Numerical study of the coupled natural convection with surface radiation in a cylindrical annular enclosure

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Abstract. The interaction of natural convection with thermal radiation of black surfaces in a cylindrical enclosure filled with air has been numerically investigated. The steady-state continuity, Navier–Stokes and energy equations were discretized using the control volume method and solved numerically via the SIMPLER algorithm. Effects of Rayleigh number (Ra), wall emissivity (ϵ_p) and height ratio parameter (X) are studied. The result shows that surface radiation significantly altered the temperature distribution and the flow patterns, especially at higher Rayleigh numbers. The total average Nusselt number has also been discussed for valuating heat transfer through the enclosure.

1 Introduction

Coupled natural convection and radiation transport processes find applications in many engineering problems, such as solar energy collectors, cooling of electronic devices and double windows. The effect of radiation on combined heat transfer with convection or conduction is more important, especially with the presence of a participating medium and/or radiative surface with large emissivities [1-8]. In this case, the energy equation provides the local temperature which determines the blackbody intensity in the radiative transfer equation (RTE). Furthermore, solving the RTE gives the divergence of the radiative flux that is a source term in the energy equation.

Several earlier studies have been carried out for rectangular enclosures differentially heated at the sidewalls [9, 11], but the focus of this paper is on square enclosures heated from below and cooled from above having walls with different emissivities. While modeling the interaction of natural convection and surface radiation in rectangular enclosures, most of the numerical studies previously reported in the literature have considered one of the vertical walls heated and the other one cooled with horizontal insulated walls. Lari K et al [12] studied numerically the coupled Natural convection with thermal surface radiation in square enclosure. They have founded that the surface radiation

modifies the temperature and velocity fields and affect the heat transfer. Webb and Viskanta [13] investigated the heat and momentum transfer by natural convection and radiation in a semitransparent fluid medium inside a rectangular cavity having one vertical transparent wall. By using air as a fluid medium, Balaji and Venkateshan [14] showed that surface radiation leads to a drop in the convective component. However, this reduction tends to be compensated by the radiative transfer that takes place between the hot and cold walls. Hence, the net effect of radiation is to increase the overall heat transfer across the enclosure when the emissivity of the hot wall is greater than 0.229. At this value of the emissivity, radiation is absent, since its “dual” effect canceled each other. Two years later, the same authors [15] generated general correlations for convection and radiation Nusselt numbers based on the numerical calculations of the coupled problem accounting for different emissivities. Good agreement between the data and the correlations has been found resulting in a maximum error of 6%. Akiyama and Chong [16] studied the problem of a square enclosure with gray surfaces filled with air. Their results demonstrated that the surface radiation has an important influence on the temperature distribution and flow patterns especially at high Rayleigh numbers (Ra). More recently, Colomer et al. [17] analyzed the natural convection phenomenon coupled with radiant exchange in a three-dimensional differentially heated cavity.

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Influence of convection and radiation on the thermal environment in an industrial building has been investigated (using a commercial computational fluid dynamics package FLUENT) by Wang et al. [18]. It has been found that radiation modified the temperature distribution and airflow through secondary convection near the sidewalls of the industrial building. Also, accurately predicting the total Nusselt number is very important to provide a comfortable thermal environment in buildings. Martyushev and Sheremet [19, 20] have analyzed numerically natural convection combined with surface thermal radiation in a square and cubical enclosures bounded by solid walls of finite thickness and conductivity with a heat source. Medebber, M. A et al [21] have studied numerically the free convection in vertical annular enclosure without radiation. They have founded that the height variation ratio altered the temperature distribution and velocity.

2 Mathematical Formulation

Convective heat transfer can be expressed mathematically by the equation of conservation of mass, the Navier-Stokes equations, and the energy equation. With above-mentioned assumptions, the equations can be written in dimensionless form as follows:

$$\frac{\partial(RU)}{R\partial R} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial R} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial R} + \frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} - \frac{U}{R^2} + \frac{\partial^2 U}{\partial Y^2} \quad (2)$$

$$U \frac{\partial V}{\partial R} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\partial^2 V}{\partial Y^2} + \frac{1}{R} \frac{\partial V}{\partial R} + \frac{\partial^2 V}{\partial R^2} + \frac{R_{aT}}{Pr} \theta \quad (3)$$

$$U \frac{\partial \theta}{\partial R} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \left(\frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R} \frac{\partial \theta}{\partial R} + \frac{\partial^2 \theta}{\partial Y^2} - \frac{\tau_0}{N_r} Q_R \right) \quad (4)$$

The dimensionless parameters are defined as follows:

$$(R, Y) = \frac{(r, y)}{L_r} \quad (U, V) = \frac{(u, v)L_r}{v} \quad P = \frac{pL_r^2}{\rho v^2}$$

$$\theta = \frac{(2T - T_c - T_f)}{2(T_c - T_f)}$$

$$A = \frac{H}{L_r} \quad a = \frac{h}{L_r} \quad X = \frac{h}{H} \quad K = \frac{r_e}{r_i}$$

$$Ra = \frac{g\beta\Delta TL_r^3}{v\alpha}, \quad i(R, Y, \vec{\Omega}) = \frac{I(r, y, \vec{\Omega})}{4\sigma T_0^4}, \quad Pr = \frac{v}{\alpha}$$

Fig. 1. Physical system.

