

Based on equation 97 and equation 98, equation 107 can be rearranged as

$$E(y_t - E(y_t)) \times E(f(x_t) - E(f(x_t))) \equiv E((y_t - E(y_t)) \times (f(x_t) - E(f(x_t)))) \quad (108)$$

Rearranging equation 108, the law of nature relationship g(y_t, f(x x_t)) follows as

$$\begin{aligned} g(y_t, f(x_t)) &\equiv \frac{E((y_t - E(y_t)) \times (f(x_t) - E(f(x_t))))}{E(y_t - E(y_t)) \times E(f(x_t) - E(f(x_t)))} \\ &\equiv \frac{\sigma(y_t, f(x_t))}{\sigma(y_t) \times \sigma(f(x_t))} \end{aligned} \quad (109)$$

Quod erat demonstrandum.

Remark 5. *Moving away from confusing and logically inconsistent theories of causation seems inevitable due to proofs provided in this publication. Especially structural equation modelling (SEM) or counterfactual claims et cetera failed to provide a coherent mathematical foundation for the analysis of cause and effect relationships. In the 1920's by Sewall Wright [see 43, p. 557] was one of the first to derive a kind of a structural equation modelling from the coefficient of correlation. Wright points out: "The present paper is an attempt to present a method of measuring the direct influence along each separate path in such a system and thus of finding the degree to which variation of a given effect is determined by each particular cause. The method depends on the combination of knowledge of the degrees of correlation among the variables in a system with such knowledge as may be possessed of the causal relations "[see 43, p. 557]. Wright himself is writing that "**The method depends on the** ... **correlation among the variables** ... "[see 43, p. 557]. The law of nature relationship $g(y_t, f(x_t))$, in contrast to Pearl's do(X=x) operator [see 44, p. 204], provide a logically consistent mathematical alternative to the structure equation modelling proposed analysis of potential causal dependencies between exogenous and endogenous variables. Even if multiplied by a sample size N, the law of nature relationship need not to change. We obtain*

$$g(y_t, f(x_t)) \equiv \frac{N \times N \times \sigma(y_t, f(x_t))}{N \times \sigma(y_t) \times N \times \sigma(f(x_t))} \quad (110)$$

4 Discussion

In general, combining classical logic and probability theory into one single mathematical framework might appear somewhat difficult and strange at first sight. It is particularly noteworthy that classical logic as such is concerned more or less with absolutely certain truths and inferences. In contrast to classical logic, probability theory deals primarily with uncertainties. In particular, these and similar difficulties should not prevent us form the possibility to recognise the relationships between events of objective reality as the same are while relying on logically consistent methods. In this context, especially causation is a live topic across a number of scientific disciplines. Thus far, what is the causal relationship k? **The causal relationship k** (see theorem 2)[1–5] enable us to evaluate cause-effect relationships including **causal chains**[see 1, p. 139-160] hypotheses in the light of empirical facts.

However, the causal relationships k should not be confused neither with Bravais [45] (1811-1863) - **Pearson's coefficient of correlation** [46, 47] nor with **Pearson's phi coefficient** [32]. Karl Pearson (1857-1936) himself "rejected causal thinking"[see 48, p. 39] as such. Following Pearson, causation is without any scientific significance, Pearson is demanding unconditionally that "...correlation ...have to replace ...causation "[see 49, p. 157]. Pearson is an advocate of anti-causality in the extreme and much more than that. "Pearson categorically denies the need for an independent concept of causal relation beyond correlation ... he *exterminated* causation from statistics before it had a chance to take root "[see 44, p. 340]. However, it is appropriate to consider that every single event ${}_{R}W_{t}$ has its own cause ${}_{R}U_{t}$. Therefore and completely in contrast to Pearson's demonstrably anti-causal statistical methods[32, 47], the causal relationship k is **defined, derived and valid at every single run of an experiment, at every single Bernoulli trial t**.

5 Conclusion

Experimental and non-experimental data can be analysed for different conditions, causal relationships and for laws of nature relationships.

