Non-linear discrete model of BLDC motor for studying the range of permissible values of the voltage vector in the state space

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Abstract. A nonlinear discrete model of a brushless direct current (BLDC) motor in a rotating coordinate system with vector control was developed. The synthesis of optimal control in the state space was performed using the Riccati equation. For practical problems of BLDC motor control, the maximum values of voltages and currents are found while maintaining the identifiability of the BLDC motor model. The study of the range of admissible values of the voltage vector in the state space for the obtained nonlinear discrete model of the BLDC motor is carried out.

1 Introduction

There are known criteria of controllability, observability and identifiability, which are considered in the classical works [1–9]. On the control of robot drives, articles and monographs have been published [10–15], in which frequency and vector control based on continuous and discrete drive models are considered. In this article, a nonlinear discrete model of a BLDC motor is developed with optimal control, taking into account the restrictions on the maximum values of current and voltage.

When developing the BLDC motor simulation program, a linear-quadratic controller was used, one of the types of optimal controllers that uses a quadratic quality functional. When synthesizing control systems that are optimal according to the quadratic quality criterion, the matrix algebraic Riccati equation is solved.

2 Development of a mathematical model of BLDC motor

The BLDC motor model is needed to identify the drive in real time for various desired angular displacements, speeds and accelerations in the state space under vector control and to calculate the maximum values of electric current and voltage under dynamic operating conditions.

The BLDC motor model in the state space is presented in the following form:

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$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \tag{1}$$

where $\dot{\mathbf{x}}$ is the time derivative of the state vector of the BLDC motor;
A is a matrix of the state of the BLDC motor;
x is a vector of state of the BLDC motor;
B is a matrix of control BLDC motor;
u is the control vector of the BLDC motor.

In general, when at least one of matrices **A**, **B** are time-dependent, the problem is nonlinear and has only particular solutions. To find the equation of state, we represent equations (1) in discrete form, with the sampling time T tending to zero, and the trajectory at each discrete section is linear.

Let us write down the solution for the nonlinear problem in discrete form, when the matrices $\mathbf{A}_k$, $\mathbf{B}_k$ are constant at time moments k, $k = 0,1,2,3, ...$

$$\frac{\mathbf{x}_{k+1} - \mathbf{x}_k}{T} = \mathbf{A}_k\mathbf{x}_k + \mathbf{B}_k\mathbf{u}_k, \tag{2}$$

or

$$\mathbf{x}_{k+1} = \widetilde{\mathbf{A}}_k\mathbf{x}_k + \widetilde{\mathbf{B}}_k\mathbf{u}_k, \tag{3}$$

where

$$\widetilde{\mathbf{A}}_k = T\mathbf{A} + \mathbf{E},$$

$$\widetilde{\mathbf{B}}_k = T\mathbf{B}_k.$$

This equation connects the transition of the system from the state $\mathbf{x}_k$ to the state $\mathbf{x}_{k+1}$. On the time interval T, we take the values of matrices $\mathbf{A}_k$, $\mathbf{B}_k$ constant. For convenience, we will remove the “wavy line” sign in subsequent entries.

The optimal solution for (3) will be in [2,3]:

$$\boldsymbol{u_k = -G_k^{-1}B_k^T K_k x_k} \tag{4}$$

where $\mathbf{K}_k$ is the Cauchy matrix, which can be found by solving the Riccati equation [2,3]:

$$\boldsymbol{-\dot{K} = Q + A^T K + K^T A - K^T B G^{-1} B^T K}, \tag{5}$$

where **Q** and **G** are positive arbitrarily defined matrices. The matrices **Q** and **G** are chosen positive definite, since affect only the scale of the solution. We will select these matrices by selection or simulation [2,3].

