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Event-triggered H_∞ state estimation for time-varying neural networks with variance-constraint and fading measurements

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Abstract. This paper addresses the event-triggered H_∞ state estimation problem for a class of discrete recurrent neural networks subject to variance-constraint and fading measurements. The phenomena of fading measurements are described by introducing a set of mutually independent random variables, which reflect that each sensor has individual missing probability. In addition, for the purpose of energy saving, an event-triggered H_∞ state estimation scheme is used for time-varying neural networks to determine whether the measurement output is transmitted to the estimator or not. Some sufficient conditions are obtained to guarantee that the estimation error system satisfies both estimation error variance constraint and prescribed H_∞ performance requirement. Finally, the feasibility of the proposed event-triggered H_∞ state estimation method is verified by a numerical example.

1 Introduction

In the past decades, the analysis and design of recursive neural networks (RNNs) have attracted great attention due to their powerful advantages, such as showing dynamic time behaviour and using internal memory to process arbitrary input sequences. Accordingly, the RNNs have been successfully applied to broad areas such as speech recognition, pattern recognition and associate memory. Hence, more and more researchers have paid attention to the state estimation problem for time varying system. In [1], an event-based recursive input and state estimator has been designed to ensure that the covariance of the estimation error has an upper bound at any time for the linear discrete time-varying systems.

Since the perfect measurements cannot be always available, especially in the unreliable network circumstances, the state estimation problems with fading measurements have aroused the extensive research. To be specific, the stochastic stability of a modified unscented Kalman filter problem has been analyzed in [2] for a class of nonlinear systems with stochastic nonlinearities and multiple fading measurements. In addition, a novel envelope-constrained performance criterion has been proposed in [3] to better quantify the transient dynamics of the filtering error process over the finite horizon. Moreover, it is worth noting that few scholars have studied the state estimation of time-varying neural networks with fading measurements.

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Different from the optimal estimation of minimum error covariance, the variance-constrained estimation method can provide a more loose technique, where the upper bound constraints are introduced to reflect the estimation accuracy. For example, in [4], the finite-horizon state estimator has been designed to provide the estimation scheme for time-varying complex networks, where both the H_∞ performance requirements and prescribed variance constraints on the estimation error can be guaranteed simultaneously. On the other hand, compared with the time-triggered control scheme, the advantage of the event triggered scheme is that it can effectively realize the sharing of communication resources. For example, a new event-triggered distributed state estimation strategy has been presented in [5], which can ensure the existence of the desired estimator gains and the exponentially stability in the mean square of the estimation error dynamics simultaneously.

Motivated by the aforementioned discussion, we handle the event-triggered H_∞ state estimation problem for discrete time-varying RNNs subject to variance-constraint and fading measurements. The main work conducted can be listed: 1) the event-triggered H_∞ state estimation problem is, for the first time, investigated for a class of discrete time-varying stochastic RNNs subject to variance-constraint and fading measurements; 2) a new solvable method is given for addressed variance-constrained state estimation problem based on the recursive linear matrix inequalities (RLMIs); and 3) the usual literature considers the augmented system satisfying both the prescribed H_∞ performance requirement and the estimation error variance constraints, but we analyse the estimation error system directly with same order of original system, which may reduce the computational complexity.

2 Problem formulation and preliminaries

In this paper, the addressed discrete time-varying neural networks are described by

$$\begin{cases} x_{k+1}=(A_k+\delta_k A_{1k})x_k+B_k f(x_k)+C_k v_{1k} \\ y_k=\Lambda_k D_k x_k+E_k v_{2k} \\ z_k=M_k x_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ and $y_k \in \mathbb{R}^m$ depict the state vector and the measurement output, $z_k \in \mathbb{R}^r$ is the controlled output, $A_k = \mathrm{diag}\{a_{1,k}, a_{2,k}, \ldots, a_{n,k}\}$ is the state coefficient matrix, A_{1k}, C_k, D_k, E_k and M_k are known real matrices with appropriate dimensions, $v_{1k} \in \mathbb{R}^{n_w}$ and $v_{2k} \in \mathbb{R}^{m_v}$ are Gaussian white noises with zero mean values and covariances $V_1 > 0$ and $V_2 > 0$, respectively. δ_k is zero mean Gaussian white noise with unity covariance. $B_k = \left[b_{ij,k}\right]_{n\times n}$ is the connection weight matrix. $f(x_k)$ is the neuron activation function. The matrix $\Lambda_k = \mathrm{diag}\{\lambda_k^1, \lambda_k^2, \ldots, \lambda_k^m\}$ describes the fading measurements phenomenon with λ_k^i $(i = 1,2,\ldots,m)$ being mutually independent random variables. Here, λ_k^i has the probability density function $P_k^i(s)$ on the interval $[\alpha_k, \beta_k]$ $(0 \le \alpha_k \le \beta_k \le 1)$ with mathematical expectation μ_k^i and variance $(\psi_k^i)^2$ $(i = 1,2,\ldots,m)$, where μ_k^i and ψ_k^i are known scalars. Subsequently, we can define $\Lambda_k = \sum_{i=1}^{m} \mu_k^i G_k^i$ and $\overline{\Lambda}_k = \sum_{i=1}^{m} \mu_k^i G_k^i$, where $G_k^i = \mathrm{diag}\{0,\ldots,0,1,0,\ldots,0\}$ with 1 being on the i-th term.