References

[1] I. Barukčić, *Die Kausalität*, 1st edn. (Wiss.-Verl., Hamburg, 1989), ISBN 3-9802216-0-1

[2] I. Barukčić, *Die Kausalität*, 2nd edn. (Scientia, Wilhelmshaven, 1997), ISBN 3-9802216-4-4

[3] I. Barukčić, *Causality: New statistical methods* (Books on Demand GmbH, Norderstedt, Germany, 2005), ISBN 978-3-8334-3645-1

[4] I. Barukčić, *The Mathematical Formula of the Causal Relationship k*, International Journal of Applied Physics and Mathematics **6**, 45 (2016)

[5] I. Barukčić, *Theoriae causalitatis principia mathematica* (Books on Demand, Norderstedt, 2017), ISBN 978-3-7448-1593-2

[6] I. Barukčić, *Die Kausalität*, reprint of first edition 1989. edn. (Books on Demand, Norderstedt, 2017), ISBN 978-3-7448-1595-6

[7] I. Barukčić, *Causal relationship k*, International Journal of Mathematics Trends and Technology IJMTT **66**, 76 (2020)

[8] I. Barukčić, O. Ufuoma, *Analysis of Switching Resistive Circuits. A Method Based on the Unification of Boolean and Ordinary Algebras*, first edition edn. (Books on Demand, Norderstedt, 2020), ISBN 978-3-7519-8474-4

[9] I. Barukčić, *Zero and infinity. Mathematics without frontiers*, 1st edn. (Books on Demand GmbH, Hamburg-Norderstedt, 2020), ISBN 978-3-7519-1873-2

[10] I. Barukčić, *Locality and Non locality*, European Journal of Applied Physics **2** (2020)

[11] I. Barukčić, *N-th index D-dimensional Einstein gravitational field equations. Geometry unchained.*, Vol. 1, 1st edn. (Books on Demand GmbH, Hamburg-Norderstedt, 2020), ISBN 978-3-7526-4490-6

[12] W. Heisenberg, *Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik*, Zeitschrift für Physik **43**, 172 (1927)

[13] I. Barukčić, *The Equivalence of Time and Gravitational Field*, Physics Procedia **22**, 56 (2011)

[14] I. Barukčić, *Anti Heisenberg-Refutation Of Heisenberg's Uncertainty Relation*, in *American Institute of Physics - Conference Proceedings* (Växjö, (Sweden), 2011), Vol. 1327, pp. 322–325, `https://aip.scitation.org/doi/abs/10.1063/1.3567453`

[15] I. Barukčić, *Anti Heisenberg – Refutation of Heisenberg's Uncertainty Principle*, International Journal of Applied Physics and Mathematics **4**, 244 (2014)

[16] I. Barukčić, *Anti Heisenberg—The End of Heisenberg's Uncertainty Principle*, Journal of Applied Mathematics and Physics **04**, 881 (2016)

[17] G. Boole, *The Mathematical Analysis of Logic – Being an Essay Towards a Calculus of Deductive Reasoning* (Macmillan, Cambrdge, 1847), `https://archive.org/details/mathematicalanal00booluoft,https://archive.org/details/mathematicalanal00bool`

[18] G. Boole, *An investigation of the laws of thought, on which are founded mathematical theories of logic and probabilities* (New York, Dover, 1854), `http://archive.org/details/bub_gb_DqwAAAAAcAAJ`

[19] G. Kröber, *Der Konditionalismus und seine Kritik in der sowjetischen Wissenschaft*, Wissenschaftliche Zeitschrift der Karl-Marx Universität Leipzig **10**, 137 (1961)

[20] D.P.v. Hansemann, *Über das konditionale Denken in der Medizin und seine Bedeutung fur die Praxis* (Hirschwald, Berlin., 1912)

[21] M. Verworn, *Kausale und konditionale Weltanschauung* (Verlag von Gustav Fischer, Jena, 1912)

[22] G.W.F.v. Leibniz, *Explication de l'arithmétique binaire, qui se sert des seuls caractères O et I avec des remarques sur son utilité et sur ce qu'elle donne le sens des anciennes figures chinoises de Fohy*, Mémoires de mathématique et de physique de l'Académie royale des sciences (1703)

[23] I. Barukčić, *The Interior Logic of Inequalities*, International Journal of Mathematics Trends and Technology IJMTT **65**, 146 (2019)

[24] I. Barukčić, *Classical Logic And The Division By Zero*, International Journal of Mathematics Trends and Technology IJMTT **65**, 31 (2019)

[25] M. Born, *Zur Quantenmechanik der Stoßvorgänge*, Zeitschrift für Physik **37**, 863 (1926)

[26] J.v. Uspensky, *Introduction To Mathematical Probability* (McGraw-Hill Company, New York (USA), 1937)

