Resilient set-membership state estimation for nonlinear complex networks with time-delay and incomplete measurements

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Abstract. Taking the incomplete measurements and the weighted try-once-discard (WTOD) protocol into account, this paper develops a novel resilient set-membership state estimation (RSMSE) method for time-varying nonlinear complex networks with time-invariant delay. A classic interval matrix technique is utilized to describe incomplete measurements. The Taylor series expansion is applied to dispose the nonlinearities, where the high-order terms of the linearization errors are described by norm-bounded uncertainties. To mitigate the communication burden, the WTOD protocol is introduced, where only one node can send updated data through a shared communication network at each certain transmission step. Using the recursive linear matrix inequalities (RLMIs), a series of ellipsoidal sets including the state vector can be determined. The desirable estimator gain and a smallest possible estimation ellipsoid can be calculated via solving the convex optimization problem. Lastly, we use an illustrative example to show the feasibility of the introduced RSMSE technique.

1 Introduction

Since the end of the 20th century, the state estimation problem of complex networks has been intensively investigated. In [1], the variance-constrained state estimation problem has been concerned for coupled nonlinear complex networks and sufficient criteria have been given to ensure the exponential boundedness of the estimation error in mean-square. Moreover, there is a special assumption that noise is random and satisfies some probability distributions in the traditional state estimation problem. However, there is another kind of noise with unknown statistical properties but bounded in the practical system. Hence, it is of practical significance to develop the set-membership state estimation (SMSE) approach. The ellipsoidal SMSE theory has been first presented in [2] and an ellipsoid has been first used to approximate the feasible set of parameters. Since then, a new set-membership filtering recursive scheme of computing the smallest ellipsoid has been exploited in [3] for nonlinear discrete-time systems.

In practice, the estimation system may be sensitive to small changes or uncertainty of parameter, and such estimator is "fragile". Therefore, a resilient/non-fragile method has been proposed to weaken the effect of gain perturbation on the estimation performance. In

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[4], a new event-triggered non-fragile state estimation strategy from partial nodes has been presented for delayed complex networks subject to randomly occurring sensor saturation and gain variations. In a networked system, all system components access a shared communication network and use network-based communication technology to update data. To mitigate the communication burden, communication networks are usually equipped with corresponding communication protocols, such as the round-robin protocol and the weighted try-once-discard (WTOD) protocol. The effects of the round-robin protocol and the WTOD protocol on the set-membership filtering problem have been considered in [5] for time-varying systems with mixed time-delays.

In response to above comprehensive analyses, this paper focuses on solving the resilient set-membership state estimation (RSMSE) problem for a class of time-varying nonlinear complex networks (TVNCNs) with time-invariant delay and incomplete measurements under the WTOD protocol. The primary contributions of this paper can be summarized as listed below: 1) the resilient state estimator (RSE) is designed by combining the methods of handling the comprehensive effect of time-invariant delay, incomplete measurements and the WTOD protocol; 2) the desirable estimator gain and a smallest possible estimation ellipsoid can be calculated by the RSMSE method and the convex optimization method. Finally, the feasibility of the proposed RSMSE method is verified via a numerical example.

2 Problem formulation and preliminaries

In this paper, we consider a class of TVNCNs with time-invariant delay composed of N coupled nodes, where the i-th node's dynamic system model is given by

$$\begin{cases} x_{i,k+1} = f\left(x_{i,k}\right) + g\left(x_{i,k-\tau}\right) + \sum_{j=1}^{N} \gamma_{ij} h\left(x_{j,k}\right) + B_{i,k}\omega_k \\ y_{i,k} = C_{i,k}x_{i,k} + D_{i,k}\upsilon_k \\ x_{i,k} = \psi_{i,k} \qquad k \in [-\tau, 0] \end{cases} \tag{1}$$