To save energy, we define the event generator function $f(\xi_k, \delta) = \xi_k^T \xi_k - \delta y_k^T y_k > 0$, where $\xi_k = y_k - y_{r_l}$ and δ is the triggering threshold. The execution is triggered as long as the condition $f(\xi_k,\delta)>0$ is satisfied. Therefore, the sequence of event-triggered instants $0\le k_0\le k_1\le\cdots\le k_k\le\cdots$ is determined iteratively by $r_{l+1}=\inf\{k\in\mathbb{N}|\ k>r_l, f(\xi(k),\delta)>0\}$.

In order to estimate the states of neurons, the following state estimator is constructed:

$$\begin{cases}\hat{x}_{k+1}=A_k\hat{x}_k+B_kf(\hat{x}_k)+K_k(y_{r_l}-\bar{\Lambda}_kD_k\hat{x}_k)\\ \hat{z}_k=M_k\hat{x}_k\end{cases} \tag{2}$$

where $\hat{x}_k$ is the estimation of neural state x_k, K_k is the estimator gain matrix to be determined. Let the estimation error be $e_k=x_k-\hat{x}_k$ and the controlled output estimation error be $\tilde{z}_k=z_k-\hat{z}_k$. The estimation error dynamics can be obtained as follows:

$$\begin{cases}e_{k+1}=(A_k-K_k\bar{\Lambda}_kD_k)e_k+\delta_kA_{1k}(\hat{x}_k+e_k)+B_kf(e_k)-K_k[\tilde{\Lambda}_kD_k(\hat{x}_k+e_k)+E_kv_{2k}-\xi_k]+C_kv_{1k}\\ \tilde{z}_k=M_ke_k\end{cases} \tag{3}$$

where $f(e_k)=f(x_k)-f(\hat{x}_k)$ and $\tilde{\Lambda}_k=\Lambda_k-\bar{\Lambda}_k$. Define the covariance matrix of the estimation error $X_k=\mathbb{E}\{e_ke_k^T\}$.

The main purpose of this paper is to design a time-varying state estimator (2), the following two requirements are satisfied simultaneously:

(i) Let the scalar $\gamma>0$, the positive define matrices W_φ and W_ϕ be given. For the initial state e_0, the estimation error $\tilde{z}_k$ satisfies:

$$J_1:=\mathbb{E}\left\{\sum_{k=0}^{N-1}(\|\tilde{z}_k\|^2-\gamma^2\|v_k\|^2_{W_\varphi})\right\}-\gamma^2\mathbb{E}\left\{e_0^TW_\phi e_0\right\}<0 \tag{4}$$

where $\|v_k\|^2_{W_\varphi}=v_k^TW_\varphi v_k$.

(ii) The estimation error covariance satisfies the bounded constraint $J_2:=\mathbb{E}\{e_ke_k^T\}\le\Theta_k$, where $\Theta_k\ (0\le k<N)$ is a set of pre-defined known matrix.