[27] C. Huygens, F. van Schooten, *De ratiociniis in ludo alae: In: Exercitationum mathematicarum liber primus [- quintus]* (ex officina Johannis Elsevirii, Lugdunum Batavorum (Leiden, The Netherlands), 1657), `https://www.e-rara.ch/zut/doi/10.3931/e-rara-8813`

[28] W.A. Whitworth, *Choice and Chance. With 1000 Exercises*, fifth eidtion edn. (Deighton, Bell & Co., Cambridge, 1901), `https://archive.org/details/choicechancewith00whituoft`

[29] B. Efron, *R. A. Fisher in the 21st century (Invited paper presented at the 1996 R. A. Fisher Lecture)*, Statistical Science **13**, 95 (1998), publisher: Institute of Mathematical Statistics

[30] R.A. Fisher, *XV.—The Correlation between Relatives on the Supposition of Mendelian Inheritance.*, Transactions of The Royal Society of Edinburgh **52**, 399 (1918), publisher: Royal Society of Edinburgh Scotland Foundation

[31] A.N. Kolmogorov, *Foundations of the theory of probability*, second english edition. translated by nathan morrison edn. (Chelsea Publishing Company, New York, 1956), ISBN 978-0-486-82159-7

[32] K. Pearson, *Mathematical contributions to the theory of evolution. XIII. On the theory of contingency and its relation to association and normal correlation*, Biometric Series I (Dulau and Co., London, 1904)

[33] J. Bernoulli, *Ars conjectandi, Opus posthumus: Accedit Tractatus de seriebus infinitis ; et epistola Gallice scripta De Ludo Pilae Reticularis* (Impensis Thurnisiorum [Tournes], fratrum, Basileae (Basel, Suisse), 1713)

[34] A.d. Moivre, *The Doctrine of Chances or a Method of Calculating the Probability of Events in Play* (printed by W. Pearson for the author, London, 1718)

[35] A. Kolmogoroff, *Grundbegriffe der Wahrscheinlichkeitsrechnung* (Springer Berlin Heidelberg, Berlin, Heidelberg, 1933), ISBN 978-3-642-49596-0

[36] F. Yates, *Contingency Tables Involving Small Numbers and the Chi square Test*, Supplement to the Journal of the Royal Statistical Society **1**, 217 (1934)

[37] I. Barukčić, *The P Value of likely extreme events*, International Journal of Current Science Research **5**, 1841 (2019)

[38] F.R. Helmert, *Ueber die Wahrscheinlichkeit der Potenzsummen der Beobachtungsfehler und über einige damit im Zusammenhange stehende Fragen*, Zeitschrift für Mathematik und Physik **21**, 102 (1876)

[39] K. Pearson, *X. On the criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can be reasonably supposed to have arisen from random sampling*, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science **50**, 157 (1900)

[40] I.J. Bienaymé, *Considérations a l'appui de la découverte de Laplace sur la loi de probabilité dans la méthode des moindres carrés*, Comptes rendus des séances de l'Académie des Sciences des Paris **37**, 309 (1853)

[41] P.L. Tschébychef, *Des valeurs moyennes*, Journal de Mathématiques Pures et Appliquées **12**, 177 (1867)

[42] G.S. Kienle, *Cum hoc, ergo propter hoc* (2020)

[43] S. Wright, *Correlation and causation*, Journal of Agricultural Research **20** (1921)

[44] J. Pearl, *Causality: models, reasoning, and inference* (Cambridge University Press, Cambridge, U.K. ; New York, 2000), ISBN 978-0-521-89560-6

[45] A. Bravais, *Analyse mathématique sur les probabilités d es erreurs de situation d'un point*, Mémoires Présentées Par Divers Savants À L'Académie Royale Des Sciences De L'Institut De France **9**, 255 (1846)

[46] F. Galton, *Typical Laws of Heredity*, Nature **15**, 492 (1877)

[47] K. Pearson, *VII. Mathematical contributions to the theory of evolution.—III. Regression, heredity, and panmixia*, Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character **187**, 253 (1896)

[48] H.M. Blalock, *Causal inferences in nonexperimental research*, 6th edn. (Univ. of North Carolina Press, Chapel Hill, NC, 1972), ISBN 978-0-8078-0917-4

[49] K. Pearson, *The Grammar of Science* (Adam and Charles Black, London (GB), 1911), ISBN 978-1-108-07711-8