2.2. Boundary Conditions

Boundary conditions are stated as follows:

$$R = \frac{1}{K-1} \quad \theta = \frac{1}{2} \quad i_{wall_1} = \left(1 + \frac{\theta_w}{\theta_0}\right)^4 \cdot \epsilon_w$$

$$U=V=0$$

$$R = \frac{K}{K-1} \quad \theta = -\frac{1}{2}$$

$$i_{wall_2} = \left(1 + \frac{\theta_w}{\theta_0}\right)^4 \cdot \epsilon_w \quad U=V=0 \quad (5)$$

$$Y=0, A \quad \frac{\partial \theta}{\partial Y} = 0$$

$$i_{wall_{3,4}} = \left(1 + \frac{\theta_w}{\theta_0}\right)^4 \epsilon_w + (1 - \epsilon_w) Q_n^{Inc}$$

$$U=V=0$$

$$Y=XA \text{ et } 0 \leq R \leq \frac{1}{K-1} \quad \theta = \frac{1}{2}$$

$$i_{wall_2} = \left(1 + \frac{\theta_w}{\theta_0}\right)^4 \cdot \epsilon_w \quad U=V=0$$

2.3. Heat Transfer

To determine the steady state heat transfer characteristics at either of the two vertical walls, contributions of both convection and radiation should be taken into consideration. In the present study the total average Nusselt number ($Nu=Nu_c+Nu_R$) is introduced as:

$$Nu_c = \frac{-\lambda \frac{\partial T}{\partial r} L_r}{\lambda \Delta T} = -\frac{\partial \theta}{\partial R} \quad (6)$$

$$\overline{Nu}_c = \frac{1}{A} \int_0^A Nu(R=1, Y) dY \quad (7)$$

$$Nu_R = q_R \frac{L_r}{\lambda \Delta T}$$

$$\overline{Nu}_R = \frac{1}{A} \int_0^A Nu_R(R=1, Y) dY \quad (8)$$

2.4. Validation

The numerical study is validated with numerical results in the literature by Dua and Cheng [26]. The results presented above are applied to axisymmetric radiative transfer through orthogonal cylindrical enclosure with absorbing-emitting medium. The comparison of nondimensional wall heat $(q_{wR}^R / \sigma T_{ref}^4)$ between present numerical study and available results in the literature are shown in Fig. 2. All bounding walls are cold ($T_w=0$ K) and black ($\epsilon_w=1$), whereas the enclosed medium is hot ($T_w=100$ K) and has three absorption coefficients of 0.1, 1, and 5 m^{-1} . The numerical investigation has good agreement between the numerical and the literature results

Fig. 2. Comparison of nondimensional radial heat aux distribution for three optical thicknesses.
 (a) $\kappa_a=5.0$ m^{-1} , (b) $\kappa_a=1.0$ m^{-1} , and (c) $\kappa_a=0.1$ m^{-1}

(a) $X=0.0$

(b) $X=0.25$

(c) $X=0.75$

(d) $X=1.0$

Fig. 3. Effects of Rayleigh number on the isotherm and streamline for $(\epsilon_1=\epsilon_2=\epsilon_3=\epsilon_4=1, Pl=0.02)$

(a) $X=0.0$, (b) $X=0.25$, (c) $X=0.75$, (d) $X=1.0$

3 Results and Discussion

In the present work, a parametric study was conducted and computations were carried out for a wide range of: $10^3 \leq Ra \leq 10^6$, $0 \leq X \leq 1$, $A=1$, $Pr=0.7$ and $K=2$. The heat results along with isotherms and flow fields have been also obtained.

3.1. Effect of the Height Ratio (X)

The influence of height ratio (X) on the isotherms and streamlines is reported in Fig. 5. The values of Rayleigh number, Prandtl number, and radius ratio are, respectively, fixed at $Ra=10^6$, $Pr=0.7$, and $K=2$, while four different height ratio ($X=0.0, 0.25, 0.75$ and 1) are considered. For $X=0$, a significant vertical thermal gradient adjacent to the horizontal heater surface of the inner cylinder is observed. The fluid is rising along the horizontal hot wall, moving toward the outer wall, and then the flow descends down along the cold wall as a plume. For the adiabatic walls, the isotherms are not orthogonal to the horizontal walls when radiation is present. The corresponding streamlines form a single cellular pattern filling the entire annular space. When height ratio increase for $X=0.25$, we observe a significant heat transfer in the upper region of the inner cylinder ($0 \leq R \leq 1$) due to the flow acceleration in the upper region of the hot inner cylinder.

For $X \geq 0.75$, the fluid motion is fully accelerated in the boundary layers near the vertical wall of the annulus gap ($1 \leq R \leq 2$), and it is stagnated at above of the inner cylinder $0 \leq R \leq 1$.

3.2. Heat transfer

In the case of the interaction of convection and thermal radiation, the overall Nusselt number Nu is a function of Ra (Figure 6) and other parameters pertaining to radiation. The appearance of radiation raises Nu and. Convection becomes the dominant mode with the increase of Ra . Radiation weakens the temperature gradient near the walls, so Nu , becomes smaller at higher Ra .

Fig. 4. Variation of the total average Nusselt number according to Rayleigh number for various values of X

4. Conclusions

The combined natural convection with radiation by black surfaces in a cylindrical annular cavity filled with air has been numerically investigated. The effect of radiation on the flow field, temperature distribution, and heat transfer is predicted. The appearance of surface radiation altered

the temperature distribution in the vicinity of insulated walls so that the temperature and the flow field in the enclosure are affected. The thermal boundaries near the side walls are relatively weakened by surface radiation. In addition, the heat transfer raise with increasing of height ratio X . The average convection Nusselt number increases with the increase of Ra . The presence of surface radiation can change the value of average convection Nusselt number, but only little variation can be observed with the black surface ($\epsilon_w=1$). On the other hand, the average radiation Nusselt number rises quickly with the increase of emissivity ($\epsilon_w=1$), and radiation heat transfer plays an important part in overall heat flux at larger emissivity.

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