3 Main results

Theorem 1: Consider the estimation error dynamics (3), assume that the scalar $\gamma>0$ and $\lambda>0$, matrices $W_\varphi>0$ and $W_\phi>0$, state estimator gain matrix K_k in (2) are given. If $Q_0\le\gamma^2W_\phi$ and there exists a series of positive-definite real-value matrices $\{Q_k\}_{1\le k\le N+1}$ satisfying the following recursive matrix inequality:

$$\Psi=\begin{bmatrix}\Gamma_{11} & 0\\ * & \Gamma_{22}\end{bmatrix}<0 \tag{5}$$

where

$$\Gamma_{11}=\begin{bmatrix}\Psi_{11} & \Psi_{12} & \Psi_{13}\\ * & \Psi_{22} & -R_{2k}^T\hat{x}_k\\ * & * & \Psi_{33}\end{bmatrix},\ \Gamma_{22}=\mathrm{diag}\{\Psi_{44},\Psi_{55},\Psi_{66}\},\ \Psi_{12}=A_k^TQ_{k+1}B_k-R_{2k}$$

$$\Psi_{11}=4A_k^TQ_{k+1}A_k+4D_k^T\overline{\Lambda}_k^TK_k^TQ_{k+1}K_k\overline{\Lambda}_kD_k+2A_{1k}^TQ_{k+1}A_{1k}+2\lambda\delta D_k^T\overline{\Lambda}_k^T\overline{\Lambda}_kD_k+2\lambda\delta\sum_{i=1}^{m}(\psi_k^i)^2D_k^T(G_k^i)^TG_k^iD_k$$
$$+2\sum_{i=1}^{m}(\psi_k^i)^2D_k^T(G_k^i)^TK_k^TQ_{k+1}K_kG_k^iD_k+M_k^TM_k-Q_k-R_{1k}$$
$$\Psi_{22}=4B_k^TQ_{k+1}B_k-I\ ,\ \Psi_{44}=4K_k^TQ_{k+1}K_k-\lambda I\ ,\ \Psi_{66}=E_k^TK_k^TQ_{k+1}K_kE_k+\lambda\delta E_k^TE_k-\gamma^2W_\varphi$$
$$\begin{aligned}\Psi_{33}&=2\hat{x}_k^TA_{1k}^TQ_{k+1}A_{1k}\hat{x}_k+2\sum_{i=1}^{m}(\psi_k^i)^2\hat{x}_k^TD_k^T(G_k^i)^TK_k^TQ_{k+1}K_kG_k^iD_k\hat{x}_k+2\lambda\delta\hat{x}_k^T\\&\times D_k^T\overline{\Lambda}_k^T\overline{\Lambda}_kD_k\hat{x}_k+2\lambda\delta\sum_{i=1}^{m}(\psi_k^i)^2\hat{x}_k^TD_k^T(G_k^i)^TG_k^iD_k\hat{x}_k-\hat{x}_k^TR_{1k}\hat{x}_k\end{aligned}\ ,\ \Psi_{55}=C_k^TQ_{k+1}C_k-\gamma^2W_\varphi$$

then, (4) can be achieved for all nonzero v_k under the initial condition.

Theorem 2: Consider the estimation error dynamics (3). Let the estimator gain matrix K_k in (2) be given. Under the initial condition $P_0=X_0$, if there exists a set of positive-definite matrices $\{P_k\}_{1\le k\le N+1}$ satisfying:

$$P_{k+1}\ge\Theta(P_k) \tag{6}$$

where

$$\begin{aligned}\Theta(P_k)&=4A_kP_kA_k^T+4K_k\overline{\Lambda}_kD_kP_kD_k^T\overline{\Lambda}_k^TK_k^T+2A_{1k}\hat{x}_k\hat{x}_k^TA_{1k}^T+2A_{1k}P_kA_{1k}^T+4\mathrm{tr}(P_k)B_kYB_k^T\\&+2\sum_{i=1}^{m}(\psi_k^i)^2K_kG_k^iD_kx_kx_k^TD_k^T(G_k^i)^TK_k^T+2\sum_{i=1}^{m}(\psi_k^i)^2K_kG_k^iD_kP_kD_k^T(G_k^i)^TK_k^T+K_kE_kV_2E_k^TK_k^T\\&+4\delta\mathrm{tr}(\overline{\Lambda}_kD_k\hat{x}_k\hat{x}_k^TD_k^T\overline{\Lambda}_k^T)K_kK_k^T+4\delta\mathrm{tr}(\overline{\Lambda}_kD_kP_kD_k^T\overline{\Lambda}_k^T)K_kK_k^T+4\delta\mathrm{tr}(\overline{\Lambda}_kD_k\hat{x}_ke_k^TD_k^T\Lambda_k^T)K_kK_k^T\\&+4\delta\mathrm{tr}(\overline{\Lambda}_kD_ke_k\hat{x}_k^TD_k^T\Lambda_k^T)K_kK_k^T+4\delta\mathrm{tr}\left[\sum_{i=1}^{m}(\psi_k^i)^2G_k^iD_k\hat{x}_ke_k^TD_k^T(G_k^i)^T\right]K_kK_k^T\\&+4\delta\mathrm{tr}\left[\sum_{i=1}^{m}(\psi_k^i)^2G_k^iD_ke_k\hat{x}_k^TD_k^T(G_k^i)^T\right]K_kK_k^T+4\delta\mathrm{tr}\left[\sum_{i=1}^{m}(\psi_k^i)^2G_k^iD_k\hat{x}_k\hat{x}_k^TD_k^T(G_k^i)^T\right]K_kK_k^T\\&+4\delta\mathrm{tr}\left[\sum_{i=1}^{m}(\psi_k^i)^2G_k^iD_kP_kD_k^T(G_k^i)^T\right]K_kK_k^T+4\delta\mathrm{tr}(E_kV_2E_k^T)+C_kV_1C_k^T\end{aligned}$$
$$Y=(\rho+\rho^{-1})U_1^2/2(1-\rho)+U_2^2/\rho(1-\rho)$$