where $x_{i,k} \in \mathbb{R}^n$ and $y_{i,k} \in \mathbb{R}^m$ are the state vector and the normal measurement output for the i-th node at the step k, respectively. $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ are some known nonlinear functions, which are assumed to be continuously differentiable. $h(\cdot)$ denotes the inner-coupling function between two coupled nodes. Assume that the inner-coupling function of each node is exactly the same. $W = \left[\gamma_{ij}\right]_{N\times N}$ stands for the coupling configuration matrix. τ represents the time-invariant delay, which is a known positive integer. $B_{i,k}$, $C_{i,k}$ and $D_{i,k}$ are known properly dimensional time-varying matrices. $\psi_{i,k}$ $\left(k \in [-\tau, 0]\right)$ are the initial conditions of the i-th node. The vectors $\omega_k \in \mathbb{R}^{r_1}$ and $\upsilon_k \in \mathbb{R}^{r_2}$ are the process noise and measurement noise, which are constrained to the ellipsoidal sets $\mathcal{W}_k = \left\{\omega_k : \omega_k^T S_k^{-1} \omega_k \le 1\right\}$ and $\mathcal{V}_k = \left\{\upsilon_k : \upsilon_k^T R_k^{-1} \upsilon_k \le 1\right\}$ with S_k and R_k being known positive definite matrices.

In this paper, the following model is chosen to depict the incomplete measurements:

$$\tilde{y}_{i,k} = \Delta_k^i C_{i,k} x_{i,k} + D_{i,k}\upsilon_k \tag{2}$$

where $\Delta_k^i = \text{diag}\left\{\sigma_{1,k}^i, \sigma_{2,k}^i, \cdots, \sigma_{m,k}^i\right\}$, $\sigma_{l,k}^i \in \left[\underline{\sigma}_l^i, \overline{\sigma}_l^i\right]$ with $0 \le \underline{\sigma}_l^i \le \overline{\sigma}_l^i \le 1$ $\quad (l = 1, 2, \cdots, m)$. Define

$\Delta_0^i = \text{diag}\left\{\frac{\overline{\sigma}_1^i + \underline{\sigma}_1^i}{2}, \frac{\overline{\sigma}_2^i + \underline{\sigma}_2^i}{2}, \cdots, \frac{\overline{\sigma}_m^i + \underline{\sigma}_m^i}{2}\right\}$ and $\Delta_1^i = \text{diag}\left\{\frac{\overline{\sigma}_1^i - \underline{\sigma}_1^i}{2}, \frac{\overline{\sigma}_2^i - \underline{\sigma}_2^i}{2}, \cdots, \frac{\overline{\sigma}_m^i - \underline{\sigma}_m^i}{2}\right\}$. When $\tilde{\sigma}_{l,k}^i \in \left[-\frac{\overline{\sigma}_l^i - \underline{\sigma}_l^i}{2}, \frac{\overline{\sigma}_l^i - \underline{\sigma}_l^i}{2}\right]$ $(l = 1, 2, \cdots, m,\ i = 1, 2, \cdots, N)$, we can obtain $\Delta_k^i = \Delta_0^i + \tilde{\Delta}_k^i$ with $\tilde{\Delta}_k^i = \text{diag}\left\{\tilde{\sigma}_{1,k}^i, \tilde{\sigma}_{2,k}^i, \cdots, \tilde{\sigma}_{m,k}^i\right\}$. Then, set $x_k = \text{col}_N\left\{x_{i,k}\right\} \in \mathbb{R}^{Nn}$, $\tilde{y}_k = \text{col}_N\left\{\tilde{y}_{i,k}\right\} \in \mathbb{R}^{Nm}$ and $\psi_k = \text{col}_N\left\{\psi_{i,k}\right\} \in \mathbb{R}^{Nn}$. Based on (1) and (2), we have

$$\begin{cases} x_{k+1} = \overline{f}(x_k) + \overline{g}(x_{k-\tau}) + (W \otimes I)\overline{h}(x_k) + B_k \omega_k \\ \tilde{y}_k = \left(\Delta_0 + \tilde{\Delta}_k\right) C_k x_k + D_k \upsilon_k \\ x_k = \psi_k \quad k \in [-\tau, 0] \end{cases} \tag{3}$$

