Numerical simulation of conjugated heat exchange in the turbulent motion of fluid in an oil well

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Abstract. The mathematical model of conjugate heat exchange is proposed for the turbulent movement of reservoir oil in a vertical well section. The motion of the medium is described using a two-dimensional axisymmetric stationary formulation and boundary layer equations. The movement of the turbulent flow of reservoir oil due to reservoir energy is presented. The liquid medium is a mixture of reservoir oil with dissolved gas and formation water. The results of the numerical modeling are presented in the form of dependences of the changing flow rate, temperature, and mass fraction of paraffin deposits that occur along the full vertical extent of the well. The results obtained describe the thermobaric state of the well under the condition of conjugate heat exchange between the fluid flow and the production pipe.

1 Introduction

Much attention is paid to the problem of the uninterrupted operation of oil wells. In many respects, the uninterrupted operation of wells is determined by possible deposits of oil components - paraffin deposits - on the inner wall of the well. The presence of paraffins in the extracted oil depends on many factors and can reach 70% of the mass of the transported fluid. Initially, paraffin is dissolved in the oil. This is due to the high temperature in the zone of productive layers, the temperature of which is higher than the melting point of paraffin. However, with further movement of the fluid, the temperature decreases along the well and paraffin deposits are formed; paraffin is mostly deposited on the inner wall of the well. When paraffin is deposited on the inner wall of the well, the flow section of the well decreases and the well flow rate decreases as well. Thus, the problem of determining the intensity of the occurrence of paraffin deposits in a well is an important scientific and technical task.

It is known that the intensity of the formation of the paraffin deposits largely depends on the hydrodynamic and thermal parameters of the extraction process. In this regard, modern research is related to identifying criteria for the assessment of the influence of temperature and hydrodynamic conditions on the well's operating conditions.

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Such examples are the studies of operating modes of high-temperature wells [1], and the studies of the paraffin crystallization with an isobaric decrease in the well temperature [2]. Mathematical modeling methods are widely used [3] for the ‘well – formation’ system, model problems are solved as stationary [4] and non-stationary [5] problems. Asymptotic methods for determining a temperature field are developed taking into account the hydrodynamic regime of the fluid flow in a well [6, 7]. However, in most cases, when problems are set and solved, empirical relations for calculating a heat transfer coefficient are used as closing dependencies in the system of energy equations [8]. In [9, 10], the solutions are presented for the problems of the conjugate heat transfer in the motion of laminar and turbulent flows of media. When studying the flow of fluids in a well, many mathematical models are constructed with the assumption of the constancy of the velocity and temperature profile along the well cross-section, which distorts the real picture of the process. In this regard, it is important to conduct numerical parametric studies of the temperature distribution in the ‘well - reservoir’ system in the conditions of conjugate heat exchange between the turbulent flow of a fluid and a well.

The aim of the present work is to construct a mathematical model of conjugate heat exchange during the turbulent movement of a fluid in an oil well and to estimate the intensity of the paraffin deposit formation.

2 Problem statement

Figure 1 shows the design diagram of a well section. It is presented as a vertical tube of length \( L \), internal radius \( r_1 \) and external radius \( r_2 \). The reservoir conditions outside the pipe are known: the reservoir pressure \( P_{pl} \) and temperature \( T_{pl} \). The temperature of the outer wall of the pipe \( T_m \), through which the oil fluid moves, is equal to the ambient temperature \( T_{pl} \) and is determined by geothermal conditions. The heat exchange between the hot fluid and the well penetrated strata occurs through the metal wall of the pipe. The moving flow of the medium has the following initial values: pressure \( p_l \), temperature \( T_l \), flow rate \( \rho u_l \), and thermal properties.

![Design diagram of the production section of the well](image_url)
3 Mathematical model

\[ \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = -g \rho - \frac{dp}{dx} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho (v_t + v) \frac{\partial u}{\partial r}) \]  \quad (1)

\[ \frac{\partial (pu)}{\partial x} + \frac{\partial (pv)}{\partial r} = 0 \]  \quad (2)

\[ c_p u \frac{\partial T}{\partial x} + c_v \frac{\partial T}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} (r \rho \left( \frac{v_t}{Pr} + \frac{v}{Pr} \right) \frac{\partial T}{\partial r}) + \dot{m} \cdot Q_p \]  \quad (3)

\[ \rho u \frac{\partial Y_p}{\partial x} + \rho v \frac{\partial Y_p}{\partial r} = \dot{m} \]  \quad (4)

The one-parameter model of turbulence of Ni and Kovazhny (1968) [11] and Sekundov (1971) was written for the calculation of the kinematic turbulent viscosity \( \nu_t \):

\[ \rho u \frac{\partial \nu_t}{\partial x} + \rho v \frac{\partial \nu_t}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} (r \rho (\gamma V_t + v) \frac{\partial \nu_t}{\partial r}) + \alpha \rho \nu_t \left| \frac{\partial u}{\partial r} \right| - \nu_t (\beta V_t + v) \]  \quad (5)

The specific mass rate of the paraffin formation is found using the heat-balance equation: \( \dot{m} = \left[ \frac{c_p \mu \Delta T^*}{Q_p} \right] \), where the reduced heat flow is determined by the following relation:

\[ \Delta T^* = \begin{cases} 0, & \text{if } T > T_p \\ \left[ \frac{T(x + \Delta x) - T(x)}{\Delta x} \right], & \text{if } T(x + \Delta x) < T_p \end{cases} \]  \quad (6)

\( T \) is the current value of the liquid temperature, \( T_p \) is the temperature of the oil saturation with paraffin.