and other elements are defined in Theorem 1, then we have $P_k\ge X_k\ (\forall k\in 1,2,\dots,N+1)$.

Theorem 3: Given the scalar γ, matrices $W_\varphi>0$, $W_\phi>0$ and a set of pre-defined variance upper bound matrices $\{\Theta_k\}_{0\le k\le N+1}$, under the initial conditions $\begin{cases}Q_0\le\gamma^2W_\phi\\ \mathbb{E}\{e_0e_0^T\}=P_0\le\Theta_0\end{cases}$, if there exist the following series of positive definite matrices $\{Q_k\}_{1\le k\le N+1}$, $\{P_k\}_{1\le k\le N+1}$ and matrix $\{K_k\}_{0\le k\le N}$ with appropriate dimensions satisfying the following RLMIs:

$$\begin{bmatrix}\Xi_{11} & \Xi_{12}\\ * & \Pi_{22}\end{bmatrix}<0 \tag{7}$$

$$\begin{bmatrix}-P_{k+1} & \Upsilon_{12}\\ * & \Upsilon_{22}\end{bmatrix}<0 \tag{8}$$

$$G_{2,k+1}-\Theta_{k+1}\le 0 \tag{9}$$

with the parameter updated by $\bar{Q}_k = Q_k^{-1}$, where

$$\Xi_{11}=\begin{bmatrix}\Pi_{11} & \Pi_{12} & \Pi_{13}\\ * & \Pi_{22} & \Pi_{23}\\ * & * & \Pi_{33}\end{bmatrix},\ \Xi_{12}=\begin{bmatrix}\Pi_{14} & \Pi_{15} & 0 & 0 & 0\\ 0 & 0 & \Pi_{26} & \Pi_{27} & 0\\ 0 & 0 & 0 & 0 & \Pi_{38}\end{bmatrix},\ \Xi_{22}=\mathrm{diag}\{\Pi_{44},\Pi_{55},\Pi_{66},\Pi_{77},\Pi_{88}\}$$

$$\Upsilon_{12}=\begin{bmatrix}\Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & \Phi_{17} & \Phi_{18} & \Phi_{19}\end{bmatrix},$$

$$\Upsilon_{22}=\mathrm{diag}\{\Phi_{22},\Phi_{33},\Phi_{44},\Phi_{55},\Phi_{66},\Phi_{77},\Phi_{88},\Phi_{99}\}$$

$$\Pi_{11}=-R_{1k}-Q_k,\ \Pi_{12}=\begin{bmatrix}-R_{2k} & -R_{1k}\hat{x}_k\end{bmatrix},\ \Pi_{13}=\begin{bmatrix}0 & 0 & 0 & A_k^T & A_{1k}^T\end{bmatrix},\ \Pi_{55}=\mathrm{diag}\{-I,-\bar{Q}_{k+1},-I\}$$

$$\Pi_{14}=\begin{bmatrix}\sqrt{2}A_k^T & 2D_k^T\bar{\Lambda}_k^T K_k^T & \sqrt{2\lambda\delta}D_k^T\bar{\Lambda}_k^T\end{bmatrix},\ \Pi_{44}=\mathrm{diag}\{-\bar{Q}_{k+1},-\bar{Q}_{k+1},-I\},\ \Pi_{77}=\mathrm{diag}\{-I,-I\}$$

