

# Simple mathematical model to predict the amount of energy produced in wind turbine - preliminary study

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**Abstract.** The manuscript presents a simple mathematical model for predicting the amount of energy produced in a wind turbine. As part of the own research, the data obtained from the SCADA program for the Enercon E-82 wind turbine was analyzed. It has been shown that it is possible to build a mathematical model to determine the amount of energy produced from the average wind speed. This method will be primarily useful for forecasting the volume of production as well as electricity demand, with particular emphasis on renewable energy sources. The application of the developed method in practice will facilitate and accelerate the implementation of the decision-making process in electricity production systems, while reducing the risk of error. This model can also be used to make repowering decisions.

## 1 Introduction

The high share of wind energy in the national power system creates many problems. They are caused by unstable operation of wind farms due to changing wind speed. During normal use, there are also periods without electricity production. Due to the planned demand for electricity in the energy system, such a situation is unfavorable. A constant production of electricity must be maintained in the energy system. It must correspond to the demand. In the case of high variability of electricity production, it is necessary to maintain other sources. An "energy mix" is being created based on electricity from renewable sources and fossil fuels. Energy storage facilities based on hydropower plants and accumulators are also maintained. A system constructed in this way must ensure a high potential for reliability of electricity supply. Maintaining such a complex system incurs high costs. They are related to construction and maintenance [1, 2].

Wind farm repowering is the process of replacing existing wind turbines with new turbines. New turbines are of higher rated power or higher efficiency. Such a reconstruction results in an increase in net electricity generated.

The use of wind energy has grown significantly in the world in recent times. The wind energy technology offered today has improved significantly. This is due to the increase in the size of the wind turbines. The process of forecasting and modeling wind resources is also improved. Better knowledge of the behavior of the wind results in an optimal wind farm design. Electricity generation costs have been reduced [3]. Wind power plants in the best locations are competitive with respect to conventional power generation technology.

Due to technological changes, there is a process of technological aging of wind turbines. At the same time, difficulties in finding new sites for the construction of wind turbines are visible. This situation causes interest in repowering. It is the process of replacing existing wind turbines with new turbines with higher rated power or greater efficiency. Replacing an old turbine with a new generation turbine results in an increase in net energy production. The

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repowering process also includes the modernization of the installation with an increase in power and efficiency [4].

However, making decisions requires conducting analyzes that will clearly indicate the legitimacy of the change [5]. The process of making decisions about repowering depends to a large extent on identifying potential benefits. One of the premises may be the estimation of the difference between the amount of energy produced by the current turbine and the newly planned one. Having a mathematical model of energy production by a new turbine makes it possible to indicate the benefits for the investor.

The physical wear of the turbine is also important for the repowering process. During the use of the wind turbine, the coatings of the turbine blades are destroyed and the resistance increases. These resistances change the power curve for the turbine. It is also possible to use the resulting models to analyze the wear of the turbine blades. Blade failure can also increase harmful noise and vibration levels [6, 7, 8, 9].

The aim of the manuscript is to present preliminary analytical research on the construction of a simple mathematical model that allows for the prediction of electricity production depending on the average wind strength. The model is built on the basis of historical data obtained from the Enercon SCADA program [10].

## 2 Methodology

The object of the analysis was the Wicko wind farm. It consists of five Enercon E-82 wind turbines. As part of the research, historical data was obtained and subjected to statistical analysis. The main goal of the research was to develop regression models. These models are to enable the determination of average daily amounts of electricity produced by wind turbines depending on the average daily wind speed. The designated models are based on historical values obtained from the SCADA system. Therefore, they are suitable for the analyzed wind turbine. This is due to the lack of analysis of other factors that may depend on the location of the wind turbine. Such factors include the roughness of the terrain, daytime temperature and other disturbing factors [11, 12, 13].

As part of the analyzes, two linear regression models were developed. In the first case, the influence of the average daily wind speed on the amount of energy produced during the month was analyzed. The analysis was carried out for the entire year 2019. For the second model, the impact of the average daily wind speed on the amount of energy produced during the week was analyzed. The analysis was carried out for the entire year 2019.

The designated regression models were developed on the basis of operational data obtained from the Enercon SCADA system. The actual data obtained concerned the average daily wind speed [ $\text{m s}^{-1}$ ] and the average daily electricity production [kWh] in the considered time period. The wind speed is obtained from a meter located on the nacelle of the wind turbine.

## 3 Analysis of the results

### 3.1 Linear regression model developed for individual months of 2019

The analysis was based on data obtained from the Enercon SCADA system. Table 3 shows the actual data for the considered wind turbine. For the construction of the model for individual months, the data was based on Average daily energy - column 2, Average daily wind speed - column 1. On their basis, a linear regression model was developed. It determines the dependence of the average daily amount of electricity produced on the average daily wind speed.

Formula (1) shows the linear regression function developed for individual months of 2019. Tables 1 and 2 present the statistics of the developed regression model.

$$E_m = 4398.12 v_m - 14404.94 \quad (1)$$

where:

$E_m$  – average daily energy [kWh],  
 $v_m$  – average daily wind speed [ $m\ s^{-1}$ ].