The quality criterion that minimizes the energy of control and displacement, in this case, is a quadratic form:

$$\boldsymbol{I = \frac{1}{2}\int_{t_0}^{t_f} (x^T Q x + u^T G u)\, dt, Q \geq 0, G > 0}, \tag{6}$$

A model of the BLDC motor based on differential equations has been developed. The electrical part of the BLDC motor model is described by a system of equations associated with the rotor:

$$\boldsymbol{\frac{d}{dt} i_d = -\frac{R}{L_d} i_d + \frac{L_q}{L_d} p\omega i_q + \frac{1}{L_d} U_d} \tag{7}$$

$$\boldsymbol{\frac{d}{dt} i_q = -\frac{R}{L_q} i_q - \frac{L_d}{L_q} p\omega i_d - \frac{\psi}{L_q} p\omega + \frac{1}{L_q} U_q} \tag{8}$$

$$\boldsymbol{M_e = 3p\psi i_q/2 + (L_d - L_q) p i_d i_q} \tag{9}$$

where L_q, L_d are stator inductance along the q and d axes;
R is the active resistance of the stator winding;
i_q, i_d are projections of the stator current on the q and d axes;
U_q, U_d are stator voltage projections on the q and d axes;
ω is the angular speed of the rotor;
ψ is the magnetic flux induced by permanent magnets in the stator winding;
p is the number of pole pairs;
M_e is the electromagnetic moment of the BLDC motor.

Provided L_q=L_d

$$\frac{d}{dt}i_d = -\frac{R}{L_d}i_d + p\omega i_q + \frac{1}{L}U_d \tag{10}$$

$$\frac{d}{dt}i_q = -p\omega i_d - \frac{R}{L_q}i_q - \frac{\psi}{L}p\omega + \frac{1}{L}U_q \tag{11}$$

$$M_e = 3p\psi i_q/2 \tag{12}$$

The mechanical part of the BLDC motor model is described by a system of equations:

$$\frac{d}{dt}\omega = \frac{1}{J}(M_e - F\omega - M_L) \tag{13}$$

$$\frac{d}{dt}\theta = \omega \tag{14}$$

where ω is the angular speed of the BLDC motor rotor;
J is the total moment of inertia of the rotor and load;
$F$is the coefficient of viscous friction of the rotor and the load;
θ is the angle of the BLDC motor rotor position;
M_e is the electromagnetic moment of the BLDC motor;
M_L is the moment of the load.

An analog and discrete model of a BLDC motor has been developed, where a variable load torque is present in the state matrix:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{i}_d \\ \dot{i}_q \\ \dot{\omega} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & p\omega & 0 & 0 \\ -p\omega & -\frac{R}{L} & -\frac{K_m}{L} & 0 \\ 0 & \frac{K_m}{J} & -\frac{F}{J}-\frac{M_L}{J\omega} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \omega \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_d \\ u_q \end{bmatrix} \tag{15}$$

$$\mathbf{x}(k+1) = \begin{bmatrix} i_d(k+1) \\ i_q(k+1) \\ \omega(k+1) \\ \theta(k+1) \end{bmatrix} = \begin{bmatrix} 1-T\frac{R}{L} & Tp\omega(k) & 0 & 0 \\ -Tp\omega & 1-T\frac{R}{L} & -T\frac{K_m}{L} & 0 \\ 0 & T\frac{K_m}{J} & 1-T\frac{F}{J}-T\frac{M_L(k)}{J\omega(k)} & 0 \\ 0 & 0 & T & 1 \end{bmatrix} \begin{bmatrix} i_d(k) \\ i_q(k) \\ \omega(k) \\ \theta(k) \end{bmatrix} + \begin{bmatrix} \frac{T}{L} & 0 \\ 0 & \frac{T}{L} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_d(k) \\ u_q(k) \end{bmatrix} \tag{16}$$

where $i_d(k), i_q(k), \omega(k), \theta(k)$ are the measured value of currents, angular velocity and displacement;
$\omega(k+1)$ is the planned value of the angular speed;
$\theta(k+1)$ is the planned value of the angular position of the motor shaft;