$$\Pi_{15}=\begin{bmatrix}\sqrt{2\lambda\delta}\sum_{i=1}^{m}\psi_k^i D_k^T(G_k^i)^T K_k^T & \sqrt{2}\sum_{i=1}^{m}\psi_k^i D_k^T(G_k^i)^T K_k^T & M_k^T\end{bmatrix},\ \Pi_{66}=\mathrm{diag}\{-\bar{Q}_{k+1},-\bar{Q}_{k+1},-\bar{Q}_{k+1}\}$$

$$\Pi_{22}=\begin{bmatrix}-I & -2R_{2k}^T\hat{x}_k\\ * & -\hat{x}_k^T R_{1k}\hat{x}_k\end{bmatrix},\qquad \Pi_{23}=\begin{bmatrix}0 & 0 & 0 & B_k^T & 0\\ 0 & 0 & 0 & 0 & A_{1k}^T\end{bmatrix},$$

$$\Pi_{33}=\mathrm{diag}\{-\lambda I,-\gamma^2 W_\varphi,-\gamma^2 W_\varphi,-\bar{Q}_{k+1},-\bar{Q}_{k+1}\}$$

$$\Phi_{19}=\begin{bmatrix}2\sqrt{\lambda\delta}\,\mathrm{tr}(E_k V_2 E_k^T)K_k & K_k E_k V_2\end{bmatrix},\ \Phi_{22}=\mathrm{diag}\{-P_k,-P_k,-1\},\ \Phi_{33}=\mathrm{diag}\{-P_k,-\mathrm{tr}(P_k)I,-1\}$$

$$\Pi_{26}=\begin{bmatrix}\sqrt{2}B_k^T & 0 & 0\\ 0 & \hat{x}_k^T A_{1k}^T & \sqrt{2}\sum_{i=1}^{m}\psi_k^i\hat{x}_k^T D_k^T(G_k^i)^T K_k^T\end{bmatrix},$$

$$\Pi_{27}=\begin{bmatrix}0 & 0\\ \sqrt{2\lambda\delta}\hat{x}_k^T D_k^T\bar{\Lambda}_k^T & \sqrt{2\lambda\delta}\sum_{i=1}^{m}\psi_k^i\hat{x}_k^T D_k^T(G_k^i)^T\end{bmatrix}$$

$$\Pi_{38}=\begin{bmatrix}2K_k^T & 0 & 0 & 0\\ 0 & C_k^T & 0 & 0\\ 0 & 0 & E_k^T K_k^T & \sqrt{\lambda\delta}E_k^T\end{bmatrix},\ \Phi_{13}=\begin{bmatrix}\sqrt{2}A_{1k}P_k & 2\mathrm{tr}(P_k)B_k Y^{\frac{1}{2}} & \sqrt{2}\sum_{i=1}^{m}\psi_k^i K_k G_k^i D_k\hat{x}_k\end{bmatrix}$$

$$\Phi_{12}=\begin{bmatrix}2A_kP_k & 2K_k\bar{\Lambda}_k D_k P_k & \sqrt{2}A_{1k}\hat{x}_k\end{bmatrix},\ \Phi_{18}=\begin{bmatrix}2\sqrt{\lambda\delta}\,\mathrm{tr}\left(\sum_{i=1}^{m}(\psi_k^i)^2 G_k^i D_k e_k\hat{x}_k^T D_k^T(G_k^i)^T\right)K_k & C_kV_1\end{bmatrix}$$

$$\Phi_{14}=\begin{bmatrix}\sqrt{2}\sum_{i=1}^{m}\psi_k^i K_k G_k^i D_k P_k & 2\sqrt{\lambda\delta}\,\mathrm{tr}(\bar{\Lambda}_k D_k\hat{x}_k\hat{x}_k^T D_k^T\bar{\Lambda}_k^T)K_k\end{bmatrix},\ \Pi_{88}=\mathrm{diag}\{-\bar{Q}_{k+1},-\bar{Q}_{k+1},-\bar{Q}_{k+1},-I\}$$