where $B_k = \left[B_{1,k}^T \quad B_{2,k}^T \quad \cdots \quad B_{N,k}^T\right]^T$, $C_k = \text{diag}\left\{C_{1,k}, C_{1,k}, \cdots, C_{N,k}\right\}$, $D_k = \left[D_{1,k}^T \quad D_{2,k}^T \quad \cdots \quad D_{N,k}^T\right]^T$, $\overline{f}(x_k) = \left[f^T(x_{1,k}) \quad f^T(x_{2,k}) \quad \cdots \quad f^T(x_{N,k})\right]^T$, $\overline{g}(x_{k-\tau}) = \left[g^T(x_{1,k-\tau}) \quad g^T(x_{2,k-\tau}) \quad \cdots \quad g^T(x_{N,k-\tau})\right]^T$, $\overline{h}(x_k) = \left[h^T(x_{1,k}) \quad h^T(x_{2,k}) \quad \cdots \quad h^T(x_{N,k})\right]^T$, $\Delta_0 = \text{diag}\left\{\Delta_0^1, \Delta_0^2, \cdots, \Delta_0^N\right\}$, $\tilde{\Delta}_k = \text{diag}\left\{\tilde{\Delta}_k^1, \tilde{\Delta}_k^2, \cdots, \tilde{\Delta}_k^N\right\}$.

Let $\theta_k \in \{1, 2, \cdots, N\}$ represent the chosen node licensed to employ the shared communication network at the step *k*. Under the WTOD protocol, the value of θ_k is determined via the selection rule $\theta_k = \min\left\{\arg\max_{1 \le i \le N}\left(\tilde{y}_{i,k} - \tilde{y}_{i,k}^*\right)^T Q_i \left(\tilde{y}_{i,k} - \tilde{y}_{i,k}^*\right)\right\}$, where $\tilde{y}_{i,k}^*$ represents the last transmitted measurement output of the *i*-th node before the step *k*. Q_i is a known positive definite matrix which represents the weight matrix of the *i*-th node. Denote by $\overline{y}_{i,k}$ the measurement output after updated via the communication network under the WTOD protocol. The renewal mechanism of $\overline{y}_{i,k}$ $(i = 1, 2, \cdots, N)$ can be given as follows:

$$\overline{y}_{i,k} = \begin{cases} \tilde{y}_{i,k}, & i = \theta_k \\ \overline{y}_{i,k-1}, & i \ne \theta_k \end{cases} \tag{4}$$

Here, other nodes without gaining the permission of employing the shared communication network at the step *k* continue to use the measurement output at the step *k*-1 by the zero-order holder. Define $\overline{y}_k = \text{col}_N\left\{\overline{y}_{i,k}\right\} \in \mathbb{R}^{Nm}$ and $\Phi_{\theta_k} = \text{diag}\left\{\delta_{\theta_k 1}, \delta_{\theta_k 2}, \cdots, \delta_{\theta_k N}\right\}$, where $\delta(\cdot)$ is the Kronecker delta function. It can be obtained from (4) that

$$\begin{cases} \overline{y}_k = \left(\Phi_{\theta_k} \otimes I\right)\tilde{y}_k + \left[\left(I - \Phi_{\theta_k}\right) \otimes I\right]\overline{y}_{k-1} \\ \overline{y}_k = \varphi_k \quad k \in [-\tau, 0] \end{cases} \tag{5}$$

where φ_k $(k \in [-\tau, 0])$ represent the initial conditions of $\overline{y}_k$.

Denote $\overline{x}_k = \left[x_k^T \quad \overline{y}_{k-1}^T\right]^T \in \mathbb{R}^{N(n+m)}$ and $\overline{\upsilon}_k = \left[\omega_k^T \quad \upsilon_k^T\right]^T \in \mathbb{R}^{r_1 + r_2}$. The combination of (3) and (5) yields the following augmented system subject to the WTOD protocol:

$$\begin{cases} \overline{x}_{k+1} = \left(\overline{A}_{\theta_k} + \tilde{A}_{\theta_k}\right)\overline{x}_k + \tilde{f}(\overline{x}_k) + \tilde{g}(\overline{x}_{k-\tau}) + \overline{W}\tilde{h}(\overline{x}_k) + \overline{B}_{\theta_k}\overline{\upsilon}_k \\ \overline{x}_k = \Psi_k \quad k \in [-\tau, 0] \end{cases} \tag{6}$$