T - sampling interval, time between *k*+1 and *k* samples;
R is the active resistance of the stator winding of the BLDC motor;
L_q, L_d are inductance of the BLDC motor stator along the q and d axes;
p is the number of pairs of poles of the BLDC motor;
ψ is the magnetic flux induced by permanent magnets in the stator winding;
F is coefficient of viscous friction in the BLDC motor;
J is moment of inertia;
M(*k*) is the electromagnetic moment of the BLDC motor;
$i_d(k+1), i_q(k+1), \omega(k+1), \theta(k+1)$ are projection of the stator current on the d-axis, projection of the stator current on the q axis, the angular velocity, angular displacement of the BLDC motor at time *k*+1;
U_q, U_d are stator voltage projections on the q and d axes.

The electromagnetic moment of the BLDC motor is calculated by the formula

$$\boldsymbol{M_L(k)} = \frac{\boldsymbol{\omega(k+1)-\omega(k)}}{\boldsymbol{T}}\boldsymbol{J}, \tag{17}$$

$$\boldsymbol{\omega(k+1)} = \frac{\boldsymbol{\theta(k+1)-\theta(k)}}{\boldsymbol{T}}. \tag{18}$$

The rotation speed ω(*k*+1) in (18) denotes the target point on the trajectory, which makes it possible to calculate the required torque at each control step using formula (17), which we substitute into the state matrix (16).

The study of the influence on the identifiability of the control model, i.e. compliance of the control model with the control object by specifying the trajectory errors and state measurement errors. The criterion for the identifiability of the BLDC motor model is the rank of the extended matrix, which determines the identifiability in the theoretical sense

$$\boldsymbol{min}\,\mathbf{det}\left[\mathbf{C}_k^\mathbf{T} \vdots \mathbf{A}_k^\mathbf{T}\mathbf{C}_k^\mathbf{T} \vdots \left(\mathbf{A}_k^\mathbf{T}\right)^2\mathbf{C}_k^\mathbf{T} \vdots \left(\mathbf{A}_k^\mathbf{T}\right)^3\mathbf{C}_k^\mathbf{T}\right] > \boldsymbol{\gamma}, \tag{19}$$

where $\mathbf{C}_k^T$ is the transported measurement matrix, taking into account the accuracy class of the sensors;

γ - threshold value of determinants determined by the identification object and close to zero.

3 BLDC motor simulation

The modeling of the BLDC motor was carried out with the parameters indicated in table 1 in MatLab software.

The vector current regulator is constructed in a rotating coordinate system *dq* and consists of controllers *d* and *q* projections. With the help of the q-component of the current, in accordance with the formula (12), the required torque is provided, and the *d*-component is maintained equal to zero, which ensures the desired orientation of the current vector. As a feedback, the regulator uses the real stator current vector measured and converted into the *dq* system.

The optimal stator voltage vector is calculated based on the Riccati equation (4). Then, using coordinate transformations, the stator voltage vector is translated into a fixed coordinate system associated with the stator, where it is implemented using Pulse-width modulation (PWM).

Table 1. Technical characteristics of JK42BLS01 BLDC motor.

BLDC motor technical characteristics	Parameter value
Weight, kg	0.3
Length, mm	41
Diameter, mm	20
Supply voltage, V	24
Winding resistance, Ohm	19
Winding inductance, H	0.0018
Idling speed, rpm	7400
No-load current, A	0.05
Rated current, A	1.8
Rated moment, N·m	0.0625
Moment of inertia, kg·m^2	0.0000024
Constant torque coefficient, N/A	0.039
Constant back-EMF coefficient, V/rpm	0.0041
Power, W	26
Rated angular speed, rpm	4000
Number of poles	8
Magnetic flux,Wb	0.001

The dependence of the control energy on the resistance of the motor winding is shown in the figure 1.