$$\Phi_{16}=\begin{bmatrix}2\sqrt{\lambda\delta}\,\mathrm{tr}(\bar{\Lambda}_k D_k e_k\hat{x}_k^T D_k^T\bar{\Lambda}_k^T)K_k & 2\sqrt{\lambda\delta}\,\mathrm{tr}\left(\sum_{i=1}^{m}(\psi_k^i)^2 G_k^i D_k\hat{x}_k\hat{x}_k^T D_k^T(G_k^i)^T\right)\end{bmatrix}$$

$$\Phi_{17}=\begin{bmatrix}2\sqrt{\lambda\delta}\,\mathrm{tr}\left(\sum_{i=1}^{m}(\psi_k^i)^2 G_k^i D_k P_k D_k^T(G_k^i)^T\right)K_k & 2\sqrt{\lambda\delta}\,\mathrm{tr}\left(\sum_{i=1}^{m}(\psi_k^i)^2 G_k^i D_k\hat{x}_k e_k^T D_k^T(G_k^i)^T\right)K_k\end{bmatrix}$$

$$\Phi_{44}=\mathrm{diag}\{-P_k,-\mathrm{tr}(\bar{\Lambda}_k D_k\hat{x}_k\hat{x}_k^T D_k^T\bar{\Lambda}_k^T)I\},\ \Phi_{55}=\mathrm{diag}\left\{-\mathrm{tr}(\bar{\Lambda}_k D_k P_k D_k^T\bar{\Lambda}_k^T)I,-\mathrm{tr}(\bar{\Lambda}_k D_k\hat{x}_k e_k^T D_k^T\bar{\Lambda}_k^T)I\right\}$$

$$\Phi_{66}=\mathrm{diag}\left\{-\mathrm{tr}(\bar{\Lambda}_k D_k e_k\hat{x}_k^T D_k^T\bar{\Lambda}_k^T)I,-\mathrm{tr}\left(\sum_{i=1}^{m}(\psi_k^i)^2 G_k^i D_k\hat{x}_k\hat{x}_k^T D_k^T(G_k^i)^T\right)I\right\}$$

$$\Phi_{77}=\mathrm{diag}\left\{-\mathrm{tr}\left(\sum_{i=1}^{m}(\psi_k^i)^2 G_k^i D_k P_k D_k^T(G_k^i)^T\right),-\mathrm{tr}\left(\sum_{i=1}^{m}(\psi_k^i)^2 G_k^i D_k\hat{x}_k e_k^T D_k^T(G_k^i)^T\right)\right\}$$

$$\Phi_{88}=\mathrm{diag}\left\{-\mathrm{tr}\left(\sum_{i=1}^{m}(\psi_k^i)^2 G_k^i D_k e_k\hat{x}_k^T D_k^T(G_k^i)^T\right),-V_1\right\},\ \Phi_{99}=\mathrm{diag}\left\{-\mathrm{tr}(E_kV_2E_k^T)I,-V_2\right\}$$

then, the H_∞ performance requirements and the upper constraints of estimation error covariance can be satisfied simultaneously.

4 A numerical example

In this section, the parameters of time-varying RNNs (1) are given as follows:

$$A_k = \begin{bmatrix} -0.1\sin(2k) & 0 \\ 0 & -0.1 \end{bmatrix},\ A_{1k} = \begin{bmatrix} -0.2 & 0.2 \\ -0.1 & -0.1\sin(2k) \end{bmatrix},\ B_k = \begin{bmatrix} -0.1\sin(2k) & 0.2 \\ -0.1 & -0.14 \end{bmatrix}$$

$$C_k = \begin{bmatrix} -0.1 & -0.3\sin(2k) \end{bmatrix}^T,\ D_k = \begin{bmatrix} -0.5\sin(k) & 1.5 \\ -0.1 & -0.14 \end{bmatrix},\ E_k = \begin{bmatrix} -0.27\sin(2k) & 0.15 \end{bmatrix}^T$$

$$M_k = \begin{bmatrix} -0.02 & -0.08\sin(3k) \end{bmatrix},\ \Omega_1 = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix},\ \Omega_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \lambda = 0.0.1,\ \delta = 0.6,\ \rho = 0.45$$