where $\bar{A}_{\theta_k}=\begin{bmatrix}0 & 0\\ \left(\Phi_{\theta_k}\otimes I\right)\Delta_0 C_k & \left(I-\Phi_{\theta_k}\right)\otimes I\end{bmatrix}$, $\tilde{A}_{\theta_k}=\begin{bmatrix}0 & 0\\ \left(\Phi_{\theta_k}\otimes I\right)\tilde{\Delta}_k C_k & 0\end{bmatrix}$, $\tilde{f}(\bar{x}_k)=\begin{bmatrix}\bar{f}(x_k)\\ 0\end{bmatrix}$, $\tilde{g}(\bar{x}_{k-\tau})=\begin{bmatrix}\bar{g}(x_{k-\tau})\\ 0\end{bmatrix}$, $\tilde{h}(\bar{x}_k)=\begin{bmatrix}\bar{h}(x_k)\\ 0\end{bmatrix}$, $\bar{W}=\begin{bmatrix}W\otimes I & 0\\ 0 & 0\end{bmatrix}$, $\bar{B}_{\theta_k}=\begin{bmatrix}B_k & 0\\ 0 & \left(\Phi_{\theta_k}\otimes I\right)D_k\end{bmatrix}$, $\Psi_k=\begin{bmatrix}\psi_k\\ \varphi_k\end{bmatrix}$.

In terms of (6), $\bar{y}_k$ can be also reformulated as $\bar{y}_k=\left(\bar{C}_{\theta_k}+\tilde{C}_{\theta_k}\right)\bar{x}_k+\bar{D}_{\theta_k}\bar{\upsilon}_k$, where $\bar{C}_{\theta_k}=\left[\left(\Phi_{\theta_k}\otimes I\right)\Delta_0 C_k \quad \left(I-\Phi_{\theta_k}\right)\otimes I\right]$, $\tilde{C}_{\theta_k}=\left[\left(\Phi_{\theta_k}\otimes I\right)\tilde{\Delta}_k C_k \quad 0\right]$, $\bar{D}_{\theta_k}=\left[0 \quad \left(\Phi_{\theta_k}\otimes I\right)D_k\right]$.

Design the following RSE:

$$\begin{cases}\hat{x}_{k+1}=\bar{A}_{\theta_k}\hat{x}_k+\tilde{f}(\hat{x}_k)+\tilde{g}(\hat{x}_{k-\tau})+\bar{W}\tilde{h}(\hat{x}_k)+\left(K_k+\Delta K_k\right)\left(\bar{y}_k-\bar{C}_{\theta_k}\hat{x}_k\right)\\ \hat{x}_k=0 \quad k\in[-\tau,0]\end{cases} \tag{7}$$

where $\hat{x}_k$ denotes the estimate of $\bar{x}_k$ and the real-valued matrix K_k represents the estimator gain to be determined. ΔK_k stands for the estimator gain perturbation satisfying $\Delta K_k=M_{1,k}\bar{E}_k N_{1,k}$ with $\bar{E}_k^T\bar{E}_k\le I$ and $\bar{E}_k$ being unknown time-varying matrix. $M_{1,k}$ and $N_{1,k}$ denote known appropriately dimensional time-varying matrices. By applying the Taylor series expansion, the nonlinear functions can be linearized as $\tilde{f}(\bar{x}_k)=\tilde{f}(\hat{x}_k)+F_k(\bar{x}_k-\hat{x}_k)+o(|\bar{x}_k-\hat{x}_k|)$, $\tilde{h}(\bar{x}_k)=\tilde{h}(\hat{x}_k)+H_k(\bar{x}_k-\hat{x}_k)+o(|\bar{x}_k-\hat{x}_k|)$ and $g(\bar{x}_{k-\tau})=\tilde{g}(\hat{x}_{k-\tau})+G_k(\bar{x}_{k-\tau}-\hat{x}_{k-\tau})+o(|\bar{x}_{k-\tau}-\hat{x}_{k-\tau}|)$, where F_k, H_k and G_k being the first order partial derivatives of $\tilde{f}(\bar{x}_k)$, $\tilde{h}(\bar{x}_k)$ and $\tilde{g}(\bar{x}_{k-\tau})$ at $\hat{x}_k$ and $\hat{x}_{k-\tau}$. Here, the high-order terms can be approximatively reformulated by $o(|\bar{x}_k-\hat{x}_k|)\approx M_{2,k}E_kN_{2,k}(\bar{x}_k-\hat{x}_k)$, $o(|\bar{x}_{k-\tau}-\hat{x}_{k-\tau}|)\approx M_{3,k}E_kN_{3,k}(\bar{x}_{k-\tau}-\hat{x}_{k-\tau})$ and $o(|\bar{x}_k-\hat{x}_k|)\approx M_{4,k}E_kN_{4,k}(\bar{x}_k-\hat{x}_k)$, where $M_{2,k}$, $M_{3,k}$ and $M_{4,k}$ are known state-dependent scaling matrices. $N_{2,k}$, $N_{3,k}$ and $N_{4,k}$ denote known tuning matrices. E_k is an unknown time-varying matrix satisfying $E_k^TE_k\le I$.

