MRT-LBM simulation of mixed convection in a horizontal channel heated from below by sinusoidal temperature profile

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Abstract. In this work, a numerical study is carried out to investigate the effect of Rayleigh number on mixed convection in a horizontal channel heated from below by sinusoidal temperature profile and cooled from above. The multiple relaxation time double population thermal lattice Boltzmann method (MRT-TLBM) is used. A validation of our code is done by comparing our results with those found in the literature. Particle oscillation amplitude and temperature profile are plotted for $Ra = 5 \cdot 10^3$, while the Reynolds number, the aspect ratio and Prandtl number are fixed at $Re = 10$, $B = 10$, $Pr = 0.667$ respectively. The effect of Rayleigh number on the heat transfer is also discussed.

Nomenclature

B	Aspect ratio
c	Microscopic velocity
F	Body force
f,g	Distribution function
H	Height of the domain
L	Length of the domain
Pr	Prandtl number
Ra	Rayleigh number
Ri	Richardson number
Re	Reynolds number
$\overline{Nu}$	Average Nusselt number
Nu	Local Nusselt number
M,N	Transformation matrices
m,n	Moment vectors
T	Dimensionless temperature
S	Diagonal relaxation matrix
t_T	Arbitrary dimensionless time
Ux, Uy	Dimensionless velocity components
u	Velocity vector
w	Weighting factor

Greek symbols

α	Thermal diffusivity
υ	Fluid kinematics viscosity
τ	Relaxation time
ψ	Stream function

Superscipt / Superscipts

eq	Equilibrium state
c	Cold
h	Hot
i	Direction for lattice
max	Maximum value

1 Introduction

The problem of mixed convection in rectangular channels with moving boundaries is of theoretical as well as of practical interest, for example, materials and metallurgical processing, electronic cooling, solar ponds, food processing and lakes [1].

Sivasankaran et al [2] did a numerical investigation of mixed convection in a square enclosure with sinusoidal boundary temperatures at both sidewalls with the presence of a magnetic field. They observed an increase in the heat transfer rate with the phase deviation up to $\pi/2$ and decreases afterward in the remaining range of these parameters.

The lattice Boltzmann method (LBM), originated from the lattice gas automata [3], is a powerful technique, which is used to simulate fluid flows, and associated transport phenomena. Various numerical simulations have been performed [4], using different models to investigate thermal systems [5]. There are different LBM models for fluid flow problems. The single relaxation time (SRT-LBM) method is a mesoscopic approach that solves the Boltzmann equation (BE) with the BGK approximation that simplifies the collision term with one relaxation time. Although it is widely used, LBGK is limited by numerical instability [6]. The approach used in this work is the multiple relaxation time double distribution function formulation (double population lattice Boltzmann method). It uses two different distribution functions; one for the flow field (velocity and density) and the other for the temperature field.

Sahraoui et al. [7] did a numerical simulation of mixed convection of helium ($Pr = 0.667$) in a horizontal duct using the thermal lattice Boltzmann method. The study was done for a Reynolds number of $Re = 10$, a Rayleigh number $Ra = 10000$ and an aspect ratio of $B = 20$.

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Dahani et al [8] did a lattice Boltzmann simulation of combined effects of radiation and mixed convection in a lid-driven cavity with cooling and heating by sinusoidal temperature profiles on one side. The single relaxation time double population method was used in this work. They found that for a given value of Reynolds number (Re) and Richardson number (Ri) varying in the range 0.01-1, the Nusselt number improves in the presence of radiation, and for a given value of Grashof number Ri decreases while the Nusselt number increases.

The objective of this paper is to investigate the effect of Rayleigh number on mixed convection in a horizontal channel heated from below by a sinusoidal temperature profile using the multiple relaxation time lattice Boltzmann method.