In addition, the activation functions can be taken as $f(x_k) = \begin{bmatrix} \tanh(x_{1,k}) & \tanh(0.8x_{2,k}) \end{bmatrix}^T$ with $x_k = \begin{bmatrix} x_{1,k} & x_{2,k} \end{bmatrix}^T$. It is easy to obtain that $\Sigma_0 = \mathrm{diag}\{0.2, 0.2\}$, $\Sigma_1 = \mathrm{diag}\{0.3, 0.3\}$ and $\Sigma_2 = \mathrm{diag}\{0.5, 0.5\}$. Let the disturbance attenuation level $\gamma = 0.9$ and $N = 100$, the weighted matrices $W_\varphi(1) = W_\varphi(2) = 1$, upper bound matrices $\{\Theta_k\}_{1 \le k \le N} = \mathrm{diag}\{0.3, 0.3\}$ and covariances as $V_1 = V_2 = 1$. Then, by solving RLMIs (7)-(9), K_k is designed as follows:

$$K_1 = \begin{bmatrix} -0.0013 & 0.0954 \\ 0.0970 & 0.0984 \end{bmatrix},\ K_2 = \begin{bmatrix} -0.0266 & 0.0878 \\ 0.0811 & 0.0827 \end{bmatrix},\ K_3 = \begin{bmatrix} -0.0145 & 0.0898 \\ 0.1077 & 0.0786 \end{bmatrix}$$

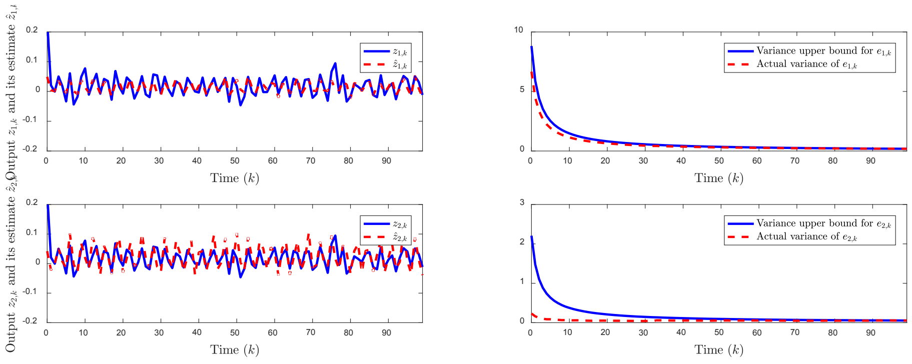

Fig. 1. (a) the controlled output $z_{1,k}$, $z_{2,k}$ and their estimations; (b) X_k and its upper bound.

Set the initial states $x_0 = \begin{bmatrix} -1.2 & 0.3 \end{bmatrix}^T$ and $\hat{x}_0 = \begin{bmatrix} -0.2 & 0.6 \end{bmatrix}^T$. Let $\mu_k^1 = 0.55$, $\mu_k^2 = 0.65$, $\psi_k^1 = 0.3905$ and $\psi_k^2 = 0.4153$. In addition, the simulation results are shown in Fig. 1, which illustrate that the proposed event-triggered H_∞ state estimation method is effective.

5 Conclusion

In this paper, we have discussed the event-triggered H_∞ state estimation problem for a class of discrete RNNs subject to variance-constraint and fading measurements. The phenomena of fading measurements have been described by introducing a set of mutually independent random variables. Applying the RLMI technique, some criteria have been established to guarantee the prescribed H_∞ performance and the estimation error covariance constraints of the estimation error system. Finally, the feasibility of the proposed method has been verified by a numerical example.

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References

1. L. Hu, Z. Wang, Q. Han, X. Liu, Event-based input and state estimation for linear discrete time-varying systems, Int. J. Control, **91**, 1, 101-113 (2018)
2. L. Li, D. Yu, Y. Xia, H. Yang, Stochastic stability of a modified unscented Kalman filter with stochastic nonlinearities and multiple fading measurements, J. Frankl. Inst.-Eng. Appl. Math., **354**, 2, 650-667 (2017)
3. D. Ding, Z. Wang, L. Lam, B. Shen, H. Dong, Envelope-constrained H_∞ filtering with fading measurements and randomly occurring nonlinearities: the finite horizon case, Automatica, **55**, 9, 37-45 (2015)
4. H. Dong, N. Hou, Z. Wang, W. Ren, Variance-constrained state estimation for complex networks with randomly varying topologies, IEEE Trans. Neural Netw. Learn. Syst., **29**, 7, 2757-2768 (2018)
5. H. Dong, Z. Wang, F.E. Alsaadi, B. Ahmad, Event-triggered robust distributed state estimation for sensor networks with state-dependent noises, Int. J. Gen. Syst., **44**, 2, 254-266 (2015)