The energy equation for the metal of the pipe wall has the form:

\[ \frac{1}{r} \frac{\partial}{\partial r} (r \lambda_m \frac{\partial T}{\partial r}) = 0 \]  \quad (7)

The effective thermophysical parameters of the medium consisting of formation water and oil with associated gas dissolved in the oil are determined through volume fractions, as given in [9].

The initial conditions: \( u = u_0, \ v = 0, \ T_i = T_{i0}, \ T_m = T_{m0}, \ Y_p = 0 \).

The boundary conditions:

\[ x = 0: \]

\[ 0 < r < r_1, \quad T_i = T_{i1}, \rho u = \rho_0 u_0, \ Y_p = 0, \ p = p_{pl}; \]

\[ r_1 < r < r_2, \quad T_m = T_{pl}; \]

\[ x = L: \]

\[ 0 < r < r_1, \quad \frac{\partial T}{\partial x} = 0, \ \frac{\partial u}{\partial x} = 0, \ \frac{\partial v}{\partial x} = 0, \ \frac{\partial Y_p}{\partial x} = 0, \ \frac{\partial p}{\partial x} = 0; \]

\[ r_1 < r < r_2, \quad \frac{\partial T}{\partial x} = 0; \]

\[ 0 < x < L : \]
\[ r = 0 \quad \frac{\partial T}{\partial r} = 0, \quad \frac{\partial u}{\partial x} = 0, \quad \frac{\partial v}{\partial x} = 0, \quad \frac{\partial Y_p}{\partial x} = 0, \quad \frac{\partial p}{\partial x} = 0; \]
\[ r = r_1, \quad -\lambda_l \frac{\partial T_l}{\partial r} = -\lambda_m \frac{\partial T}{\partial r}, \quad T_l = T_m, \quad u = 0, \quad v = 0; \]
\[ r = r_2, \quad T_m = T_{pl}. \]

The system of equations (1) – (7) is solved by the finite difference method. To determine the velocity field and the pressure gradient, the algorithm of L. M. Simuni was used.

4 Results of the numerical calculations

For the calculations, let us assume the following: the inner radius of the pipe \( r_1 = 70 \) mm, the outer radius \( r_2 = 90 \) mm and the well depth \( L = 1500 \) m. The ambient temperature is determined by the natural geotherm. The pipe material is alloy steel. The thermophysical parameters of the fluid are calculated with regard to the changes of the temperature and pressure in the well. The formation pressure \( p_{pl} = 15 \) MPA, the formation temperature \( T_{pl} = 47 \) ºC. The water cut of the formation fluid is 10% over the volume. At the pipe entrance, the two-dimensional fluid-velocity profile is given. The properties of the fluid are determined according to [12-14]. The calculations have been made for the sections with the length from 1 to 10 m, 2 to 30 m and 3 to 50 m from the level of the producing reservoir. The initial fluid velocity is 1 m/s, the paraffin concentration in the oil \( C_p = 15\), wt %.

Figure 2 shows the fluid velocity profile in the well.

![Profile of the fluid flow rates in the well](image)

It can be seen that in the sections of 1 to 10 m in length, the progress in the turbulent velocity profile 1 takes place. At further movement of the medium to 30 m, the turbulent velocity profile becomes established, profile 2 and 3. The velocity near the inner surface of the pipe decreases, and near the axis of the pipe it increases on the average by 30% compared with the initial velocity.

Figure 3 presents the temperature distribution over the well radius.
The system of equations (1) – (7) is solved by the finite difference method. To determine the velocity field and the pressure gradient, the algorithm of L. M. Simuni was used.

### Results of the numerical calculations

For the calculations, let us assume the following: the inner radius of the pipe $r_1 = 702$ mm, the outer radius $r_2 = 902$ mm and the well depth $L = 1500$ m. The ambient temperature is determined by the natural geotherm. The pipe material is alloy steel. The thermophysical parameters of the fluid are calculated with regard to the changes of the temperature and pressure in the well. The formation pressure $p_{p} = 15$ MPa, the formation temperature $T_{f} = 47$ºC. The water cut of the formation fluid is 10% over the volume. At the pipe entrance, the two-dimensional fluid-velocity profile is given. The properties of the fluid are determined according to [12-14]. The calculations have been made for the sections with the length from 1 to 10 m, 2 to 30 m and 3 to 50 m from the level of the producing reservoir. The initial fluid velocity is 1 m/s, the paraffin concentration in the oil $C_{p}$, wt %.

**Figure 2.** Profile of the fluid flow rates in the well

It can be seen that as the fluid moves up in the vertical pipe the temperature decreases. In the section with the length to 10 m, the discrepancy in the temperature of the fluid and geothermal temperature is negligible. When the fluid is passing 30 m, its temperature drops almost by one degree. From the beginning of the fluid movement up to the level of 50 m, the difference in the fluid temperature and geothermal temperature is two degrees.

Figure 4 displays the distribution of the paraffin mass concentration in the fluid flow.

**Figure 4.** Distribution of the mass concentration of paraffin in the fluid flow

It can be seen that paraffin begins to form in the fluid flow near the inner surface of the well wall. The growth of the paraffin mass fraction takes place in the direction from the inner surface of the wall to the pipe axis and correlates with the change of the temperature field in the radial direction. In the section from 1 to 10 m in length the paraffin concentration is 0.3% by mass and in the section close to 30 m the paraffin mass concentration increases by a factor of 2.5; in the section close to 50 m the paraffin mass fraction near the wall is 1.4%.

### 5 Conclusion

In the present paper, a mathematical model is offered for the conjugate heat exchange during the turbulent movement of the fluid in the oil well. The model allows to parametrically analyze the intensity of the formation of paraffin deposits depending on the channel geometry and the thermophysical properties of the substances and to determine the distribution of the fluid flow velocity and temperature at the given parameters of the well operation.

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