2 Problem statement

The investigated configuration shown in figure 1, consists of a two-dimensional channel with a length (L) and a height (H), whose upper wall is maintained at a cold temperature ($T_c = 0$) and the lower wall is subject to a sinusoidal spatial variation of the form $T(x) = T_c - \Delta T sin\left(\frac{2\pi x}{H}\right)$.
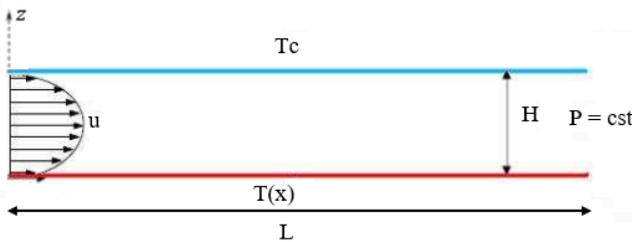

Fig. 1. Physical configuration

The fluid entering the channel, which is Helium (Pr = 0.667) in this case, is assumed to be Newtonian.
Rayleigh number was varied in the range $5.10^3 < Ra < 10^5$ while Reynolds number and aspect ratio were fixed at Re = 10 and B = 10 respectively.

3 Lattice Boltzmann approach

A multiple relaxation time lattice Boltzmann model for thermal fluids in two-dimensions is considered. The double population approach is used with two sets of independent distribution function; f for the flow field and g for the temperature field. The two-dimensional nine velocities model (D2Q9) is used for the flow field and the five-velocity model (D2Q5) is used for the temperature field.

The evolution equation is written in general as the following:

$$\mathbf{f}(x_j + c_i\delta t, t_n + \delta t) = \mathbf{f}(x_j, t_n) + \mathbf{Q}(x_j, t_n) + \mathbf{F}(x_j, t_n)$$

Where:

$$\mathbf{Q}(x_j, t_n) = -M^{-1}.S.[\mathbf{m}(x_j, t_n) - \mathbf{m}^{(eq)}(x_j, t_n)]$$

Where f is the distribution function for the thermal field and m are the corresponding moments. m^{eq} is the vector

of the equilibrium moments, F is the vector of external forcing and M the transformation matrix.
The ordering of the moment m is:

$$\mathbf{m} = (\rho, j_x, j_y, e, p_{xx}, p_{xy}, q_x, q_y, \varepsilon)$$

Where ρ is the mass density, $\mathbf{j} = \rho\mathbf{u}$ is the flow momentum and **u** is the flow velocity; e is the second-order moment corresponding to energy; P_{xx} and P_{xy} are two independent components of the stress tensor (diagonal and off-diagonal, respectively). Q_x and Q_y are the third-order moments corresponding to the x and y components of the energy flux, respectively; and ε is the fourth-order moment [9,10]. The diagonal relaxation matrix is given by:

$$S = diag\ (0,1,1, s_e, s_v, s_q, s_q, s_\varepsilon)$$

Where: $s_e, s_v, s_q, s_q, s_\varepsilon$ are the relaxation rates for the dynamic field.

For the temperature field, the D2Q5 model is used to simulate the advection–diffusion equation for the temperature:

$$\begin{aligned}\mathbf{g}(x_j + c_i\delta t, t_n + \delta t) \\ = \mathbf{g}(x_j, t_n) - N^{-1}.Q.[\mathbf{n}(x_j, t_n) \\ - \mathbf{n}^{eq}(x_j, t_n)]\end{aligned}$$

Where: g is the distribution function for the thermal field and n are the corresponding moments , n^{eq} are the equilibrium moments for the D2Q5 model, and N the transformation matrix that maps the distribution function $\{g_i|i =1, 2... 5\}$ to the corresponding moments $\{n_i|i =1, 2... 5\}$.