**Table 1.** Statistics of the linear regression model developed for individual months of 2019.

Coefficient	Value of the coefficient
Correlation coefficient R	0.9938
R <sup>2</sup>	0.9875
Adjusted R <sup>2</sup>	0.9863
Standard error	379.5719
Number of observations	12

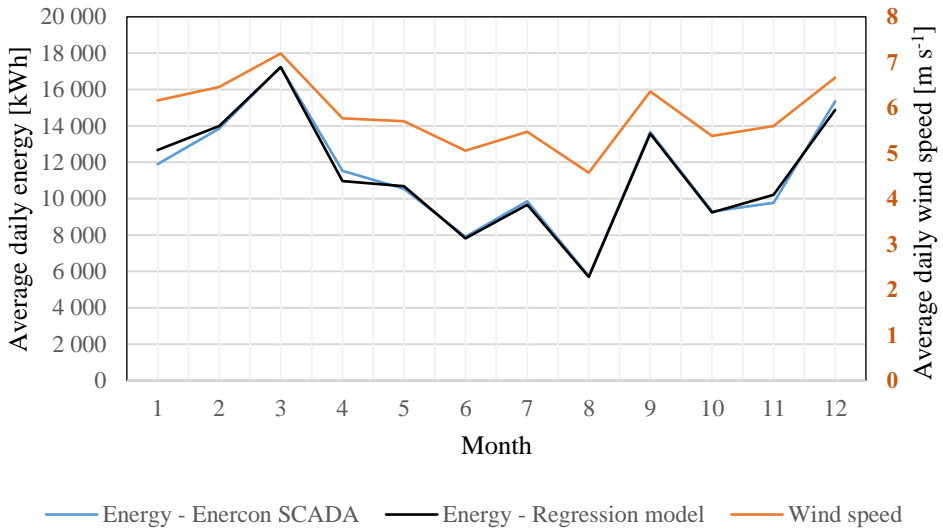
**Table 2.** Results of the statistical analysis of the coefficients of the linear regression function developed for individual months of 2019.

Coefficient	Value of the coefficient	Standard error	t Stat	p- Value	Lower 95%	Upper 95%
a	4 398.12	156.16	28.16	7.39892E-11	4 050.17	4 746.06
b	- 14 404.94	922.31	- 15.62	2.37016E-08	- 16 459.98	- 12 349.89

Then, for the linear regression model represented by the formula (1), the values of the average daily amount of electricity produced were determined depending on the average daily wind speed. The obtained results are presented in Table 3, column 4 (Average daily energy - Regression model) and in Figure 1.

**Table 3.** Average daily energy [kWh] generated by the Wicko wind farm in the following months of 2019 determined on the basis of real data (Enercon SCADA) and for the linear regression model.

Month	Average daily wind speed [ $m\ s^{-1}$ ] Enercon SCADA	Average daily energy [kWh] Enercon SCADA	Average daily energy [kWh] Regression model
1	6.16	11 894.92	12 680.13
2	6.46	13 855.39	13 994.33
3	7.19	17 246.77	17 233.13
4	5.77	11 526.37	10 972.20
5	5.71	10 540.13	10 692.71
6	5.05	7 899.03	7 820.21
7	5.47	9 850.87	9 657.02
8	4.57	5 740.19	5 698.71
9	6.36	13 665.17	13 567.09
10	5.38	9 299.77	9 245.58
11	5.60	9 775.40	10 209.86
12	6.66	15 354.97	14 878.01



**Fig. 1.** Average monthly energy [kWh] generated by the Wicko wind farm in the following months of 2019 determined on the basis of real data (Enercon SCADA) and for the linear regression model.

**3.2 Linear regression model developed for each week of 2019**

The weekly analysis was based on the data obtained from the Enercon SCADA system. Table 6 shows the actual data for the considered wind turbine. For the construction of the model for individual months, the data was based on Average daily energy - column 2, Average daily wind speed - column 1. On their basis, a linear regression model was developed. It determines the dependence of the average daily amount of electricity produced on the average daily wind speed.

Formula (2) presents the linear regression function developed for individual weeks of 2019. Tables 4 and 5 present the statistics of the developed regression model.

$$E_m = 5022.27 v_m - 17960.18 \tag{2}$$

where:

$E_m$  – average daily energy [kWh],  
 $v_m$  – average daily wind speed [m s<sup>-1</sup>].