Assumption 1: The initial data Ψ_k $\left(k\in[-\tau,0]\right)$ satisfy the constraint condition $\Psi_k^TP_k^{-1}\Psi_k\le 1$, where P_k $\left(k\in[-\tau,0]\right)$ are known positive definite matrices.

This paper aims to cope with the RSMSE problem for the augmented system (6). More specifically, we need to find a better estimator gain K_k to achieve the following objectives.

Objective 1: For a given sequence of constraint matrices $P_{k+1}>0$, the augmented vector $\bar{x}_{k+1}$ is contained in the following prescribed ellipsoidal set:

$$\left(\bar{x}_{k+1}-\hat{x}_{k+1}\right)^T P_{k+1}^{-1}\left(\bar{x}_{k+1}-\hat{x}_{k+1}\right)\le 1 \tag{8}$$

Objective 2: Minimize the trace of P_{k+1} via properly selecting the estimator gain K_k.

3 Main results

Theorem 1: For the augmented system (6), suppose that the augmented vector $\bar{x}_k$ is contained in the pre-determined estimation ellipsoid $\left(\bar{x}_k-\hat{x}_k\right)^T P_k^{-1}\left(\bar{x}_k-\hat{x}_k\right)\le 1$. If there are matrices $P_{k+1}>0$ and K_k, scalars $\varepsilon_s>0$ $(s=1,2,\cdots,7)$ and $\lambda_i>0$ $(i=1,2,\cdots,N)$ such that

$$\begin{bmatrix} -\Xi_k & \bar{\Pi}_k^T & 0 & \varepsilon_6\mathcal{N}_k^T & 0 & \varepsilon_7\tilde{\mathcal{N}}_k^T \\ * & -P_{k+1} & \mathcal{M}_k & 0 & -M_{1,k} & 0 \\ * & * & -\varepsilon_6 I & 0 & 0 & 0 \\ * & * & * & -\varepsilon_6 I & 0 & 0 \\ * & * & * & * & -\varepsilon_7 I & 0 \\ * & * & * & * & * & -\varepsilon_7 I \end{bmatrix} < 0 \tag{9}$$

where $\Xi_k = \Sigma_k + \Gamma_k^T \sum_{i=1}^{N} \lambda_i \bar{Q}\left[\left(\Phi_i - \Phi_{\theta_k}\right) \otimes I\right]\Gamma_k$, $\Gamma_k = \left[\bar{C}_k \hat{x}_k \ \ \bar{C}_k L_k \ \ 0 \ \ \tilde{D}_k \ \ I \ \ I\right]$, $\tilde{D}_k = \left[0 \ D_k\right]$, $\Sigma_k = \mathrm{diag}\left\{\Sigma_{1,k}, \Sigma_{2,k}, \varepsilon_2 I, \varepsilon_3\Omega_k^{-1}, \varepsilon_4 I, \varepsilon_5 I\right\}$, $\Sigma_{1,k} = 1 - \varepsilon_1 - \varepsilon_2 - 2\varepsilon_3 - \varepsilon_4 \hat{x}_k^T \mathcal{C}_k^T \Delta_1^T \Delta_1 \mathcal{C}_k \hat{x}_k$, $\Omega_k = \mathrm{diag}\left\{S_k, R_k\right\}$, $\Sigma_{2,k} = \varepsilon_1 I - \varepsilon_5 L_k^T \mathcal{C}_k^T \Delta_1^T \Delta_1 \mathcal{C}_k L_k$, $\Delta_1 = \mathrm{diag}\left\{\Delta_1^1, \Delta_1^2, \cdots, \Delta_1^N\right\}$, $\bar{C}_k = \left[\Delta_0 C_k \ \ -I\right]$, $\mathcal{A}_{\theta_k} = \bar{A}_{\theta_k} + F_k + \bar{W}H_k$, $\mathcal{C}_k = \left[C_k \ \ 0\right]$, $\bar{Q} = \mathrm{diag}\left\{Q_1, Q_2, \cdots, Q_N\right\}$ $\bar{\Pi}_k = \left[0 \ \ \left(\mathcal{A}_{\theta_k} - K_k \bar{C}_{\theta_k}\right)L_k \ \ G_k L_{k-\tau} \ \ \bar{B}_{\theta_k} - K_k \bar{D}_{\theta_k} \ \ \bar{\Pi}_{2,k} \ \ \bar{\Pi}_{2,k}\right]$,