The macroscopic fluid variables density ρ and velocity $\boldsymbol{u}$ are obtained from the moments of the distribution functions as follows:

$$\rho = \sum_{i=1}^{9} f_i$$

$$\rho_0\boldsymbol{u} = \sum_{i=1}^{9} \boldsymbol{e}_i f_i$$

The temperature T is the sole conserved quantity in the D2Q5 model and it is given by:

$$T = \sum_{i=1}^{5} g_i$$

In addition to the open boundary conditions (OBC) used at the outlet [7, 11], the Zou and He boundary condition is applied [12] to impose a constant pressure at the outlet as well as a parabolic velocity profile at the inlet.
For the temperature field, the Dirichlet boundary condition was used for T_c and was realized by the anti-bounce back boundary condition:

$$g_{\bar{i}}(x_j, t_n + \delta t) = -g_i^*(x_j, t_n)$$

Where $g_{\bar{i}}$ corresponds to $c_{\bar{i}} = -c_i$, g_i^* denotes the post collision value of g_i.
The local Nusselt number was calculated using following expression:

$$Nu = \left|\frac{\partial T}{\partial n}\right|$$

The average Nusselt number was determined by integrating the local Nusselt number over the hot wall and is given by:

$$\overline{Nu} = \frac{1}{\frac{L}{H}} \int_0^{L/H} Nu \, dx$$

4 Validation

A code based on the double population multiple relaxation time thermal lattice Boltzmann was developed. The validation of this code is done by comparing our results with those of the literature, in the case of mixed convection in a horizontal channel heated uniformly from below. The parameters used in this test case are: an aspect ratio of B = 20, a Peclet number Pe = 20/3, a Reynolds number Re = 10 and a Rayleigh number Ra = 10^4.

**Table 1.** $\overline{Nu}$ comparison with the literature for B = 20, Re = 10, Ra = 10^4 and Pe = 20/3

Reference	Present	Sahraoui et al[7]	Evans et al [11]	Abassi et al[13]
$\overline{Nu}$	2.487	2.550	2.558	2.536

The comparison of the obtained results with other references is shown in Table 1; demonstrate good agreement of our numerical code, where the maximum error is 2.77% for [11].

5 Results and discussion

The oscillation amplitude of a particle on the horizontal mid-plan (for Ra = 5.10^3) is shown in figure 2, at a time t_T which correspond to a minimum in the temperature at the mid-outlet of the duct. We like to point out that the rest of the plots reported in this work are obtained at this value of time.
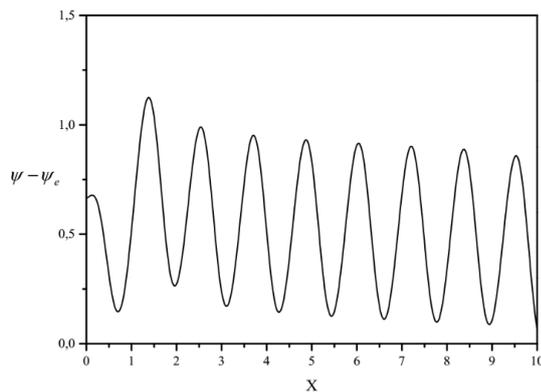
Fig. 2. Oscillation amplitude of a particle on the horizontal mid-plan (for Ra = 5.10^3)

A close examination of the plot shows that the particle displacement oscillates in an asymptotic state, which reveals the periodic and unstable nature of the flow.

The examination of isotherms contours, shown in figure 3, for different Rayleigh numbers, reveals the apparition of plumes carrying the heat to the center of the channel, which is driven to the outlet by the cold fluid entering the channel.
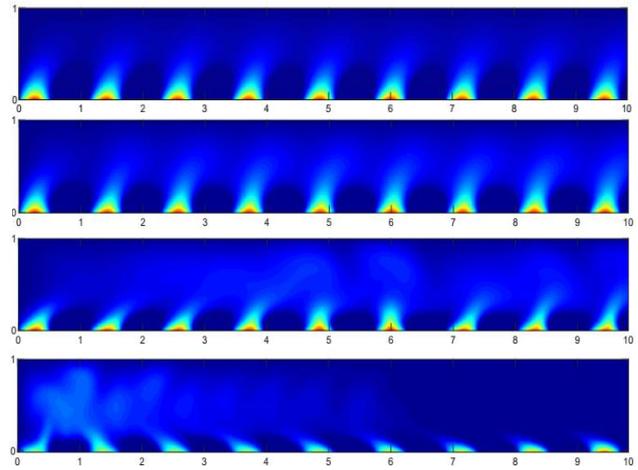