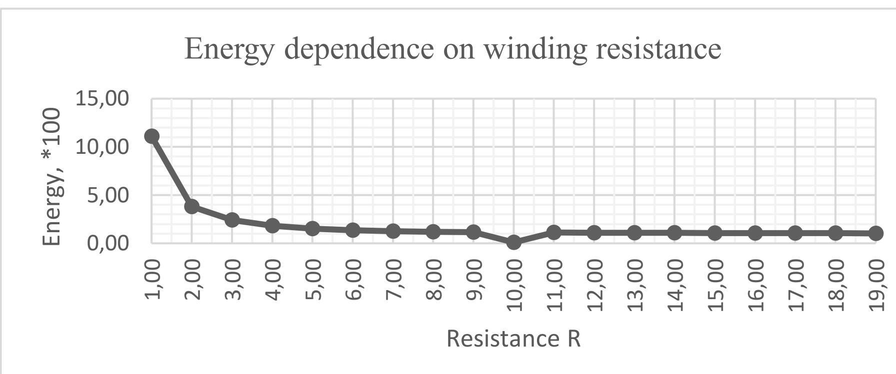

Fig. 1. Dependence of the control energy on the resistance of the motor winding.

With an increase in the angular velocity, the amplitude of the stator current of the BLDC motor is limited by the growth of the EMF generated by the rotor. Consequently, with an increase in the angular velocity, the maximum torque attainable by the engine is also limited.

The maximum possible amplitude of the stator voltage vector is limited by the capabilities of the power converter provided by the U_{smax}.

The maximum limiting possible value of the BLDC motor torque depending on the angular speed [16].

$$\mathbf{U}_{smax}^2 = I_q^2 \omega^2 L_q^2 + (E + I_q R_s)^2 \tag{20}$$

We transform this record and get a quadratic equation for the stator current.

Solving it with respect to the current I_q, we get the maximum value of I_{qmax}.

The torque is limited at the level of a certain overload torque, which is selected based on the required dynamics of the BLDC motor and the permissible current I_q of the power converter and the motor:

$$\boldsymbol{M_{max}} = \frac{3}{2}\boldsymbol{Z_p I_{qmax}\psi} \tag{21}$$

The simulation of the optimal discrete control of the BLDC motor with the matrix $\mathbf{A}_k$, k=0,1,2,...N, with an increase in the torque up to the overload torque within the limits of the permissible current and possible acceleration is carried out. The simulation result is the optimal voltage (4). At each stage of enumerating the moment for a fixed angular speed of rotation of the shaft and for the maximum moment, we obtain the values U_q and U_d, according to which we calculate the function of the minimum determinants of the extended matrix (16):

$$\mathbf{f}(\boldsymbol{U_q}, \boldsymbol{U_d}|\boldsymbol{M}, \boldsymbol{\omega}) = \mathbf{mindet}\left[\mathbf{C}_k^{\mathrm{T}} \vdots \mathbf{A}_k^{\mathrm{T}}\mathbf{C}_k^{\mathrm{T}} \vdots \left(\mathbf{A}_k^{\mathrm{T}}\right)^2\mathbf{C}_k^{\mathrm{T}} \vdots \left(\mathbf{A}_k^{\mathrm{T}}\right)^3\mathbf{C}_k^{\mathrm{T}}\right](\boldsymbol{U_q}, \boldsymbol{U_d}|(\pm\Delta\boldsymbol{Mn} < |\boldsymbol{M_{max}}|), \Delta\boldsymbol{\omega k}),$$

$$\boldsymbol{k} = \mathbf{0}, \mathbf{1}, \mathbf{2}, ..; \boldsymbol{n} = \mathbf{0}, \mathbf{1}, \mathbf{2}, ... \tag{22}$$

4 Conclusion

The article has developed a nonlinear discrete model of the BLDC motor in matrix-vector form based on optimal control. The state matrix takes into account the torque that must be provided by the diagnostics. For practical problems of BLDC motor control, the maximum values of voltages and currents are found while maintaining the identifiability of the BLDC motor model. The study of the range of admissible values of the stress vector in the state space for the obtained nonlinear discrete model of the BLDC motor is carried out.

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