$\bar{\Pi}_{2,k} = \bar{\Phi}_{\theta_k} - K_k\left(\Phi_{\theta_k} \otimes I\right)$, $\mathcal{M}_k = \left[M_{2,k} \ \ M_{3,k} \ \ \bar{W}M_{4,k}\right]$, $\mathcal{N}_k = \begin{bmatrix} 0 & N_{2,k}L_k & 0 & 0 & 0 & 0 \\ 0 & 0 & N_{3,k}L_{k-\tau} & 0 & 0 & 0 \\ 0 & N_{4,k}L_k & 0 & 0 & 0 & 0 \end{bmatrix}$,

$\tilde{\mathcal{N}}_k = \left[0 \ \ N_{1,k}\bar{C}_{\theta_k}L_k \ \ 0 \ \ N_{1,k}\bar{D}_{\theta_k}L_k \ \ N_{1,k}\left(\Phi_{\theta_k} \otimes I\right) \ \ N_{1,k}\left(\Phi_{\theta_k} \otimes I\right)\right]$, $\bar{\Phi}_{\theta_k} = \left[0 \ \ \left(\Phi_{\theta_k} \otimes I\right)^T\right]^T$, with L_k being the Cholesky factorization of $P_k = L_k L_k^T$, then the augmented vector $\bar{x}_{k+1}$ is contained in the estimation ellipsoid (8).

Theorem 2: For the augmented system (6), the minimized matrix $P_k > 0$ (in the sense of matrix trace) is determined if there are real-valued matrix K_k , scalars $\varepsilon_s > 0$ $(s = 1,2,\cdots,7)$ and $\lambda_i > 0$ $(i = 1,2,\cdots,N)$ by settling the convex optimization problem $\begin{cases} \min\limits_{P_{k+1}, K_k, \varepsilon_s, \lambda_i} \mathrm{tr}\left\{P_{k+1}\right\} \\ \text{subject to } (9) \end{cases}$.

4 A numerical example

The TVNCNs (1) with time-invariant delay and incomplete measurements is made up of three nodes, where corresponding parameters for each node are listed below:

$$B_{1,k} = \left[0.6 \ \ 0.3\right]^T, B_{2,k} = \left[0.4 \ \ 0.5\right]^T, B_{3,k} = \left[0.8 \ \ 0.5\right]^T, D_{1,k} = 0.3, D_{2,k} = 0.5, D_{3,k} = 0.2$$

$$C_{1,k} = \left[0.5\cos(0.5k) \ \ -0.4\right], C_{2,k} = \left[0.3\cos(0.5k) \ \ 0.2\right], C_{3,k} = \left[0.4\cos(0.5k) \ \ -0.5\right]$$

$$W = \begin{bmatrix} -0.2 & 0.1 & 0.1 \\ 0.1 & -0.2 & 0.1 \\ 0.1 & 0.1 & -0.2 \end{bmatrix}, \ f\left(x_{i,k}\right) = \left[\sin\left(0.5x_{i,k}^{(1)}\right) \ \ \sin\left(0.5x_{i,k}^{(2)}\right)\right]^T$$

$$g\left(x_{i,k-\tau}\right) = \left[\cos\left(0.5x_{i,k-\tau}^{(1)}\right) \ \ \cos\left(0.5x_{i,k-\tau}^{(2)}\right)\right]^T, \ h\left(x_{i,k}\right) = \left[\left(x_{i,k}^{(1)}\right)^2 \ \ \left(x_{i,k}^{(2)}\right)^2\right]^T \ \ (i = 1,2,3)$$