**Table 4.** Statistics of the linear regression model developed for individual weeks of 2019.

Coefficient	Value of the coefficient
Correlation coefficient R	0.9832
R <sup>2</sup>	0.9667
Adjusted R <sup>2</sup>	0.9660
Standard error	1 255.5474
Number of observations	52

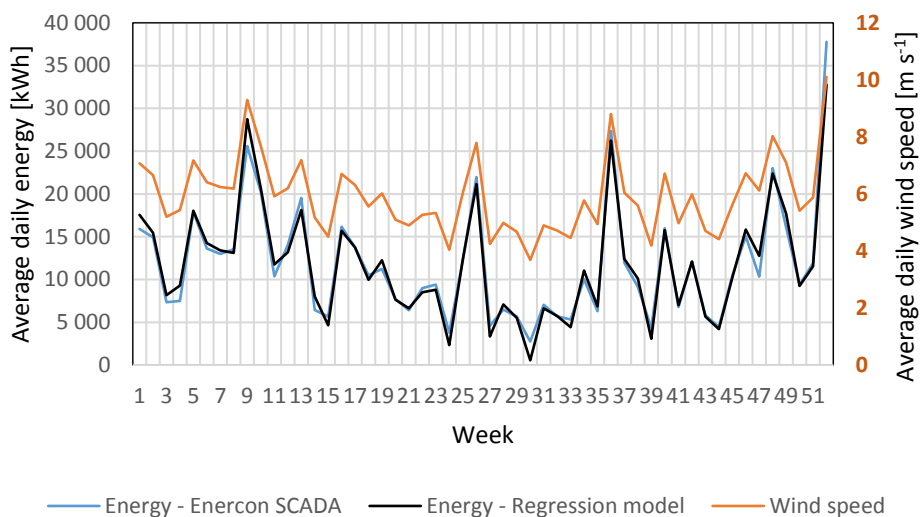
**Table 5.** Results of the statistical analysis of the coefficients of the linear regression function developed for individual weeks of 2019.

Coefficient	Value of the coefficient	Standard error	t Stat	p- Value	Lower 95%	Upper 95%
a	5 022.27	131.84	38.09	1.32128E-38	4 757.46	5287.08
b	- 17 960.18	800.38	-22.44	9.24282E-28	- 19 567.80	- 16 352.55

**Table 6.** Average daily energy [kWh] generated by the Wicko wind farm in the following weeks of 2019 determined on the basis of real data (Enercon SCADA) and for the linear regression model.

Week	Average daily wind speed [m/s] Enercon SCADA	Average daily energy [kWh] Enercon SCADA	Average daily energy [kWh] Regression model
(1)	(2)	(3)	(4)
1	7.07	15 906.86	17 554.45
2	6.66	14 940.29	15 473.79
3	5.20	7 357.86	8 155.63
4	5.43	7 486.71	9 303.58
5	7.17	17 892.43	18 056.67
6	6.41	13 610.00	14 254.10
7	6.24	12 996.14	13 393.14
8	6.19	13 497.71	13 106.15
9	9.30	25 598.43	28 746.93
10	7.67	20 186.57	20 567.81
11	5.91	10 382.29	11 742.96
12	6.20	14 029.43	13 177.90
13	7.19	19 536.00	18 128.42
(1)	(2)	(3)	(4)
14	5.17	6 455.57	8 012.13
15	4.50	5 675.29	4 640.04
16	6.70	16 180.71	15 689.03
17	6.31	13 728.29	13 751.87
18	5.56	10 491.00	9 949.30
19	6.01	11 237.86	12 245.19
20	5.10	7 709.86	7 653.40
21	4.90	6 390.86	6 648.95
22	5.27	8 969.71	8 514.36
23	5.33	9 407.57	8 801.35
24	4.04	3 786.71	2 344.14
25	6.06	12 364.29	12 460.43
26	7.79	21 969.29	21 141.78
27	4.24	4 603.00	3 348.60
28	4.99	6 480.29	7 079.43
29	4.67	5 675.00	5 501.00
30	3.69	2 746.00	550.48
31	4.90	7 058.00	6 648.95
32	4.71	5 688.00	5 716.24
33	4.46	5 329.86	4 424.80
34	5.77	10 044.43	11 025.50
35	4.94	6 303.29	6 864.19
36	8.80	27 346.14	26 235.80
37	6.04	11 929.29	12 388.68

38	5.59	9 132.29	10 092.79
39	4.19	4 356.29	3 061.61
40	6.71	15 990.14	15 760.78
41	4.97	6 767.71	7 007.68
42	5.99	12 105.00	12 101.70
43	4.70	5 806.14	5 644.49
44	4.41	4 466.14	4 209.56
45	5.61	10 428.86	10 236.28
46	6.73	15 034.14	15 832.53
47	6.11	10 325.14	12 747.42
48	8.03	23 015.43	22 361.48
49	7.10	16 091.00	17 697.94
50	5.41	9 383.43	9 231.83
51	5.87	11 915.00	11 527.72
52	10.10	37 768.00	32 764.75



**Fig. 2.** Average monthly energy [kWh] generated by the Wicko wind farm in the following weeks of 2019 determined on the basis of real data (Enercon SCADA) and for the linear regression model.

## 4 Summary

The research results presented above are the first stage of the work carried out, the aim of which is to develop a predictive method based on solutions in the field of artificial intelligence and expert knowledge. This method will be primarily useful for forecasting the volume of production as well as electricity demand, with particular emphasis on renewable energy sources. The proposed method will be developed with the use of predictive models that enable the analysis of time series (stationary and non-stationary), such as the ARMA and ARIMA models, as well as neural networks. The application of the developed method in practice will facilitate and accelerate the implementation of the decision-making process in electricity production systems, while reducing the risk of making a mistake (making wrong decisions). It is also possible to use the developed models to indicate the benefits of repowering.

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