Fig. 3. Isotherms contours for Ra = $5.10^3, 10^4, 5.10^4$ and 10^5

When increasing the Rayleigh number, the convective heat transfer becomes more significant and the plume region becomes more obvious.

The temperature profile on the horizontal mid-plan (for Ra = 5.10^3) is illustrated in figure 4. We notice that the fluid temperature increases from the entrance until X = 0.5, which represents the first contact between the cold fluid entering the channel and the hot fluid coming from the bottom wall, after a certain distance the profile becomes periodic due to the sinusoidal temperature profile at the bottom wall.

The effect of the Rayleigh number on heat transfer is illustrated in figure 5. It can be seen that the average Nusselt number increases with the increase of the Rayleigh number, which can be explained by the progressive predominance of the natural convection over the forced convection on the mixed convention phenomenon.
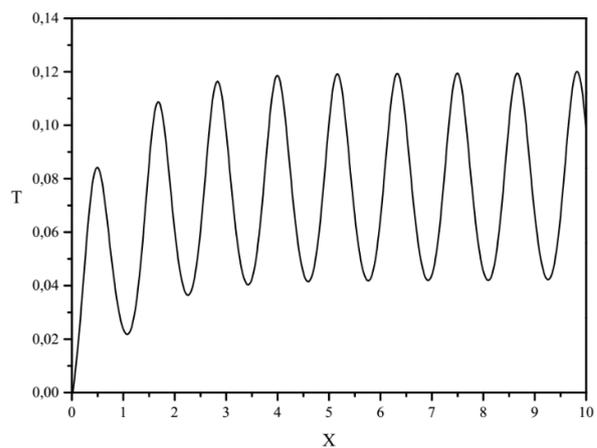
Fig. 4. Temperature profile on the horizontal mid-plan (for Ra = 5.10^3)
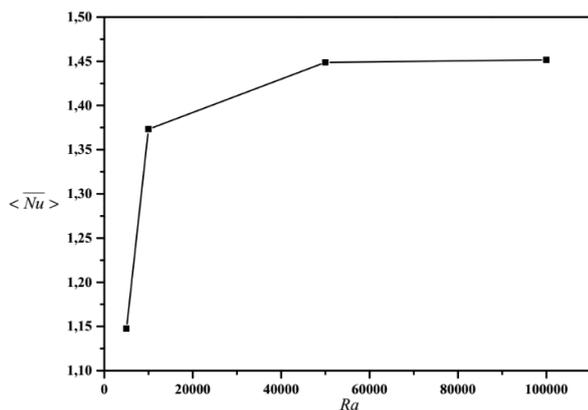
Fig. 5. Average Nusselt number variation on the bottom wall for Ra = 5.10^3 to 10^5

6 Conclusion

The effect of Rayleigh number on the mixed convection in a horizontal channel heated from below by a sinusoidal temperature profile was investigated using the thermal lattice Boltzmann method. The double population multiple relaxation time lattice Boltzmann method was applied to simulate fluid flow and heat transfer. A validation of our code was done, that demonstrated good agreement between the results obtained with our code and the results found on the literature. The results expressed in terms of isotherms, particle oscillation amplitude, temperature profile and average Nusselt number, reveal the effect of the Rayleigh number on the heat transfer and show that the latter is enhanced when the Rayleigh number increases, in the particular case of the constant parameters : Re = 10, B = 10 and Pr = 0.667.

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