Let the time delay $\tau = 2$. The process noise and the measurement noise are set as $\omega_k = 0.9\cos(k)$ and $\upsilon_k = 0.9\sin(k)$, respectively. It is obvious that the matrices S_k and R_k can be selected as $S_k = R_k = 0.81$. The parameter matrices of the estimator gain perturbation are chosen as $M_{1,k} = [0.1\ 0.1\ 0.1\ 0.1\ 0.1\ 0.1\ 0.1\ 0.1\ 0.1]^T$, $N_{1,k} = [0.1\ 0.1\ 0.1]$, $\bar{E}_k = \sin(0.5k)$. In the WTOD protocol, the weighted matrices are taken to be $Q_1 = 0.6$, $Q_2 = 1.2$ and $Q_3 = 0.8$. Then, we set the initial values of states and constraint matrices as

$$x_{1,j} = [0 \ \ 0]^T, \ x_{2,j} = [0 \ \ 0]^T, \ x_{3,j} = [0 \ \ 0]^T, \ P_j = \mathrm{diag}\{1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1\} \ \ (j = -1,-2)$$

$$x_{1,0}=[2\ \ 0]^T,\ x_{2,0}=[-1.5\ \ 0]^T,\ x_{3,0}=[2\ \ 0]^T,\ P_0=\mathrm{diag}\{18,\ 18,\ 30,\ 30,\ 10,\ 10,\ 4,\ 4,\ 1\}$$

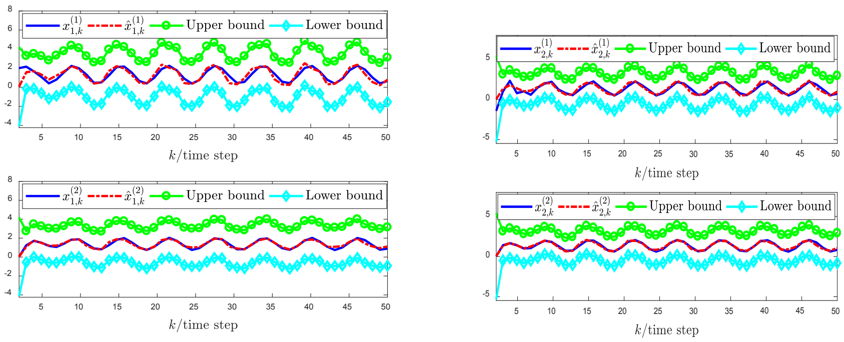

Fig. 1. The trajectories of $x_{i,k}$, $\hat{x}_{i,k}$ and their bounds for node i with $\sigma^i_{1,k}\in[0.7,1]$ ($i=1,2$).

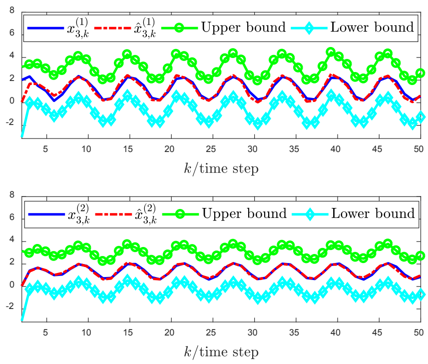

Fig. 2. The trajectories of $x_{3,k}$, $\hat{x}_{3,k}$ and its bounds for node 3 with $\sigma^3_{1,k}\in[0.7,1]$.

On the strength of the above information, the simulation outcomes are presented in Fig. 1 and Fig. 2, which show the trajectory cures of the real states $x_{i,k}$, the state estimations $\hat{x}_{i,k}$, as well as their upper and lower bounds of *i*-th node with $\sigma^i_{1,k}\in[0.7,1]$ $(i=1,2,3)$. It is fully demonstrated from Fig. 1 and Fig. 2 that the designed RSE performs well under the WTOD protocol and the incomplete measurements.

5 Conclusion

In this paper, the RSMSE problem has been tackled for a class of TVNCNs with time-invariant delay and incomplete measurements under the WTOD protocol. To reduce unnecessary transmission of measured data, the WTOD protocol has been utilized to determine which nodes have the highest priority of transmitting updated data at a certain transmission step. By means of the RLMIs, a sufficient criterion has been derived to guarantee that the state vector is confined to the ellipsoidal set. The designed estimator gain has been obtained by solving the convex optimization problem. A numerical example has been provided to demonstrate the feasibility of the proposed RSE design scheme.

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